Isoscaling in Nuclear Reactions

Betty Tsang
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Outline

• What is isoscaling?
• Where is it observed?
  – From multifragmentation to binary reactions
• What is the origin of isoscaling?
• What can we learn from it?
  – Density dependence of symmetry energy.
  – Isospin diffusion.
Isoscaling constructed from Measured Isotopic yields

T.X. Liu et al. PRC 69, 014603

$dM/d\Omega_{cm}$ (sr$^{-1}$)

$124_{\text{Sn}} + 124_{\text{Sn}}$; E/A=50 MeV

$112_{\text{Sn}} + 112_{\text{Sn}}$

Li Be B C N O

Multifragmentation
Isoscaling from Relative Isotope Ratios

\[ R_{21} = \frac{Y_2}{Y_1} \]

\[ \propto e^{N\Delta \mu_n / T + Z\Delta \mu_p / T} \]

\[ \propto (\hat{\rho}_n)^N (\hat{\rho}_p)^Z \]

Factorization of yields into \( p \) & \( n \) densities

Cancellation of effects from sequential feedings

MB Tsang et al. PRC 64,054615
$^{112}\text{Sn} + ^{58}\text{Ni}$ and $^{124}\text{Sn} + ^{64}\text{Ni}$ at 35 AMeV; Central collisions, CHIMERA-REVERSE experiment

Simple derivation of the isoscaling law

• Basic trends from Grand Canonical ensemble:
  - Yields $\propto$ term with exponential dependence on the chemical potentials.

$$Y(N,Z)_{HOT} \propto \exp\left(\left[\mu_n N + \mu_p Z + B(N,Z)\right]/T\right) \cdot Z_{int}(N,Z)$$

where $Z_{int} = \sum_i (2J_i + 1) \exp(-E_i^* / T)$

$$Y(N,Z)_{COLD} = Y(N,Z)_{HOT} \ast f(N,Z)$$

• Ratios to reduce sensitivity to secondary decays:

$$R_{21}(N,Z) = \frac{Y_2(N,Z)}{Y_1(N,Z)} \approx C \cdot e^{N\Delta\mu_n / T + Z\Delta\mu_p / T}$$

$$= C \cdot e^{\alpha N + \beta Z} = C \cdot \hat{\rho}_n^N \cdot \hat{\rho}_p^Z ; \hat{\rho} = \rho_2 / \rho_1 , \rho \propto e^{\mu / T}$$

• Scaling parameters $C, \hat{\rho}_n, \hat{\rho}_p$ or $C, \alpha = \Delta\mu_n / T, \beta = \Delta\mu_p / T$
Temperature Dependence of $\alpha$
$p, \text{ He induced reactions on }^{112}\text{Sn and }^{124}\text{Sn}$

Botvina et al, PRC 65 044610

Reasonable agreements with isoscaling
Isoscaling observed in many reactions

\[ \frac{Y_2}{Y_1} \propto e^{(N\Delta \mu_n + Z\Delta \mu_p)/T} \]

More Data

- $^{124}$Sn+$^{124}$Sn
- $^{112}$Sn+$^{112}$Sn
- E/A=35 MeV

- $^{58}$Ni+$^{58}$Ni
- $^{58}$Fe+$^{58}$Fe
- E/A=30,40,47

- $^{4}$He+$^{116}$Sn
- $^{4}$He+$^{112}$Sn
- E/A=50 MeV
  - $\theta=160^\circ$

- p+$^{4}$He+$^{116}$Sn
- p+$^{4}$He+$^{124}$Sn
- E/A>1 GeV
  - Botvina, Trautmann (2002)

- $^{16}$O+$^{232}$Th
- $^{16}$O+$^{197}$Au
- E/A=8.6 MeV

- $^{86}$Kr+$^{116}$Sn, $^{124}$Sn
- $^{86}$Kr+$^{58}$Ni, $^{64}$Ni
- E/A=35 MeV

PRL, 86, 5023 (2001)
$R_{21} \propto \exp\left[\frac{(-\Delta S_n \cdot N - \Delta S_p \cdot Z)}{T}\right]$ 

**Q Value, Separation Energy**

$E_{\text{Coul}} \ E_{\text{sym}}$

**Evaporation**

$R_{21} \propto \exp\left[\frac{(-\Delta S_n + \Delta f_n^* \cdot N + (-\Delta S_p + \Delta f_p^* + \Delta \Phi) \cdot Z)}{T}\right]$ 

**Separation Energy**

$E_{\text{Coul}} \ E_{\text{sym}}$

**Multifragmentation**

$R_{21} \propto \exp\left[\frac{(-\Delta \mu_n \cdot N - \Delta \mu_p \cdot Z)}{T}\right]$ 

