# NUCLEAR STRUCTURE -- THEORY

# Limits on the Presence of Scalar and Induced-scalar Currents in Superallowed β-Decay

*W.E. Ormand[a], B.A.Brown, and B.R. Holstein[b]*

It is a long-standing hypothesis that the vector part of the weak current is a conserved quantity[1]. The principal test of this proposition lies in the study of $J^\pi = 0^+$, T = 1→ $0^+$, T = 1 nuclear β-transitions. Namely, their transition rates should be nucleus-independent and given by the relation

$$ft = \frac{K}{G_V^{2\prime}|M_F|^2} \quad (1)$$

where $t$ is the partial half-life, $f$ is the statistical rate function, $K = 8.1201\times10^{-7}$ is a product of fundamental constants, $G'_V$ is the effective vector coupling constant for nucleon β-decay [measured in units of $\hbar c)^3$], and $M_F$ is the Fermi matrix element, $M_F = (\psi_f|T_+|\psi_i)$.

Two classes of nucleus-dependent corrections, however, must be applied to Eq. (1). The first is radiative corrections[2] to the statistical rate function $f$, denoted by $\delta_R$, giving $f_R = f(1+\delta_R)$. The second is corrections to the nuclear matrix element due to the presence of isospin-nonconserving (INC) forces in nuclei[3], and is denoted by $\delta_C$; that is,

$$|M_F|^2 = |M_{F0}|^2(1-\delta_C),$$

where $M_{F0} = [T(T+1)-T_{z_i}T_{z_f}]^{\frac{1}{2}}\delta_{if}$. With $\delta_R$ and $\delta_C$, the "nucleus-independent" $ft$-value for $0^+, T = 1\rightarrow 0^+, T = 1$ transitions is

$$Ft = ft(1+\delta_R)(1-\delta_c) = \frac{K}{2(G'_V)^2} \quad (2)$$

Using these $Ft$-values, it is possible to test the conserved vector current (CVC) hypothesis. Further, by comparing $G'_V$ with the vector-coupling constant obtained from muon β-decay, the Kobayashi-Maskawa (KM) mixing angle between $u$ and $d$ quarks $v_{ud}$ can be determined[4,5]. Until recently, the $Ft$-values for the most accurately measured transitions $^{14}$O, $^{26m}$Al, $^{34}$Cl, $^{38m}$K, $^{42}$Sc, $^{46}$V, $^{50}$Mn, $^{54}$Co failed to yield constant values. Both nucleus-dependent corrections $\delta_R$[6] and $\delta_C$[7] have been reinvestigated, leading to $Ft$-values that are now constant to the level of ≈.1%, with $Ft_{avg} = 3077.3\pm1.5$ s, with $\chi^2/\nu = 0.87$.

There are, however, two other "exotic" processes that could, in principle, contribute to these transitions, and it is important to establish upper limits on their presence. These are

(i) an induced-scalar current that is present only if the vector current is not conserved, and
(ii) the possibility that in addition to the standard V-A components of the electro-weak interaction, there is also a scalar coupling.

The former is also forbidden in the Standard model by the absence of second-class currents, but could arise via a quark doubling mechanisim[8]. Likewise, a direct scalar coupling lies outside the conventional Weinberg-Salam-Glashow picture, but could arise from the exchange of a charged Higgs boson if two or more charged Higgs doublets are introduced[9].

The most general weak Hamiltonian that would contribute to $0^+, T = 1\rightarrow 0^+, T = 1$ transitions is then:[10]

$$H_\beta = \frac{iG_S}{\sqrt{2}}(\bar{u}_n u_p)(\bar{u}_\nu(1+\gamma_5)v_e) + \frac{iG'_V}{\sqrt{2}}\left(\bar{u}_n\left[f_V\gamma_\mu - f_S\left(\frac{\partial}{\partial x_\mu} - ieA_\mu\right)\right]u_p\right)(\bar{u}_\nu\gamma_\mu(1+\gamma_5)v_e)$$

where $G_S$ is the scalar coupling strength, $f_V$ and $f_S$ are the vector and induced-scalar form factors (note that for CVC, $f_V = 1$ and $G_S = f_S = 0$), and $A_\mu$ is the potential for the static electric field of the nuclear charge, and is included in order to preserve gauge invariance. Limits on $G_S$[11] and the ratio $f_S/f_V$[12] have been established previously. However, since these studies, the experimental *ft*-values have been measured with greater precision[13], and the corrections $\delta_R$ and $\delta_C$ have been re-evaluated, and in the case of $\delta_R$ a major error has been corrected[6].

