Extracting Density dependence of Symmetry Energy in Heavy Ion Reactions

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Extracting Density dependence of Symmetry Energy in Heavy Ion Reactions

Micha Kilburn

$E_{sym}$ (MeV)

$\rho/\rho_0$

F1
F3
FSUGold
AV18
skm*
NL3

Sn124 on Sn124 t=00 fm/c

Micha Kilburn
Highest density reached by central collisions depends on incident beam energy.

Types of fragment formed depend on emission times and density.
Constraining the EOS at high densities by laboratory collisions

• Experiment: measure collective flow (emission patterns) of particles emitted in Au+Au collisions from \((E/A \sim 1-8 \text{ GeV})\).

• Transport model (BUU) relates the measurements to pressure and density.

Equation of State of Nuclear Matter

\[ E/A(\rho, \delta) = E/A(\rho,0) + \delta^2 \cdot S(\rho); \quad \delta = (\rho_n - \rho_p)/(\rho_n + \rho_p) = (N-Z)/A \]


• Transport model includes constraints in momentum dependence of the mean field and NN cross-sections
Equation of State of Nuclear Matter

\[ \frac{E}{A}(\rho, \delta) = \frac{E}{A}(\rho, 0) + \delta^2 S(\rho) ; \quad \delta = \frac{(\rho_n - \rho_p)}{(\rho_n + \rho_p)} = \frac{(N-Z)}{A} \]


Newer calculation and experiment are consistent with the constraints.
Experimental Techniques to probe the symmetry energy with heavy ion collisions at E/A<100 MeV

Isospin degree of freedom

\[ B = a_v A - a_s A^{2/3} + \delta - a_c \frac{Z(Z-1)}{A^{1/3}} - a_{sym} \frac{(A-2Z)^2}{A} \]

- Vary the N/Z compositions of projectile and targets:
  \(^{124}\text{Sn} + ^{124}\text{Sn}, \ ^{124}\text{Sn} + ^{112}\text{Sn}, \ ^{112}\text{Sn} + ^{124}\text{Sn}, \ ^{112}\text{Sn} + ^{112}\text{Sn}\)
- Measure N/Z compositions of emitted particles:
  - n & p
  - isotopes
  - \(\pi^+ \) & \(\pi^-\) at higher incident energy
Heavy Ion collision: $^{124}$Sn+$^{124}$Sn, $E/A=50$ MeV

Around incident energy: $E/A<100$ MeV:

Reaction mechanism depends on impact parameters

$n$ & $p$ are emitted throughout

Charged fragments ($Z=3-20$) are formed at subnormal density

$E_{\text{sym}} = 12.7(\rho/\rho_0)^{2/3} + 19(\rho/\rho_0)^{\gamma_i}$
Classes of models used to interpret experimental results

I. Transport models:
Describe dynamical evolution of the collision process
• Self consistent mean field
• n-n collisions,
• Pauli exclusion
Uncertainties
Semi-classical
Approximations needed to make computation feasible.

II. Statistical models:
Describe longer time scale decays from single source.
• nuclear mass,
• level densities,
• decay rates
Uncertainties
Source parameters: A₀, Z₀, E₀, V₀, J
Information obtained is for finite nuclei, not for infinite nuclear matter

Symmetry energy included in the nuclear EOS for infinite nuclear matter at various density.
Symmetry energy included in the form of fragment masses – finite nuclei & valid for ρ₀ only. EOS extrapolated using statistical model is questionable.

Theory must predict how reaction evolves from initial contact to final observables
Nuclear Collisions simulations with Transport Models – Nuclear EOS included from beginning of collisions

**BUU models:**

*Semiclassical solution of one-body distribution function.*

**Pros**

Derivable, approximations better understood.

