A-VIII. Shadow Bar Beam Time

Let R_1 and R_2 be the data acquisition rates without and with shadow bars, respectively. Also let T_1 and T_2 be the length of time spent taking data without and with shadow bars. So $T = T_1 + T_2$ is the total time spent taking data. Assuming T, R_1, and R_2 are constants, an equation will be derived that indicates how T should be divided between T_1 and T_2 such as to minimize the fractional uncertainty of the data after subtraction of the shadow bar data.

Assume for a given channel that N_1 is the number of counts from the data taken without a shadow bar and N_2 is the number of counts taken with the shadow bar in place. Then the corrected number of counts is

$$N = N_1 - N_2(T_1/T_2)$$

The uncertainty in the corrected number of counts is given by

$$\delta N = \left\{(\delta N_1)^2 + (\delta N_2 T_1/T_2)^2\right\}^{\frac{1}{2}}$$

$$= \left\{R_1 T_1 + (T_1/T_2)^2 R_2 T_2\right\}^{\frac{1}{2}}$$

having substituted $(\delta N_i)^2 = (\sqrt{N_i})^2 = R_i T_i$ for $i = 1, 2$. Then the fractional uncertainty is

$$\frac{\delta N}{N} = \frac{\{R_1 T_1 + (T_1/T_2)^2 R_2 T_2\}^{\frac{1}{2}}}{R_1 T_1 - (T_1/T_2) R_2 T_2}$$

$$= \frac{1}{R_1 - R_2}\left\{\frac{R_1}{T_1} + \frac{R_2}{T - T_1}\right\}^{\frac{1}{2}}$$

having rearranged and substituted T - T_1 for T_2. To minimize $\delta N/N$ with respect to T_1, it is sufficient to minimize only that which is contained within the braces in the last relation, so

$$0 \underset{\text{set}}{=} \frac{\partial}{\partial T_1}\left\{\frac{R_1}{T_1} + \frac{R_2}{T - T_1}\right\}$$

$$= \frac{-R_1}{T_1^2} - \frac{R_2(-1)}{(T - T_1)^2}$$

Upon resubstituting T_2 for T - T_1, this solves to give

$$\frac{T_1}{T_2} = \frac{\sqrt{R_1}}{\sqrt{R_2}}$$

which is the desired relation.