Given these considerations, it is perhaps timely to re-establish the limits at which these "exotic" effects may be present. Further, in this note, we also investigate the absolute upper-limits that can be established from the *Ft*-values, given both experimental and theoretical uncertainties. The consequences pertaining to the Kobayashi-Maskawa mixing angles will also be discussed.

In general, the electro-weak Hamiltonian may include both the scalar and induced-scalar terms. However, operationally their effect in Fermi transitions is identical, because writing the induced scalar interaction in the form:

$$H_{IS} = -\frac{iG'_V}{\sqrt{2}}\bar{u}_n f_S(\partial_\mu - eA_\mu)u_p\bar{u}_\nu\gamma_\mu(1+\gamma_5)\psi_e$$

where $\psi_e$ is a Coulomb solution to the Dirac equation, we have

$$H_{IS} = \frac{iG'_V}{\sqrt{2}}m_e f_S\bar{u}_n u_p\bar{u}_\nu(1+\gamma_5)\psi_e,$$

which is of the same form as the direct scalar coupling, provided one makes the substitution $m_e f_S G'_V/\sqrt{2} \to G_S/\sqrt{2}$. The *Ft*-values are modified in the presence of a scalar current by[14]:

$$Ft = (Ft)_0(1 - 2b\gamma\langle W^{-1}\rangle) \tag{4}$$

with $(Ft)_0 = K/2G'^2_V$, $\gamma = \sqrt{1-\alpha^2Z^2}$, $b = G_S G'_V f_V/((G'_V f_V)^2 + G_S^2) \approx G_S/G'_V = m_e f_S/f_V$, and $\langle W^{-1}\rangle$ is the average of $W^{-1}$ over the allowed spectrum, i.e.

$$\langle W^{-1}\rangle = \frac{\int dW\, p(W-W_0)^2F(Z,W)}{\int dW\, pW(W-W_0)^2F(Z,W)},$$

where $W$ is the total β energy in units of $m_e c^2$, $W_0$ is the β end-point energy, $p$ the momentum (units of $m_e c$), and $F(Z,W)$ is the Fermi function.

Table I lists the experimental *ft*-values[13], the corrections $\delta_R$[6] and $\delta_C$[7], the corresponding *Ft*-values, the β end-point energies[13], and the factors $\gamma\langle W^{-1}\rangle$ for each of the eight superallowed transitions. Performing a least-squares fit to the *Ft*-values, we find $(Ft)_0 = 3078.4 \pm 4.3$ s and $b = (0.6\pm2.5)\times10^{-3}$, $\chi^2/\nu = 0.93$. This is in good agreement with the results obtained previously, $(Ft)_0 = 3077.3\pm1.5$ s, in which $G_S = 0.0$ was assumed[7]. Further, we note that the limit given here is only slightly tighter than that established by Hardy and Towner[11] of $b = (0.5\pm3.0)\times10^{-3}$, and is slightly weaker than that found by Szybiscz and Silbergleit[12] of $b = (0.8\pm2.1)\times10^{-3}$.

TABLE I: List of experimental *ft*-values, the corrections $\delta_R$ and $\delta_C$, the corresponding *Ft*-values, the beta end-point energies $E_0$, and the quantity $\gamma W^{-1}$ for the eight superallowed transitions.