**Cons**

Mean field → no fluctuations

BUU does not predict cluster formation

**QMD:**

*Molecular dynamics with Pauli blocking.*

**Pros**

Predicts cluster production

**Cons**

Cluster properties (masses, level densities) approximate

Need sequential decay codes to de-excite the hot fragments

Code used: ImQMD

At high incident energies, cluster production is weak → the two models yield the same results. Clusters are important in low energy collisions.
Experimental Observables: n/p yield ratios

- n and p potentials have opposite sign.
- n & p energy spectra depend on the symmetry energy \( \rightarrow \) softer density dependence emits more neutrons.

\[ E_{\text{sym}} = 12.7(\rho/\rho_0)^{2/3} + 19(\rho/\rho_0)\gamma_i \]

- More n’s are emitted from the n-rich system.

\[ F_1 = 2u^2/(1+u) \]
\[ F_2 = u \]
\[ F_3 = \sqrt{u} \]

\( u = \rho/\rho_0 \)

\( \delta = 0.3 \) stiff
\( \gamma_i = 0.5, 2 \)

ImQMD

\( ^{112}\text{Sn} + ^{112}\text{Sn} \)
\( ^{124}\text{Sn} + ^{124}\text{Sn} \)
n/p Double Ratios (central collisions)

Double Ratio \[ \frac{^{124}\text{Sn} + ^{124}\text{Sn}; Y(n)/Y(p)}}{^{112}\text{Sn} + ^{112}\text{Sn}; Y(n)/Y(p)} \]

minimize systematic errors

Data : Famiano et al. PRL 97 (2006) 052701
n/p Double Ratios (central collisions)

Double Ratio $\frac{^{124}\text{Sn}+^{124}\text{Sn}; Y(n)/Y(p)}{^{112}\text{Sn}+^{112}\text{Sn}; Y(n)/Y(p)}$ minimize systematic errors

Data suggest soft EOS
Will repeat experiment for better accuracy


$E_{\text{sym}} = 12.7\left(\frac{\rho}{\rho_0}\right)^{2/3} + 19\left(\frac{\rho}{\rho_0}\right)^{\gamma_i}$

$\gamma_i = 0.5, 2$
N/Z ratios from bound fragments (Z=3-8) → complementary to n/p ratios

\[ \sum \frac{N}{Z} = 12.7 \left( \frac{\rho}{\rho_o} \right)^{2/3} + 19 \left( \frac{\rho}{\rho_o} \right)^{\gamma_i} \]

Effects are small

Hot fragments produced in calculations.
Sequential decay effects are important

Data consistent with soft EOS

N/Z ratios from bound fragments (Z=3-8) → complementary to n/p ratios

\[ E_{\text{sym}} = 12.7 \left( \frac{\rho}{\rho_0} \right)^{2/3} + 19 \left( \frac{\rho}{\rho_0} \right)^{\gamma_i} \]
Isospin Diffusion

- Probe the symmetry energy at subsaturation densities in peripheral collisions, e.g. \(^{124}\text{Sn} + ^{112}\text{Sn}\)
- Isospin “diffuse” through low-density neck region
- Symmetry energy drives system towards equilibrium.
  - stiff EOS \(\rightarrow\) small diffusion
  - soft EOS \(\rightarrow\) fast equilibrium

- Require measurements of collisions of nuclei with no isospin asymmetries for scaling due to existence of non-isospin diffusion effects:
  - Pre-equilibrium emissions
  - Sequential decays
  - Coulomb effects
Isospin Diffusion

No diffusion

Complete mixing

from isoscaling

from $Y(^7\text{Li})/Y(^7\text{Be})$

Sn+Sn data

Target $^{112}\text{Sn}$

Projectile $^{124}\text{Sn}$

Degree of Asymmetry

$b (\text{fm})$

$y/y_{\text{beam}}$
Isospin Diffusion

Data consistent with soft EOS!
Consistent analysis of four observables with ImQMD model

Central collisions

Peripheral collisions

\[ E_{\text{sym}} = 12.7 \left( \frac{\rho}{\rho_o} \right)^{2/3} + 19 \left( \frac{\rho}{\rho_o} \right)^{0.5} \]
Consistent analysis of four observables with ImQMD model

\[ E_{\text{sym}} = 12.7 \left( \frac{\rho}{\rho_0} \right)^{2/3} + 19 \left( \frac{\rho}{\rho_0} \right)^{-0.5} \]
Consistent interpretation of present measurements can be obtained using the IQMD model.

$$E_{\text{sym}} = 12.7 \left( \frac{\rho}{\rho_o} \right)^{2/3} + 19 \left( \frac{\rho}{\rho_o} \right)^{-0.5}$$

Concerns: Current result is model dependent and additional data sets are needed.

