Determining the symmetry energy of asymmetric nuclear matter

What have we learned about symmetry energy at low density?
What are the opportunities offered at high density?

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Symmetry Energy in Nuclei

\[ B = a_v A - a_s A^{2/3} + \delta - a_c \frac{Z(Z-1)}{A^{1/3}} - a_{\text{sym}} \frac{(A-2Z)^2}{A} \]

\[ (a_v^{\text{sym}} A - a_s^{\text{sym}} A^{2/3}) \frac{(A-2Z)^2}{A^2} \]

Inclusion of surface terms in symmetry
\[ B = a_v A - a_s A^{2/3} + \delta - a_c \frac{Z(Z-1)}{A^{1/3}} - C_{\text{sym}} \frac{(A - 2Z)^2}{A} \]

\[ C_{\text{sym}} \text{ is adjusted to reproduce experimental } \alpha \]

Reduction of \( \alpha \) values can be accomplished with more accurate mass formula obtained from fitting empirical masses rather than to change \( C_{\text{sym}} \) values!
How to obtain the information about EoS?

Laboratory observables constrain the EoS, $\varepsilon(\rho,T,\delta)$ indirectly.

Experiments:

**Accelerator:** Projectile, target, energy

**Detectors:** Information of emitted particles – identity, spatial info, energy, yields → construct observables

Models

**Input:** Projectile, target, energy.

**Simulate the collisions with the appropriate physics**

**Success depends on the comparisons of observables.**

What are the experimental uncertainties of the constraints?

What are the model uncertainties of the constraints?
I. Transport models:
Describe dynamical evolution of the collision process
• Self consistent mean field
• $n-n$ collisions,
• Pauli exclusion

Uncertainties
Semi-classical
Approximations needed to make computation feasible.

Symmetry energy is included in the nuclear EOS for infinite nuclear matter at various density.

II. Statistical models:
Describe longer time scale decays from single source.
• nuclear mass,
• level densities,
• decay rates

Uncertainties
Source parameters: $A_o$, $Z_o$, $E_o$, $V_o$, $J$
Information obtained is for finite nuclei, not for infinite nuclear matter

Symmetry energy is included in the form of fragment masses – finite nuclei & valid around $\rho_o$ only.

Theory must predict how reaction evolves from initial contact to final observables
Strategies used to study the symmetry energy with Heavy Ion collisions

- Vary the N/Z compositions of projectile and targets: $^{124}\text{Sn} + ^{124}\text{Sn}$, $^{124}\text{Sn} + ^{112}\text{Sn}$, $^{112}\text{Sn} + ^{124}\text{Sn}$, $^{112}\text{Sn} + ^{112}\text{Sn}$

- Measure N/Z compositions of emitted particles:
  - n & p yields
  - isotope yields – isospin diffusion
  - $\pi^+$ & $\pi^-$ at high incident energy

Isospin degree of freedom

$B = a_v A - a_s A^{2/3} + \delta - a_c \frac{Z(Z-1)}{A^{1/3}} - a_{sym} \frac{(A - 2Z)^2}{A}$

Neutron Number

Proton Number

Crab Pulsar

Hubble ST
Experimental Observables: n/p yield ratios

- $n$ and $p$ potentials have opposite sign.
- $n$ & $p$ energy spectra depend on the symmetry energy $\rightarrow$ softer density dependence emits more neutrons at low density.

$$S(\rho) = 12.5(\rho/\rho_o)^{2/3} + 17.6(\rho/\rho_o)^{\gamma_i}$$

- More $n$'s are emitted from the $n$-rich system and softer iso-EOS.
n/p Experiment $^{124}\text{Sn} + ^{124}\text{Sn}; ^{112}\text{Sn} + ^{112}\text{Sn}$; $E/A = 50$ MeV

Famiano et al
n/p Double Ratios (central collisions)

Double Ratio $\frac{^{124}\text{Sn}+^{124}\text{Sn}; Y(n)/Y(p)}{^{112}\text{Sn}+^{112}\text{Sn}; Y(n)/Y(p)}$ 

minimize systematic errors

Data: Famiano et al. PRL 97 (2006) 052701

Will repeat experiment for better accuracy
n/p Experiment $^{124}\text{Sn}+^{124}\text{Sn}; ^{112}\text{Sn}+^{112}\text{Sn}; \ E/A=50 \ \text{MeV}$