| Nucleus | *ft* (s) | $\delta_R$ (%) | $\delta_C$ (%) | *Ft* (s) | $E_0$ (keV) | $\gamma\langle W^{-1}\rangle$ |
|---|---|---|---|---|---|---|
| $^{14}$O | 3038.1(23) | 1.53(1) | 0.19(9) | 3078.7(36) | 1808.44(27) | 0.438 |
| $^{26m}$Al | 3034.5(14) | 1.47(2) | 0.24(10) | 3071.7(34) | 3209.95(25) | 0.299 |
| $^{34}$Cl | 3052.0(29) | 1.45(3) | 0.48(10) | 3081.4(44) | 4470.27(18) | 0.232 |
| $^{38m}$K | 3045.1(26) | 1.44(3) | 0.49(14) | 3073.7(51) | 5020.49(56) | 0.211 |
| $^{42}$Sc | 3048.7(63) | 1.46(4) | 0.39(9) | 3081.1(71) | 5403.02(28) | 0.198 |
| $^{46}$V | 3043.7(22) | 1.46(4) | 0.21(10) | 3081.6(40) | 6028.62(69) | 0.180 |
| $^{50}$Mn | 3039.9(40) | 1.46(5) | 0.28(10) | 3075.6(53) | 6610.01(41) | 0.166 |
| $^{54}$Co | 3044.7(23) | 1.45(5) | 0.35(10) | 3077.1(42) | 7220.14(32) | 0.154 |

We note, however, that the experimental $Ft$ values used by Szybiscz and Silbergleit had considerably smaller uncertainties than those used here. This is most likely due to the use of a different data set and a somewhat smaller estimate of the uncertainty in the nuclear corrections $\delta_C$. In addition, the older Hardy and Towner survey used $Ft$-values from all known transitions, in particular $^{10}C$, although with experimental uncertainties considerably larger than those used here. In this light, however, we note that only a slight improvement on the limits of $b$ could be obtained if the $ft$-value for $^{10}C$ could be measured with the same accuracy as the eight transitions used here. For example, assuming $Ft = 3077\pm4$ and $\gamma\langle W^{-1}\rangle = 0.618$ ($E_0 = 0.8882$) keV, we would obtain $b = (0.3\pm1.5)\times10^{-3}$.

From these results we see that the superallowed β-decay data are consistent with $G_S = 0$ and $f_S = 0$. However, the fact that the limits on the "exotic" effects established by the new data are not significantly tighter than those established previously is somewhat surprising, especially given the fact that the uncertainty in the experimental $ft$-values has improved considerably, and that the $Ft$ values were obtained using the values of $\delta_R$ that contained a significant error. It is interesting to speculate at this point what are the smallest values of $G_S/G_V$ (and $f_S/f_V$) that could be extracted from the $Ft$-values. To do this, it is necessary to examine the sources of uncertainty in $Ft$.

Table II shows the contributions to the uncertainty in $Ft$ due to the experimental $ft$-values, the radiative corrections $\delta_R$, and the isospin-mixing corrections $\delta_C$ to the nuclear matrix elements. In many cases, it is seen that the contribution due to the theoretical uncertainty in $\delta_C$ is of the order or larger than the experimental error. In this regard, it is interesting to ask what limits on $G_S/G_V$ and $f_S/f_V$ could be established in the event that all $Ft$ values were constant with experimental uncertainties of zero.

TABLE II: Contributions to the uncertainty in $Ft$ due to the experimental $ft$-values and the corrections $\delta_R$ and $\delta_C$.

| Nucleus | $\delta(Ft)$ | | |
|---|---|---|---|
| | $ft$ | $\delta_R$ | $\delta_C$ |
| $^{14}O$ | 2.3 | 0.3 | 2.8 |
| $^{26m}Al$ | 1.4 | 0.6 | 3.1 |
| $^{34}Cl$ | 2.9 | 0.9 | 3.1 |
| $^{38m}K$ | 2.6 | 0.9 | 4.3 |
| $^{42}Sc$ | 6.3 | 1.2 | 2.8 |
| $^{46}V$ | 2.2 | 1.2 | 3.1 |
| $^{50}Mn$ | 4.0 | 1.5 | 3.1 |
| $^{54}Co$ | 2.3 | 1.5 | 3.1 |

In this case, we arrive at the limit $b = \pm1.9\times10^{-3}$, with the uncertainty $\delta(Ft)_0$ being $\pm3.2$ s. From this, we see that the values obtained here are very near the absolute limit that could be established from superallowed $Ft$-values given the uncertainty in the nuclear correction $\delta_C$.