OUTLOOK

• Detailed model studies to understand differences between the predictions of different transport theories.

• More precise and complete data to probe sub-saturation densities and constrain transport parameters of the transport theories.

• Extend study to higher densities (high energy collisions) -- Unique capability of heavy ion collisions – details in discussions session.
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Laboratory experiments to study properties of neutron stars

<table>
<thead>
<tr>
<th></th>
<th>n-star</th>
<th>HI collisions</th>
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</thead>
<tbody>
<tr>
<td>$\rho/\rho_0$</td>
<td>$\sim 0.1 - 10$</td>
<td>$\sim 0.1-5$</td>
</tr>
<tr>
<td>$y_e$</td>
<td>$\sim 0.1$</td>
<td>$\sim 0.38-0.5$</td>
</tr>
<tr>
<td>$T$(MeV)</td>
<td>$\sim 1$</td>
<td>$\sim 4-50$</td>
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</tbody>
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Extrapolate information from limited asymmetry and temperature to neutron stars!
Laboratory experiments to study properties of neutron stars

extrapolation from $^{208}\text{Pb}$ radius to n-star radius

\[ L = \]


**Data:**
- Collisions of Sn+Sn isotopes at E/A=50 MeV
  - \( b/b_{\text{max}} > 0.8 \)
  - \( \langle b \rangle \sim 7.2 \text{ fm} \) → from multiplicity gates.
  - \( y/y_b > 0.7 \)
- Results obtained with clusters

**IBUU04:**
- Collisions of Sn+Sn isotopes at E/A=50 MeV
  - \( b = 6 \text{ fm} \)
  - Projectile-like residue determined from density &
  - \( y/y_b > 0.5 \)
  - No clusters
$P(MeV/fm^{-3})$ vs $\rho/\rho_0$ for EOS_DD.

- RMF:NL3
- Akmal
- Fermi gas
- Flow Experiment
- Kaon Experiment
- FSU Au
- GMR Experiment

The graph shows the pressure ($P$) as a function of density ($\rho/\rho_0$) for different models and experiments.
Consistent analysis of four observables with ImQMD model

\[ E_{\text{sym}} = 12.7 \left( \frac{\rho}{\rho_o} \right)^{2/3} + 19 \left( \frac{\rho}{\rho_o} \right)^{-0.5} \]
\[ E_{\text{sym}} = 12.7 \left( \frac{\rho}{\rho_o} \right)^{2/3} + 19 \left( \frac{\rho}{\rho_o} \right) \gamma_i \]
ImQMD, $b=6\text{fm}$

- $X=\delta(v_{n,\text{frag}} > 0.7v_{\text{beam}})$
- $X=\ln(n/p)$
- $X=\delta, \; v_i > 0.7v_{\text{beam}}$
- $X=\delta, \; Z_{\text{max}} > 20$

$v > 0.7v_{\text{c.m.}}^{\text{beam}}$
Probes of the symmetry energy

\[ \frac{E}{A}(\rho, \delta) = \frac{E}{A}(\rho, 0) + \delta^2 \cdot S(\rho) ; \quad \delta = \frac{(\rho_n - \rho_p)}{(\rho_n + \rho_p)} = \frac{(N-Z)}{A} \]

Li et al., PRL 78 (1997) 1644

\[
\begin{align*}
F_1 &= 2u^2/(1+u) \\
F_2 &= u \\
F_3 &= \sqrt{u} \\
F_4 &= \text{stiff} \\
F_5 &= \text{soft}
\end{align*}
\]

\( u = \frac{\rho}{\rho_o} \)
Data consistent with soft EOS!
Experimental Observables: n/p yield ratios

• Low densities ($\rho < \rho_0$):
  – Isoscaling with statistical models
  – Isospin diffusion
  – n/p spectra and flows; $R(\text{n/p}), R(\text{t/3He})$
  – Fragment isotopic distributions, $R(\text{N/Z})$
  – Correlation function, $C(q)$
  – Neutron, proton radii, E1 collective modes.