**Chemical Potentials**

$E_{\text{Coul}} \ E_{\text{sym}} \ \rho_p \ \rho_n$
Isoscaling in spallation process

M. Andronenko & L. Andronenko (PNPI, 2004)

Found isoscaling behaviors in spallation processes for $p+^{54,56}\text{Fe}$, $^{58,60,62,64}\text{Ni}$, $^{70,76}\text{Ge}$, Rh, Ag, Cs

Detailed examinations suggest that isoscaling is not strictly adhered to. However, there is an increase in alpha with differences in $N/Z$ changes in targets.
Fragmentation of $^{40}$Ca and $^{48}$Ca

Isoscaling not strictly observed especially for high Z
Slope changes with Z, higher T for small Z
Isoscaling in deeply inelastic reactions

$^{86}$Kr$+^{124}$Sn, $^{112}$Sn

$Z \sim 36$, short reaction time, little movement of p/n from targets.

$Z$ small, long reaction time, exchange of p/n from targets.

Evolution of N/Z equilibrium
Isoscaling in Fission

Fission: Binary process with mass, charge conservation.

Fragment $z$ dependence of $\alpha(z)$ – Friedman PRC 69, 031601 (2004)

Shell effects affect isoscaling fits

M. Veselsky et al. PRC 69, 044607

Friedman:

$$\alpha(z) = 8(C_{\text{sym}}/T)(A_h - A_l)$$

$$\times [2Z_U/(A_h + A_l)]^3/(Z_U - z)$$

$$\sim (C_{\text{sym}}/T)$$

$$\times \Delta A/<A_U>^3 \times 1/(1 - z/Z_U)$$

Use isoscaling for high $Z$ to study symmetry energy information
Can we study symmetry energy from fission isoscaling?

Friedman:
\[ \alpha(z) \propto (C_{\text{sym}}/T) \Delta A/\langle A \rangle^3 \]

Promising but need more study

Observation
Non-linear increase with \( \Delta A \)
Increase with \( \langle A \rangle \)
Isoscaling in Antisymmetrized Molecular Dynamical model
Symmetry energy from AMD

\[ \alpha(\text{Gogny}) > \alpha(\text{Gogny-AS}) \]
\[ C_{\text{sym}}(\text{Gogny}) > C_{\text{sym}}(\text{Gogny-AS}) \]

Multifragmentation occurs at low density

Experimental study of the density dependence of asymmetry energy multi-fragmentation study in central collisions

PRL 86,5023(2001); PRC, C64, 051901R (2001); nucl-ex/0406008

Bulk multifragmentation models such as (ISMM, AMD, SMF) predict that soft EOS favors more symmetric fragments.

Surface emission models such as EES predict just the opposite.
New observable: isospin diffusion in peripheral collisions

Vary isospin driving forces by changing the isospin of projectile and target.

symmetric system: no diffusion

asymmetric systems

strong diffusion

weak diffusion

Examine asymmetry by measuring the scaling parameter for projectile decay

proton rich system

neutron rich system
\[ R_i = \frac{2x - x_{124+124} - x_{112+112}}{x_{124+124} - x_{112+112}} \]

Data: \( x = \alpha \)

BUU: \( x = \delta = (N - Z) / (N + Z) \)

BUU predictions

\[ E(\rho, \delta) = E(\rho, 0) + S_{sym}(\rho) \delta^2 \]

\[ S_{sym}(\rho) \propto (\rho)^\gamma \]

\[
R_i = \frac{2x - x_{124+124} - x_{112+112}}{x_{124+124} - x_{112+112}}
\]

Data: \( x = \alpha \)

BUU: \( x = \delta = (N - Z)/(N + Z) \)

Isospin diffusion using mirror Nuclei

T. Liu, poster session
Summary

- A lot of work has been done on isoscaling.
  - Robust observable
  - Seen in many different reactions
  - Promising tool to study symmetry energy with heavy ion collisions.
Acknowledgements

Bill Friedman

Isoscaling of mixed systems

\[ R_i = \frac{2x - x_{124+124} - x_{112+112}}{x_{124+124} - x_{112+112}} \]
Experimental study of the density dependence of asymmetry energy multi-fragmentation study in central collisions

\[ S(\rho) = C_{\text{sym}}(\rho/\rho_o)^\gamma \]

**EES model:** \( \alpha=0.36 \quad \Rightarrow \quad \gamma=2/3; \quad PRL 86,5023(2001) \)

**BUU+SMM model:** \( \alpha=0.36 \quad \Rightarrow \quad \gamma=2 \) agrees with data better \quad \textit{PRC, C64, 051901R (2001).} \)

**AMD model:** Gogny-AS agree with data better – \quad \textit{nucl-ex/0406008} \)

Bulk multifragmentation models such as (ISMM, AMD, SMF) predict that soft EOS favors more symmetric fragments.