Famiano et al

Mike Youngs & Dan Coupland thesis expts

$^{124}\text{Sn}+^{124}\text{Sn}; ^{112}\text{Sn}+^{112}\text{Sn}; \ E/A=120 \ \text{MeV}$

$^{48,40}\text{Ca}+^{112,124}\text{Sn}; ^{48,40}\text{Ca}+^{112,124}\text{Sn}$
Analysis of n/p ratios with ImQMD model

\[ E_{\text{sym}} = 12.5 \left( \frac{\rho}{\rho_o} \right)^{2/3} + 17.6 \left( \frac{\rho}{\rho_o} \right)^{\gamma_i} \]
Analysis of n/p ratios with ImQMD model

\[ E_{\text{sym}} = 12.5 \left( \frac{\rho}{\rho_0} \right)^{2/3} + 17.6 \left( \frac{\rho}{\rho_0} \right)^{\gamma_i} \]

Data need better measurements but the trends and magnitudes still give meaningful \( \chi^2 \) analysis at 2\( \sigma \) level
Experimental Observable: Isospin Diffusion -- Isospin Transport Ratio

Isospin diffusion occurs only in asymmetric systems A+B

No isospin diffusion between symmetric systems

Non-isospin diffusion effects → same for A in A+B & A+A; same for B in B+A & B+B

Non-isospin transport effects are “cancelled”??

\[ R_i = 2 \frac{x_{AB} - (x_{AA} + x_{BB})}{x_{AA} - x_{BB}} / 2 \]
Isotope Distribution Experiment

MSU, IUCF, WU collaboration

Sn+Sn collisions involving $^{124}$Sn, $^{112}$Sn at E/A=50 MeV

Miniball + Miniwall

4 $\pi$ multiplicity array
Z identification, A<4

LASSA
Si strip +CsI array
Good E, position, isotope resolutions

Xu et al, PRL, 85, 716 (2000)
Isotope distributions and isospin diffusions

The main effect of changing the asymmetry of the projectile spectator remnant is to shift the isotopic distributions of the products of its decay

This can be described by the isoscaling parameters $\alpha$ and $\beta$:

$$\frac{Y_2(N,Z)}{Y_1(N,Z)} = C \exp(\alpha N + \beta Z)$$

$$R_i = 2 \frac{x_{AB} - (x_{AA} + x_{BB})}{2}$$

$$x_{AA} - x_{BB}$$
Analysis of isospin diffusion data with ImQMD model

\[ R_i = 2 \frac{x_{AB} - (x_{AA} + x_{BB})/2}{x_{AA} - x_{BB}} \]

\[ S(\rho) = 12.5(\rho/\rho_0)^{2/3} + 17.6(\rho/\rho_0) \]

\[ x(\text{data}) = \alpha \]
\[ x(\text{QMD}) = \delta \]

Equilibrium \[ R_i = 0 \]

\[ b \approx 5.8 - 7.2 \text{ fm} \]

\[ b = 7 \text{ fm} \]

\[ b = 6 \text{ fm} \]

\[ 0.4 \leq \gamma_i \leq 1 \]
Isospin transport observable

$Y(\text{^7Li})$ enhanced from $^{124}\text{Sn}$

$Y(\text{^7Be})$ enhanced from $^{112}\text{Sn}$
Isospin transport observable

\[ \frac{Y(\text{Li})}{Y(\text{Be})} \]

Mainly dominated by Coulomb

\[ V_{\parallel}(\text{au}) \]

\[ 1^{124}\text{Sn} + 1^{124}\text{Sn} \]

\[ 7\text{Li} \]

\[ 7\text{Be} \]

\[ 1^{112}\text{Sn} \]

\[ 1^{124}\text{Sn} \]

\[ R_i = \frac{2x_{AB} - x_{AA} - x_{BB}}{x_{AA} - x_{BB}} \]

\[ Y(\text{Li}) \text{ enhanced from } 1^{124}\text{Sn} \]

\[ Y(\text{Be}) \text{ enhanced from } 1^{112}\text{Sn} \]