• High densities ($\rho \approx 2\rho_0$)
  – Neutron/proton spectra and flows; $C(q)$
  – $\pi^+$ vs. $\pi^-$ production, $k$, hyperon production.
Experimental Observables to probe the symmetry energy

\[ E/A(\rho, \delta) = E/A(\rho,0) + \delta^2 S(\rho) \; ; \; \delta = (\rho_n - \rho_p)/(\rho_n + \rho_p) = (N-Z)/A \]

- Collision systems: \(^{124}\text{Sn}+^{124}\text{Sn}, \; ^{124}\text{Sn}+^{112}\text{Sn}, \; ^{112}\text{Sn}+^{124}\text{Sn}, \; ^{112}\text{Sn}+^{112}\text{Sn}\)
- \(E/A=50 \text{ MeV}\)
- Low densities (\(\rho<\rho_0\)):
  - n/p spectra and flows; \(Y(n)/Y(p), \; Y(t)/Y(^3\text{He})\),
  - Fragment isotopic distributions,
    - Isoscaling: interpretation with statistical model is incorrect
    - \(<N>/<Z>\) of \(Z=3-8\) fragments
    - Isospin diffusion
  - Correlation function, \(C(q)\)
  - Neutron, proton radii, E1 collective modes.
- High densities (\(\rho \approx 2\rho_0\))
  - Neutron/proton spectra and flows; \(C(q)\)
  - \(\pi^+\) vs. \(\pi^-\) production, \(k\), hyperon production.
Exploring Bulk properties of Nuclear Matter with Heavy Ion Collisions

Low density/energy
- fragments, ratios
- isospin diffusion
- isoscaling
- migration/fractionation
- collective excitations
- surface phenomena
- phase transitions

High density/energy
- differential flow
- n/p, LIF ratios
- pions ratios
- kaon ratios
- neutron stars

Tsang, HW
BA Li, HW
Tsang
Di Toro
Aumann, Ducoin
Danielewicz
Lehaut

QF Li, Di Toro
Di Toro, Lukasic
DiToro, Reisdorf, QF Li
Prassa, QF Li
BA Li, Kubis

Hermann Wolter
Cluster effects

- Data
- Coalescence invariant

DR(n/p) vs. E_{c.m.}(MeV)

Zhang et al. PLB 664 (2008) 145
Effects of impact parameters on diffusions

IBUU04, \( E/A = 50\text{AMeV} \)

- \( x = -1 \)
- \( x = 0 \)

- MSU data

- Pwl’s method
- BaL’s method

\( b \) (fm)
Comparison of n/p double ratios

$\gamma_i = 0.5$

$\gamma_i = 2$

ImQMD; $b = 2$ fm

Data

Coalescence invariant

Data

$E_{C.M.}$ (MeV)
Constraints from Isospin Diffusion Data

Data:

- Collisions of \(^{124}\text{Sn}+^{112}\text{Sn}\) isotopes at \(E/A=50\) MeV
- \(b/b_{\text{max}} > 0.8\)
- \(<b> \sim 7.2\) fm
- From multiplicity gates.
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IBUU04:

- Collisions of \(^{124}\text{Sn}+^{112}\text{Sn}\) isotopes at \(E/A=50\) MeV
- \(b=6\) fm
- Projectile-like residue determined from density &
- \(y/y_b > 0.5\)
- No clusters
Isospin diffusion in the projectile-like region