The consequences of these results can be seen when values of the effective vector coupling constant $G'_V$ are extracted from $(Ft)_0$. The Kobayashi-Maskawa (KM) mixing matrix element $v_{ud}$ is given by

$$v_{ud} = \frac{G'_V}{G_\mu}(1+\Delta_\beta-\Delta_M)^{-1/2}$$

where $G_\mu/(\hbar c)^3 = 1.16637(2)\times10^{-5}$ GeV$^{-2}$ [15] is the vector coupling constant for muon β-decay, and $\Delta_\beta$ and $\Delta_\mu$ are the "inner" radiative corrections to both nucleon (β) and muon (μ) β-decay, with $\Delta_\beta-\Delta_\mu = (2.3\pm0.2)\times10^{-2}$ [13]. The only condition that is imposed upon the KM matrix is that it should be unitary, i.e $v^2 = v_{ud}^2+v_{us}^2+v_{ub}^2 = 1$. Deviations from unity would either indicate the possibility of another quark generation ($v^2 < 1$), or a defect in the standard form of the electro-weak interaction. Assuming $G_S = f_S = 0$, we obtain $v_{ud} = 0.9737\pm0.0010$[7]. Taking $v_{us} = 0.220\pm0.002$[17] and $v_{ub} < 0.0075$[18], we arrive at $v^2 = 0.9965\pm0.0021$. We note that at this point the uncertainty in $v^2$ is dominated by the uncertainty in $\Delta_\beta-\Delta_\mu$.

Applying the above analysis to the $Ft$-value obtained while including a scalar current, we find $v_{ud} = 0.9735\pm0.0012$ and $v^2 = 0.9961\pm0.0025$. We note that this result is not significantly different from that obtained assuming $G_S = f_S = 0$. In fact, the somewhat larger uncertainty in $(Ft)_0$ contributes an uncertainty of only 0.0012 to $v^2$, which is less than that brought about by the "inner" radiative corrections (~ 0.0019).

We conclude that it is unlikely that limits on a possible scalar coupling can be improved in this fashion even if significantly more precise *ft* data are available. There is one aspect here, however, that bears comment. If the induced scalar mechanism is relevant, or if the scalar coupling is due to Higgs exchange and is proportional to the lepton mass, then even though effects in beta decay are suppressed by $m_e/m_N E_e \sim 10^{-4}$, in a muon capture reaction, scalar effects arise at $O(m_\mu/m_N) \sim 10\%$ – a significant enhancement. Unfortunately, there is, in general, only one parameter in which to deduce the coupling strengths, i.e. the capture rate. By making the following reasonable assumptions:

(i) $q^2$ dependence of nuclear matrix elements from the impulse approximation and CVC; and

(ii) an induced pseudoscalar coupling from PCAC,

one can obtain from the capture rates measured for $^1$H and $^3$He a limit on a combination of induced-scalar ($f_S$) and induced-tensor ($f_T$) strengths[19]

$$m_e \left| f_S - \frac{f_A}{f_V} f_T \right| \leq 5\times10^{-4}$$

which is significantly stronger than that found in the $0^+ \to 0^+$ analysis. It should be remarked, honwever, that this result is strongly model-dependent.

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# Comparison of Exact and QPRA Calculations for Double-Beta Decay

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As experiments for the 2ν and 0ν modes of double beta (ββ) decay in nuclei become more precise, it becomes important to improve the theoretical calculations in order to be able to extract precise limits on the neutrino mass[1,2]. The nuclear structure in the mass regions of interest (A = 76, 82 and 128) is very complicated and the approximations which must always be made are more difficult than usual to test. The truncated shell-model calculations carried out[2] can be criticized for leaving out some of the important spin-orbit partner orbits. The QRPA has also been widely used[3,4], but can be criticized for leaving out some of the correlations present in these transitional nuclei.

A closely related topic is the suppression of the $\beta^+$ decay in heavy nuclei, which is still not understood very clearly, in spite of the fact that it has been studied for a long time. The total strength in the calculations is usually larger than experiment[5]. In previous studies, the QRPA has been frequently employed to discuss $\beta^+$ decay, because the total strength is strongly affected by the ground-state correlations. Recent QRPA calculations have shown that the strength was very sensitive to the particle-particle interaction term[6,7], which was first introduced by Cha[8]. But there may be other types of correlations that effect $\beta^+$ strength not contained in the QRPA.