Ratio \(Y(\text{Li})/Y(\text{Be})\)
Coulomb & other (preequilibrium & sequential) effects are “cancelled”

\[ R_i = \frac{2x_{AB} - x_{AA} - x_{BB}}{x_{AA} - x_{BB}} \]

Rami et al., PRL, 84, 1120 (2000)
Isospin Transport Ratio

\[ R_i = \frac{2x_{AB} - x_{AA} - x_{BB}}{x_{AA} - x_{BB}} \]

Rami et al., PRL, 84, 1120 (2000)
Analysis of rapidity dependence of $R_i$ with ImQMD model

$$R_i = 2 \frac{x_{AB} - (x_{AA} + x_{BB})/2}{x_{AA} - x_{BB}}$$

$$S = 12.5 \left( \frac{\rho}{\rho_o} \right)^{2/3} + 17.6 \left( \frac{\rho}{\rho_o} \right)^{\gamma_i}$$

$x$(data) = $\ln(Y(^7\text{Li}/^7\text{Be}))$

$x$(QMD) = $\delta$

Equilibrium $R_i = 0$

New analysis on rapidity dependence of isospin diffusion ratios – not possible with BUU type of simulations due to lack of fragments.
Consistent constraints from the $\chi^2$ analysis of three observables

$$S(\rho) = 12.5 \left( \frac{\rho}{\rho_o} \right)^{2/3} + 17.6 \left( \frac{\rho}{\rho_o} \right)^{\gamma_i}$$

$0.4 \leq \gamma_i \leq 1.05$
Earlier Constraints from Isospin Diffusion Data

IBUU04: $S(\rho) \sim 31.6(\rho/\rho_o)^{\gamma}$

$0.69 \leq \gamma \leq 1.05$

How to connect different representations of the symmetry energy

QMD: $S(\rho) = 12.5(\rho/\rho_o)^{2/3} + 19.6 (\rho/\rho_o)^{\gamma_i}$
How to connect different symmetry energy representations

Expansion around $\rho_0$:
\[ E_{sym} = a_4 + \frac{L}{3} \left( \frac{\rho_B - \rho_0}{\rho_0} \right) + \frac{K_{sym}}{18} \left( \frac{\rho_B - \rho_0}{\rho_0} \right)^2 \]

\[ L = 3\rho_0 \frac{\partial E_{sym}}{\partial \rho_B} \bigg|_{\rho_B=\rho_0} = 3 \rho_0 \frac{P_{sym}}{\rho_0} \]

Value of symmetry energy at saturation
\[ E_{sym}(\rho_0) = a_4 \]
Expansion around $\rho_0$:
\[ S = S_0 + \frac{L}{3} \left( \frac{\rho_B - \rho_0}{\rho_0} \right) + \frac{K_{\text{sym}}}{18} \left( \frac{\rho_B - \rho_0}{\rho_0} \right)^2 + \ldots \]

$\Rightarrow$ slope L & curvature $K_{\text{sym}}$

$S_0 (\text{MeV})$ vs. $L (\text{MeV})$

$S(\rho) = 12.5(\rho/\rho_0)^{2/3} + 17.6 (\rho/\rho_0)^{\gamma_i}$

$0.4 \leq \gamma_i \leq 1$

ImQMD

$S(\rho) \sim 31.6(\rho/\rho_0)^{\gamma}$

$0.69 \leq \gamma \leq 1.05$

IBUU04

$P_\text{sym} (\text{MeV}/\text{fm}^3)$

$\rho_0$

$B_{\text{sym}}$

$E_L$

$0.4 \leq \gamma_i \leq 1$

ImQMD

$\Rightarrow$ Symmetry pressure $P_{\text{sym}}$

$L = 3\rho_0 \frac{\partial E_{\text{sym}}}{\partial \rho_B} \bigg|_{\rho_B = \rho_0} = \frac{3}{\rho_0} P_{\text{sym}}$

Expansion around $\rho_0$: $\Rightarrow$ Symmetry pressure $P_{\text{sym}}$
\[ S = 12.5 \left( \frac{\rho}{\rho_0} \right)^{2/3} + C_{s,p} \left( \frac{\rho}{\rho_0} \right)^{\gamma_i} \]