**Basic ideas:**

- *Peripheral reactions*
- *Asymmetric collisions* $^{124}\text{Sn} + ^{112}\text{Sn}, ^{112}\text{Sn} + ^{124}\text{Sn} \rightarrow$ diffusion
- *Symmetric Collisions* $^{124}\text{Sn} + ^{124}\text{Sn}, ^{112}\text{Sn} + ^{112}\text{Sn} \rightarrow$ no diffusion
- *Relative change between target and projectile is the diffusion effect*
Characteristics of Heavy Ion collision: $^{124}$Sn+$^{124}$Sn

Highest density reached by central collisions depends on incident energy.

E/A=800 MeV
200 MeV
50 MeV

Types of fragment formed depends on emission times.

Pions, n, p fragments
Density region sampled depends on collision observable & beam energy

- $\rho > \rho_0$ examples:
  - Pion energy spectra
  - Pion production ratios
  - Isotopic spectra
  - Isotopic flow

- With NSCL beams, densities up to $1.7\rho_0$ are accessible
  - Beams: 50-150 MeV, 50,000 pps
  - 106Sn-126Sn, 37Ca-49Ca
Density region sampled depends on collision observable & beam energy

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  106Sn-126Sn, 37Ca-49Ca
Isospin Dependence of the Nuclear Equation of State

\[ E/A (\rho, \delta) = E/A (\rho, 0) + \delta^2 \cdot S(\rho) \]

\[ \delta = (\rho_n - \rho_p) / (\rho_n + \rho_p) = (N - P) \left( \frac{\partial E/A}{\partial \rho} \right)_{s/a} \]

- The density dependence of symmetry energy is largely unconstrained.
- Pressure, i.e., EOS is rather uncertain even at \( \rho_0 \).
ImQMD, \( b = 6 \text{fm} \)

- \( X = \delta(\nu_{\text{frag}} > 0.7\nu_{\text{beam}}) \)
- \( X = \ln(n/p) \)
- \( X = \delta, \nu_i > 0.7\nu_{\text{beam}} \)
- \( X = \delta, Z_{\text{max}} > 20 \)

\( \nu > 0.7\nu_{\text{beam}} \)
\[ \frac{b}{b_{\text{max}}} = 0.8-1.0 \]
\[ \frac{b}{b_{\text{max}}} = 0.4-0.6 \]
\[ \frac{b}{b_{\text{max}}} = 0-0.2 \]
• $\rho<\rho_0$ – created in multifragmentation process

Observables:
– n/p ratios
– Isotopic spectra
– Beams: 50 MeV, 112Sn-124Sn
Transport description of heavy ion collisions:

non-relativistic: BUU

\[
\frac{\partial f}{\partial t} + \frac{\vec{p}}{m} \nabla f - \nabla U \cdot p f = I_{coll}[f, \sigma] = \frac{1}{(2\pi)^{3/2}} \iiint dp_2 dp_3 dp_4 \frac{d\sigma}{d\Omega_{12\rightarrow 4}} \delta(p_1 + p_2 - p_3 - p_4) \\
[1 - f_1(1 - f_2)] f_3 f_4 - f f_2(1 - f_3)(1 - f_4)
\]

Vlasov eq.; mean field

Relativistic BUU

\[
[p^* \mu \partial^x + (p^*_\nu F^\mu\nu + m^* \partial^x m^*) \partial^{p*}_\mu] f(x, p^*) = I_{coll}(f, \sigma)
\]

Simulation with Test Particles:

EOS

\[m^* = m - \Sigma_v\]

loss term

gain term

2-body hard collisions

Field tensor

\[F^{\mu\nu} = \partial^\mu \Sigma^\nu - \partial^\nu \Sigma^\mu\]

effective mass

Kinetic momentum

\[p^*_\mu = p_\mu - \Sigma_\mu\]

mean p_{x(y_0)} [MeV]

Transport model comparison (tw2003 homework)

Au(100 AMeV) + Au soft eos b=3fm

same ini, \sigma=40mb