In this work we test various aspects of the application of QRPA to $\beta^+$ and ββ decay by comparing the results to exact calculations where possible. By "exact" we mean a complete full-basis shell-model calculation within a major harmonic oscillator shell. Such calculations are possible for the 0d-1s shell. When applied to the A = 16-40 mass region with Wildenthal's effective interaction[9], the results have been extremely successful in reproducing a wide variety of experimental data, and in reproducing Gamow-Teller decay data in particular[10]. All of the comparisons here will made with Wildenthal's interaction. The ultimate goal of the present work is to provide a prescription for carrying out and estimating the errors in calculations for the heavier mass regions of interest.

We investigate several type of Gamow-Teller observables within a given series of neutron-rich nuclei with a given mass $A$ and variable proton numbers $Z$, $Z+1$ and $Z+2$. In particular, we look at the $\beta^+$ matrix element defined by:

$$M(\beta^+, i) = \langle A, Z+1, J=1_i^+ || \sigma t_+ || A, Z+2, J=0^+ \rangle \qquad (1)$$

and the ββ matrix element defined by:

$$M(\beta\beta, i) = \langle A, Z+2, J=0^+ || \sigma t_- || A, Z+1, J=1_i^+ \rangle \times \langle A, Z+1, J=1_i^+ || \sigma t_- || A, Z, J=0^+ \rangle \qquad (2)$$

We will display the results in terms of the running sums

$$S(\beta^+, Ex) = \sum |M(\beta^+, i)|^2 \qquad (3)$$

$$S(\beta\beta, Ex) = \sum M(\beta\beta, i) \qquad (4)$$

where the summation runs over all states $i$ between the ground state and excitation energy $Ex$. $S(\beta^+, Ex=\infty)$ is just the total $\beta^+$ strength, and $S(\beta\beta, Ex=\infty)$ is proportional to the 2ν ββ decay matrix element in the closure approximation. (One obtains the exact 2ν matrix element by dividing the left-hand side of Eq. (4) with an appropriate energy denominator. The 0ν matrix element has an additional radial factor which brings in other intermediate state multipoles into Eq. (2).)

We have initially looked at a wide variety of nuclei in the 0d-1s shell, those with $(A,Z+2)$ = $^{22}$Ne, $^{24}$Ne, $^{24}$Mg, $^{26}$Mg, $^{28}$Mg and $^{34}$S. Because of space, I will discuss here the results for only one of these, $^{28}$Mg, which is fairly typical of the others. We will compare results from three different calculations:

FB: The exact full-basis results (solid lines in the figure),

QRPA: The QRPA results obtained with the BSC $u$ and $v$ factors (dashed lines), and

EQRPA: "Extended" QRPA results obtained with $u$ and $v$ factors from the exact full-basis calculation (dotted lines).

The formalism used in the full-basis calculation is standard. The formalism in the QRPA approach is also standard, but certain parts will be repeated here for the purpose of discussion. The intermediate states in Eq. (2) in the QRPA are not the same, starting from the $Z$ and $Z+2$ nuclei, and we use the overlap method to project one onto the other[4]. The $A$ and $B$ amplitudes in the QRPA matrix are given by

$$A(J,pn,p'n') = (\epsilon_p+\epsilon_n)\delta_{pp'}\delta_{nn'} + G_{ph}W(J,pn,p'n')(u_pv_nu_{p'}v_{n'}+v_pu_nv_{p'}u_{n'}) + G_{pp}V(J,pn,p'n')(u_pu_nu_{p'}u_{n'}+v_pv_nv_{p'}v_{n'}) \quad (5)$$

$$B(J,pn,p'n') = G_{ph}W(J,pn,p'n')(v_pu_nu_{p'}v_{n'}+u_pv_nv_{p'}u_{n'}) + G_{pp}V(J,pn,p'n')(u_pu_nv_{p'}v_{n'}+v_pv_nu_{p'}u_{n'}) \quad (6)$$

where $p$ and $n$ label the orbit occupied by the proton or neutron, and the $\epsilon$ are the quasi-particle energies. For the single-particle energies, we use 0.0, 4.0 and 7.0 MeV for the $d_{5/2}$, $s_{1/2}$ and $d_{3/2}$ orbits, respectively. $V$ are Wildenthal's two-body matrix elements expressed in the proton-neutron basis and $W$ are the particle-hole Pandya transform of these. The multiplicative factors $G_{ph}$ and $G_{pp}$ are conventionally introduced in order to discuss the results as a function of the strength associated with each part of the interaction. The matrix elements discussed here are more sensitive to $G_{pp}$ than to $G_{ph}$, thus in the following we'll set $G_{ph}$ = 1 and discuss the dependence on $G_{pp}$.