Vary \( C_{s,p} \) and \( \gamma_i \)

\[ S = S_o + \frac{L}{3} \left( \frac{\rho_B - \rho_0}{\rho_0} \right) + \frac{K_{sym}}{18} \left( \frac{\rho_B - \rho_0}{\rho_0} \right)^2 + \ldots \]

\[ L = 3 \rho_0 \left. \frac{\partial E_{sym}}{\partial \rho_B} \right|_{\rho_B = \rho_0} = \frac{3}{\rho_0} P_{sym} \]
$E_{\text{sym}} = 12.5 \left( \frac{\rho}{\rho_0} \right)^{2/3} + C_{s,p} \left( \frac{\rho}{\rho_0} \right)^{y_i}$

Constraints from masses and Pygmy Dipole Resonances

$E_{\text{sym}} = S_o + \frac{L}{3} \left( \frac{\rho_B - \rho_0}{\rho_0} \right) + \frac{K_{\text{sym}}}{18} \left( \frac{\rho_B - \rho_0}{\rho_0} \right)^2 + \ldots$

$L = 3 \rho_0 \frac{\partial E_{\text{sym}}}{\partial \rho_B} \bigg|_{\rho_B = \rho_0} = \frac{3}{\rho_0} P_{\text{sym}}$
$E_{sym} = 12.5 \left( \frac{\rho}{\rho_0} \right)^{2/3} + C_{s,p} \left( \frac{\rho}{\rho_0} \right)^{\gamma_i}$

Constraints from masses and Pygmy Dipole Resonances

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$L = 3 \rho_0 \frac{\partial E_{sym}}{\partial \rho_B} \bigg|_{\rho_B = \rho_0} = \frac{3}{\rho_0} P_{sym}$
\[ E_{\text{sym}} = 12.5 \left( \frac{\rho}{\rho_0} \right)^{2/3} + C_{s,p} \left( \frac{\rho}{\rho_0} \right)^{\gamma_i} \]

Current constraints on symmetry energy from HIC

\[ E_{\text{sym}} = S_o + \frac{L}{3} \left( \frac{\rho_B - \rho_0}{\rho_0} \right) + \frac{K_{\text{sym}}}{18} \left( \frac{\rho_B - \rho_0}{\rho_0} \right)^2 + \ldots \]

\[ L = 3 \rho_0 \left. \frac{\partial E_{\text{sym}}}{\partial \rho_B} \right|_{\rho_B = \rho_0} = \frac{3}{\rho_0} P_{\text{sym}} \]
Constraints from HIC at sub-saturation density: Sn+Sn collisions at E/A=35 and 50 MeV
N/Z ratios from bound fragments ($Z=3-8$) → complementary to n/p ratios

\[ E_{\text{sym}} = 12.7 \left( \frac{p}{p_o} \right)^{2/3} + 19 \left( \frac{p}{p_o} \right) \gamma_i \]
Chimera array

MSU+INFN, LNS Catania

$^{124}\text{Sn}+^{124}\text{Sn}$, $^{124}\text{Sn}+^{112}\text{Sn}$, $^{112}\text{Sn}+^{124}\text{Sn}$, $^{112}\text{Sn}+^{112}\text{Sn}$ at $E/A=35$ MeV

Lower energy

Longer interaction times,

more N/Z equilibrations

$N_c, E_t \rightarrow b$

$Y(^{7}\text{Li}) \& Y(^{7}\text{Be}) \rightarrow R_7$
Chimera array

- $4\pi$ array: 1192 Si + CsI telescopes
- http://www.lns.infn.it/research/chimera/
b-selection \[ \frac{b}{b_{\text{max}}} = \sqrt{\frac{\int_0^{Et} N(E) dE}{\int_0^{\infty} N(E) dE}} \]

\[ \frac{b}{b_{\text{max}}} = \sqrt{\sum_{i=1}^{n} N_i / \sum_{i=1}^{\infty} N_i} \]
Energy Spectrum of $^7$Li & $^7$Be

$\frac{dM}{dE \, d\Omega}$ (sr$^{-1}$MeV$^{-1}$)