Surface emission models such as EES predict just the opposite.
\[
E(\rho, \delta) = E(\rho, 0) + S_{\text{sym}}(\rho) \delta^2
\]

\[
S_{\text{sym}}(\rho) \propto (\rho)^\gamma
\]

\[
R_i = \frac{2x - x_{124+124} - x_{112+112}}{x_{124+124} - x_{112+112}}
\]

Data: \( x = \alpha \)

BUU: \( x = \delta = (N-Z)/(N+Z) \)

Symmetry energy potentials

\[ E_{\text{sym}}(\rho) = e_{\text{sym}}^{\text{kin}} + e_{\text{sym}}^{\text{int}} \]

\[ e_{\text{sym}}^{\text{kin}} = 12.25 \text{MeV} \left( \frac{\rho}{\rho_0} \right)^{2/3} \]

\[ e_{\text{sym}}^{\text{int}} = \begin{cases} 
14 \text{MeV} \left( \frac{\rho}{\rho_0} \right)^2 \\
14 \text{MeV} \left( \frac{\rho}{\rho_0} \right) \\
14 \text{MeV} \left( \frac{\rho}{\rho_0} \right)^{1/3} \\
38.5(\rho/\rho_0) - 21(\rho/\rho_0)^2 
\end{cases} \]
b=6.6 fm

Micha Kilburn, REU 2003
Time Evolution of Isospin Diffusion

![Graph showing the time evolution of isospin diffusion with two reactions: $^{124}\text{Sn} + ^{112}\text{Sn}$ and $^{112}\text{Sn} + ^{124}\text{Sn}$, and the skm interaction.](image)
Isospin Diffusion from BUU


\[ \rho^2 \]

\[ t \text{ (fm/c)} \]

\[ R_i \]

\[ 124^{\text{Sn}} + 112^{\text{Sn}} \text{ skm} \]

\[ 112^{\text{Sn}} + 124^{\text{Sn}} \]

\[ 124^{\text{Sn}} + 112^{\text{Sn}} \]

\[ 112^{\text{Sn}} + 124^{\text{Sn}} \]
Origin of isoscaling

- Isoscaling disappears when the symmetry energy is set to zero.
- Provides an observable to study symmetry energy.

\[ B = a_v A - a_s A^{2/3} + \delta - a_c \frac{Z(Z-1)}{A^{1/3}} - a_{sym} \frac{(A-2Z)^2}{A} \]
Isoscaling in statistical models

Primary distributions show good isoscaling
A2=186, Z2=75; A1=168, Z1=75
Effects of sequential decay on isoscaling

A₂=186, Z₂=75; A₁=168, Z₁=75

WCI statistical model working group (2004)
Isoscaling in sequential decay models

$A_2=186, Z_2=75; A_1=168, Z_1=75$

WCI statistical model working group (2004)
Density dependence of asymmetry energy

\[ S(\rho) = 23.4 (\rho/\rho_0)^\gamma \]

Results are model dependent

Strong influence of symmetry term on isoscaling

\[ \alpha = 0.36 \Rightarrow \gamma = 2/3 \]

Consistent with many body calculations with nn interactions

PRL 86, 5023 (2001)
Sensitivity to the isospin terms in the EOS

Isotope Ratios

Asy-stiff
\( \gamma \sim 2 \)

Asy-soft
\( \gamma \sim 0.5 \)

Isotone Ratios

Freeze-out source
Data

Asy-stiff term agrees with data better

\( \gamma \), 051901R (2001).
Transport model predictions

Sensitivity to the isospin terms in the EOS

\[ E(\rho, \delta) = E(\rho, 0) + S_{\text{sym}}(\rho) \delta^2 \]

\[ S_{\text{sym}}(\rho) \propto (\rho)^\gamma; \quad \gamma = 2, 1, 0.5, \text{ skm } (a\rho - b\rho^2) \]

\[ R_i = \frac{2x - x_{124+124} - x_{112+112}}{x_{124+124} - x_{112+112}} \]

\[ x = \delta = \frac{(N - Z)}{(N + Z)} \]
Symmetry energy study using fission isoscaling?

15 MeV $n+U$ isotopes

Friedman: $\alpha(z) \sim (C_{\text{sym}}/T) \times (A_h-A_l) \times [2Z/(A_h+A_l)]^3$