It is common in these calculations to base the quasi-particle occupations in Eqs. (5) and (6) on a simple pairing interaction and to use the one-parameter equation:

$$(v_j)^2(BCS) = \left(\frac{1}{2}\right)\left\{1-\frac{\epsilon_j-\lambda}{[(\epsilon_j-\lambda)^2+\Delta^2]^{1/2}}\right\} \quad (7)$$

The pairing gap was chosen to reproduce the even-even and odd-even binding-energy systematics for nuclei in the region of interest[7], which resulted in a value of $\Delta$ = 1.6 MeV for $^{28}$Mg.

As an alternative to this, we also take the occupation probabilities in Eqs. (5) and (6) as calculated for the even-even nuclei in the exact full basis:

$$(v_j)^2(\text{exact}) = \langle \text{exact} | a_j^\dagger a_j | \text{exact} \rangle \quad (8)$$

The meaning of such a procedure is questionable, but we do it to see what differences there might be. The differences are large. For example, for $^{28}$Mg, $(v_p)^2$(BSC) = 0.640, 0.045 and 0.017 for $j$ = $d_{5/2}$, $s_{1/2}$ and $d_{3/2}$, respectively, while $(v_p)^2$(exact) = 0.562, 0.169 and 0.073.

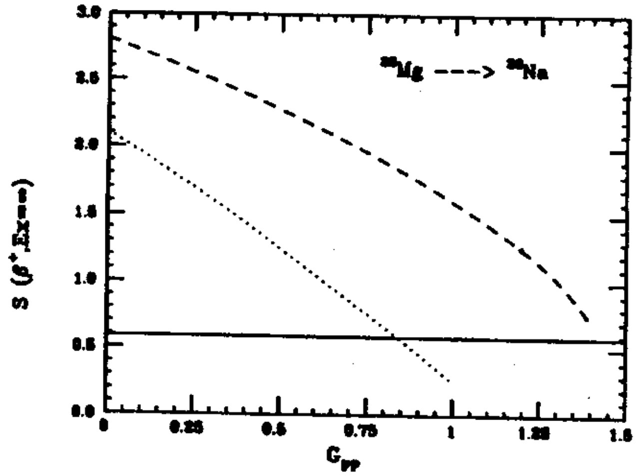


FIG. 1: The total $\beta^+$ strength $S(\beta^+, Ex=\infty)$ vs. $G_{pp}$ for $^{28}Mg \rightarrow {}^{28}Na$.

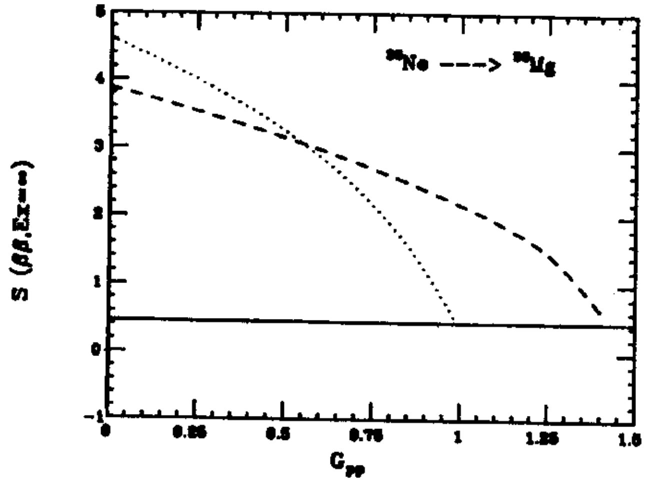


FIG. 2: $S(\beta\beta, Ex=\infty)$ vs. $G_{pp}$ for $^{28}Ne \rightarrow {}^{28}Mg$.