- $^{124}$Sn + $^{124}$Sn
- $^{112}$Sn + $^{112}$Sn

$E/A = 35$ MeV

E.C.M. / A (MeV)
Impact parameter dependence of Isospin diffusion data at E/A=35 MeV

\[ R_i = 2 \frac{x_{AB} - (x_{AA} + x_{BB}) / 2}{x_{AA} - x_{BB}} \]

\[ x = \ln(\frac{Y(^7\text{Li}/^7\text{Be}))}{\sum_{i=1}^{\infty} N_i}} \]

\[ \frac{b}{b_{\text{max}}} = \sqrt{\frac{\int_{0}^{E_t} N(E) dE}{\int_{0}^{\infty} N(E) dE}} \]

\[ \frac{b}{b_{\text{max}}} = \sqrt{\sum_{i=1}^{n} N_i / \sum_{i=1}^{\infty} N_i} \]
The magnitude of $R_{N,\text{frag}}$ depends on $\gamma_i$

Calculations predict flat dependence on $b<8$ fm and sharp rise for $b>9$ fm.

Stable values at small $b$ can be compared to data.
Comparison of ImQMD calculations to data

Data are in good agreement with $\gamma_1 \sim 0.5$, consistent with $E/A = 50$ MeV data.

No complete stopping & no isospin equilibrations in central collisions
Constraints on the density dependence of symmetry energy
Constraints on the density dependence of symmetry energy

![Graph showing constraints on the density dependence of symmetry energy](image)
Constraints on the density dependence of symmetry energy

$V_2,\text{neut}/V_2,\text{hydro}$

 weighted mean $\gamma = 0.94(21)$

$p_t$ (GeV/c)

$L$ (MeV)

$S_0$ (MeV)

$E_{\text{beam}}$ (GeV/u)

$n/p$ Squeeze-out

GDR

Sn+Sn

Au+Au

E/A=0.25–6 GeV

$\pi^+$/π$^-$

Data

$x=1$

$x=0$

$x=-1$

Au+Au

E/A=0.4 GeV
Symmetry Energy Project → International collaboration to determine the symmetry energy over a range of density

Require: New Detectors (TPC), travel money, theory support

<table>
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Determination of the Equation of State of Asymmetric Nuclear Matter

5th RIKEN PAC recommends completion of TPC in 2013.

DOE FOA proposal (12/18/08): $1.2 M includes US contributions to SAMARAI TPC

NSF PIRE proposal (9/18/2009): $3.6 M includes JREU, extended visits by PD & GS to Japan & Europe

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ImQMD Calculations

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PRL 1022, 062501 (2009)
Results from Sn+Sn at E/A=35 MeV

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Summary

The density dependence of the symmetry energy is of fundamental importance to nuclear physics and neutron star physics.

Observables in HI collisions provide unique opportunities to probe the symmetry energy over a range of density especially for dense asymmetric matter.

Calculations suggest a number of promising observables that can probe the density dependence of the symmetry energy.

Need more guidance from theory regarding observables beyond normal nuclear matter density.

The availability of intense fast rare isotope beams at a variety of energies at RIKEN, FRIB & FAIR allows increased precisions in probing the symmetry energy at a range of densities – international co-ordination of a global program.
FRIB @MSU

- Driver linac capable of E/A ≥ 200 MeV for all ions, P_{beam} ≥ 400 kW
  - Easy to implement upgrade options (tunnel can house E/A = 400 MeV uranium driver linac, ISOL, multi-user capability ...)

Diagram showing:
- Radioactive Ion Beam Post Accelerator
- Experimental Area Instrumentation
- Gas Stopper
- Fragment Separator
- Target Facilities
- Switchyard
- Cryogenics Facilities
- Superconducting Heavy Ion Driver Linac
- Infrastructure


Experimental Observable: Isospin Diffusion

- Isospin diffusion is measured with fragments emitted from the neck region.
- Probe the symmetry energy at subsaturation densities in semi-peripheral collisions, e.g. $^{124}\text{Sn} + ^{112}\text{Sn}$ at $b=6$ fm
- Isospin “diffuse” through low-density neck region
- Symmetry energy drives system towards equilibrium.
  - stiff EOS $\rightarrow$ small diffusion; $|R_i| >> 0$
  - soft EOS $\rightarrow$ fast equilibrium; $R_i \rightarrow 0$

$$R_i = 2 \frac{x_{AB} - (x_{AA} + x_{BB})}{2} \times \frac{x_{AA} - x_{BB}}{2}$$
\[ B = a_v A - a_s A^{2/3} + \delta - a_c \frac{Z(Z - 1)}{A^{1/3}} - C_{\text{sym}} \frac{(A - 2Z)^2}{A} \]

\[ C_{\text{sym}} = 22.4 \]

\[ (a_v^\text{sym} A + a_s^\text{sym} A^{2/3}) \]

\[ (a - \frac{a^2}{b}) \]

\[ (a_v^\text{sym} A + a_s^\text{sym} A^{2/3}) \]

