Can we test Statistical Equilibrium?
Measured Isotopic yields

Isotopic effects are small: How to quantify them?

$$R_{21} = \frac{Y_2}{Y_1}$$

$$\propto e^{N\Delta\mu_n + Z\Delta\mu_p}$$

$$\propto (\hat{\rho}_n)^N (\hat{\rho}_p)^Z$$

Factorization of yields into p & n densities
Isoscaling from Relative Isotope Ratios

\[ R_{21} = \frac{Y_2}{Y_1} \]

\[ \propto e^{N\Delta\mu_n + Z\Delta\mu_p} \]

\[ \propto \left(\rho_n^N\right)\left(\rho_p^Z\right) \]

Factorization of yields into p & n densities

Cancellation of effects from sequential feedings

Robust observables to study isospin effects
Origin of isoscaling

- Isoscaling disappears when the symmetry energy is set to zero
- Provides an observable to study symmetry energy

\[ B = \alpha_v A - \alpha_s A^{2/3} + \delta - \alpha_c \frac{Z(Z-1)}{A^{1/3}} - \alpha_{sym} \frac{(A-2Z)^2}{A} \]
Predicted Isotopic yields

\[ Y(N,Z) = \frac{Y^{124+124}(N,Z)}{C \cdot \exp(\alpha N + \beta Z)} \]

Agreement between data and predictions are excellent.

• extend to other systems?
Conditions for Isoscaling

\[ M_{i}^{\text{pri}}(N, Z) \propto V \cdot (2J_i + 1) \cdot e^{(N\mu_n+Z\mu_p+B(N,Z)-E_i^*)/T} \]

Experimental Yields

\[ M_{g.s.}^{\text{sec}}(N, Z)_{\text{tot}} \propto V \cdot S_{g.s.} \cdot e^{N\mu_n+Z\mu_p} \cdot e^{B(N,Z)/T} \cdot f_{N,Z}(T) \]

Isotope Ratios

\[ \frac{Y_2(N, Z)}{Y_1(N, Z)} = \frac{f_2(T)}{f_1(T)} \cdot e^{(N\Delta\mu_n+Z\Delta\mu_p)/T} \]

Isoscaling

\[ \Rightarrow \text{thermal equilibrium} \]
\[ \Rightarrow \text{cancelation of sequential decay effects} \]
\[ \Rightarrow \text{same temperature for reactions 1 & 2} \]
Generalized Isoscaling

**Isotope Ratios with same T**

\[
\frac{Y_2(N, Z)}{Y_1(N, Z)} = \frac{f_2(T)}{f_1(T)} \cdot e^{(N\Delta\mu_n + Z\Delta\mu_p)/T}.
\]

**Isotope Ratios with different T**

\[
\frac{Y_2(N, Z)}{Y_1(N, Z)} = \frac{f_2(T_2)}{f_1(T_1)} \cdot e^{(\alpha'N + \beta'Z)\cdot k} \cdot e^{B(N, Z)\cdot k}.
\]

where \( k = \frac{1}{T_2} - \frac{1}{T_1} \)

**Correct for exp(-B(N,Z)k)**

\[
\frac{Y_2(N, Z)}{Y_1(N, Z)} e^{-B(N, Z)\cdot k} \propto e^{(\alpha'N + \beta'Z)}
\]
Kr+Nb: \( E/A = 35, 70, 100, 120 \text{ MeV} \)

Ar+Sc: \( E/A = 50, 100, 150 \text{ MeV} \)

MSU 4\(\pi\) -- central collisions

Catania hodoscope -- \( T_{\text{iso}} \)

\[
<k> = \left< \frac{1}{T_2} - \frac{1}{T_1} \right>
\]
General Isoscaling

Ar+Sc: E/A=50, 100, 150 MeV
Kr+Nb: $E/A = 35, 70, 100, 120$ MeV

**Generalized isoscaling**

\[
<k> = \left< \frac{1}{T_2} - \frac{1}{T_1} \right>
\]

and $k_{\text{best fit}}$ values are consistent
**Summary**

Existence of isoscaling relations in reactions with different isospin but same temperature suggests thermal equilibrium.

- For reactions with different temperature, isoscaling relations can be restored by correcting the temperature dependence.
- Need better IMF data to resolve temperature measurements involving light and IMF isotopes.
- Can be used as a test for equilibrium.
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