In Fig. 1 we show for the FB, QRPA and EQRPA calculations defined above, the total $\beta^+$ strength $S(\beta^+, Ex = \infty)$ *vs.* $G_{pp}$ for $^{28}Mg \rightarrow {}^{28}Na$, and in Fig. 2 we show $S(\beta\beta, Ex = \infty)$ *vs.* $G_{pp}$ for $^{28}Ne \rightarrow {}^{28}Mg$. We note in both cases that in $G_{pp}$ must be increased to about 1.4 in order to get agreement between QRPA and FB. However, EQRPA and FB are in agreement with $G_{pp} \approx 1$. This is an important general conclusion from our study. Also we point out that the QRPA and EQRPA calculations become unstable just above the point that they agree with the FB[7].

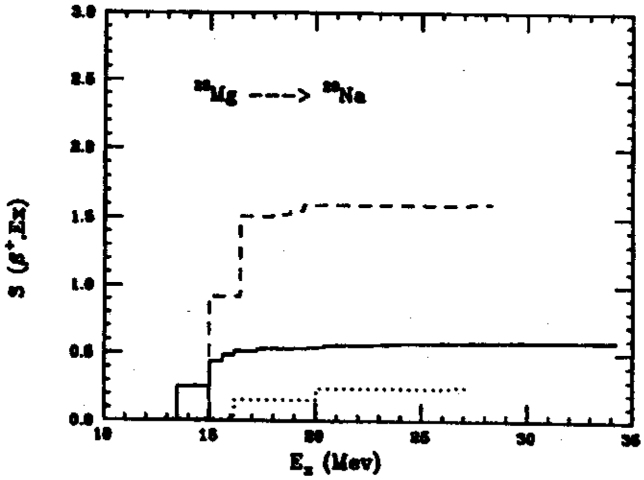


FIG. 3: $S(\beta^+, Ex)$ vs. Ex for $^{28}Mg \rightarrow {}^{28}Na$, with $G_{pp}$ = 1.

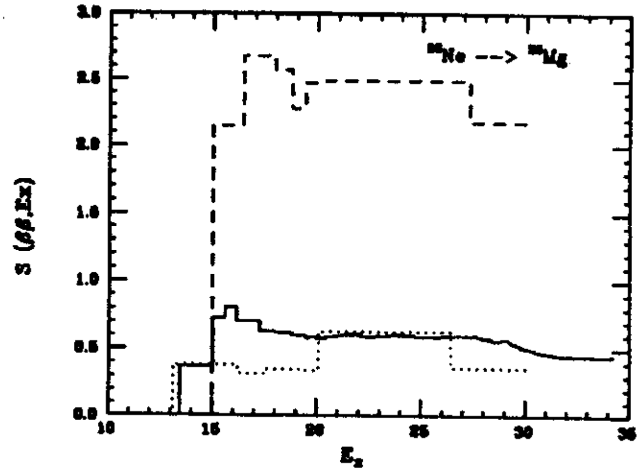


FIG. 4: $S(\beta\beta, Ex)$ vs. Ex for $^{28}Ne \rightarrow {}^{28}Mg$, with $G_{pp}$ = 1.

In Fig. 3, we show $S(\beta^+, Ex)$ *vs.* $Ex$ for $^{28}Mg \rightarrow {}^{28}Na$, and in Fig. 4 we show $S(\beta\beta, Ex)$ *vs.* $Ex$ for 28Ne→ 28Mg, both with $G_{pp}$ = 1. We note that there are only a few states in the QRPA and EQRPA calculations which tend to follow the trend of the FB calculations. In this sense the QRPA states act as "doorway" states which are mixed into the large background of $1^+$ in the FB calculation.

Our main conclusion is that the QRPA results for both $\beta^+$ and $\beta\beta$ are sensitive to the occupation probabilities and that these occupation probabilities are given rather poorly by the usual pairing-model assumption. Of course, for heavy nuclei it will be difficult or impossible to carry out FB calculations to get $v^2$. However, we plan to study the origin the difference between $v^2$(BSC) and $v^2$(FB), to understand the validity of using $\upsilon^2$(FB) in the QRPA equations, and to come up with a more reliable method for putting realistic interactions into the calculation of the $\upsilon^2$ factors for the transitional nuclei of interest.

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