Reduction of \( \alpha \) values can be accomplished with more accurate mass formula rather than to change \( C_{\text{sym}} \) values obtained from fitting empirical masses!

\( A_1 = 186 \)

\( Z_1 = 75 \)

\( A_2 = 168 \)

\( Z_2 = 75 \)

\( T = 5 \text{ MeV} \)
Physics of symmetry energy

\[ B = a_v A - a_s A^{2/3} + \delta - a_c \frac{Z(Z-1)}{A^{1/3}} - a_{sym} \frac{(A-2Z)^2}{A} \]

- masses
- n-skin of heavy nuclei such as $^{208}$Pb
- Fission barriers
- Neutron star properties
Analysis of rapidity dependence of $R_i$ with ImQMD model

$$R_i = 2 \left( \frac{x_{AB} - (x_{AA} + x_{BB})}{2} \right) / \left( x_{AA} - x_{BB} \right)$$

$$S = 12.5 \left( \rho/\rho_o \right)^{2/3} + 17.6 \left( \rho/\rho_o \right)^{\gamma_i}$$

$x$(data) = $Y(\text{Li}/\text{Be})$

$x$(QMD) = $\delta$

Equilibrium $R_i = 0$

No diffusion $R_i = 1; R_i = -1$

New analysis on rapidity dependence of isospin diffusion ratios – not possible with BUU type of simulations due to lack of fragments.
Lessons learned from LE measurements:

1. HI collision dynamics are complex but proved to be sensitive to density dependence of symmetry energy.
2. Problems still remain, e.g.
   • How to extract results to $T=0$;
   • Control of input parameters in transport models.

Low and Intermediate Energy Heavy Ion Collisions Workshop
ECT*

Constraints on the density dependence of symmetry energy
Constraints on the density dependence of symmetry energy

Au+Au experiments in high energy are not designed to measure symmetry energy

Need better experiments
Wed afternoon discussions
Inconsistencies in the constraints on symmetry energy at high and low density?
$$S = 12.5 \left( \frac{\rho}{\rho_0} \right)^{2/3} + 17.6 \left( \frac{\rho}{\rho_0} \right)^{\gamma_i}$$

$$0.4 \leq \gamma_i \leq 1$$

ImQMD

$$S = 12.5 \left( \frac{\rho}{\rho_0} \right)^{2/3} + 17.6 \left( \frac{\rho}{\rho_0} \right)^{\gamma_i}$$

$$0.4 \leq \gamma_i \leq 1$$

ImQMD

$$\delta R_{np} = \pm 0.04 \text{ fm}$$

$$\delta R_{pb} = 0.28 \text{ fm}$$

$$S \approx 31.6 \left( \frac{\rho}{\rho_0} \right)^{\gamma}$$

$$0.69 \leq \gamma \leq 1.05$$

IBUU04

$$S = S_o + \frac{L}{3} \left( \frac{\rho_B - \rho_0}{\rho_0} \right) + \frac{K_{sym}}{18} \left( \frac{\rho_B - \rho_0}{\rho_0} \right)^2 + \ldots$$

$$L \rightarrow \text{Symmetry pressure } P_{sym}$$

$$L = 3 \rho_0 \left. \frac{\partial E_{sym}}{\partial \rho_B} \right|_{\rho_B = \rho_0} = \frac{3}{\rho_0} P_{sym}$$

$$S_o \ (\text{MeV})$$

$$L \ (\text{MeV})$$

$$P_o \ (\text{MeV/fm}^3)$$

$$\rho \ (\text{fm}^3)$$

$$\delta R_{np} = \pm 0.04 \text{ fm}$$

$$\delta R_{pb} = 0.28 \text{ fm}$$

$$S \approx 31.6 \left( \frac{\rho}{\rho_0} \right)^{\gamma}$$

$$0.69 \leq \gamma \leq 1.05$$

IBUU04
Wednesday afternoon discussions: Experiments in RIBF

Sn+Sn, E/A=50 MeV

Isospin diffusions for fragments and residues with RIBF, ~2010

Riken TPC & to measure $\pi^+/\pi$, t/3He, p/n (Nebula), spectra ratios
Outlook

E/A=200-300 MeV $\rightarrow$ measure $\pi^+$, $\pi^-$ spectra ratios, p,n, t/3He spectra ratios and differential flow $\rightarrow$ determine $S(\rho)$, $m^*$, $\sigma_{nn}$, $\sigma_{pp}$, $\sigma_{np}$ at $\rho \sim 2\rho_o$

E/A=50 MeV $\rightarrow$ measure isospin diffusions for fragments and residues $\rightarrow$ determine $S(\rho)$ at $\rho < 2\rho_o$. $^{108}$Sn+$^{112,124}$Sn – RI beam used to increase $\delta$. 
Central collisions

Multifragmentation

Similar distributions

\[ R_{21}(N,Z) = \frac{Y_2(N,Z)}{Y_1(N,Z)} \]
GSI (2011) : E/A~400 – 800 MeV → measure p,n spectra ratios and differential flow → determine constraints S(ρ), m*, σ_{nn}, σ_{pp}, σ_{np} at 2.5ρ_0 < ρ < 3ρ_0

Lemmon, Russotto et al, experimental proposal to GSI PAC
Outlook

MSU (2009-2012) : E/A<100 MeV $\Rightarrow$ measure isospin diffusion, fragments, residues, p,n spectra ratios and differential flow $\Rightarrow$ improve constraints on $S(\rho)$, $m^*$, $\sigma_{nn}$, $\sigma_{pp}$, $\sigma_{np}$ at $\rho<\rho_o$

MSU (~2013) : E/A>120 MeV $\Rightarrow$ measure $\pi^+$, $\pi^-$ spectra ratios $\Rightarrow$ constraints at $\rho_o<\rho<1.7\rho_o$ -- Bickley
GSI (2011) : E/A~400 – 800 MeV $\rightarrow$ measure $p,n$ spectra ratios and differential flow $\rightarrow$ determine constraints $S(\rho)$, $m^*$, $\sigma_{nn}$, $\sigma_{pp}$, $\sigma_{np}$ at $2.5\rho_o < \rho < 3\rho_o$

Lemmon, Russotto et al, experimental proposal to GSI PAC
Outlook

Precision measurements in FRIB
Detectors needed:

- n detectors: NSCL/pre-FRIB; GSI; RIBF/Riken
- Pions/kaons & p, t, \(^3\)He detectors: TPC

**NSCL Dual purpose AT-TPC:** proposal to be submitted to DOE

**RIBF TPC:** SUMARAI magnet funded, TPC – Japan-US collaboration: proposal to be submitted to DOE.

**AT-TPC: FRIB**

- NSCL: \(\sim 1.2 \text{ M}\)
- TPC: \(\sim 0.78 \text{ M}\)
- Travel: \(0.37 \text{ M}\)

Density region sampled depends on collision observable & beam energy
- $\rho > \rho_0$ examples:
  - Pion energy spectra
  - Pion production ratios
  - Isotopic spectra
  - Isotopic flow
- With NSCL beams, densities up to $1.7\rho_0$ are accessible
- Beams: 50-150 MeV, 50,000 pps
  - $^{106}\text{Sn}-^{126}\text{Sn}$, $^{37}\text{Ca}-^{49}\text{Ca}$
Density region sampled depends on collision observable & beam energy
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  - \(^{106}\text{Sn}-^{126}\text{Sn}, \; {^{37}\text{Ca}}-{^{49}\text{Ca}}\)
Experimental Observable: Isospin Diffusion

- Probe the symmetry energy at subsaturation densities in peripheral collisions, e.g. $^{124}{\text{Sn}} + ^{112}{\text{Sn}}$

- Isospin “diffuse” through low-density neck region

$$R_i = 2 \frac{x_{AB} - (x_{AA} + x_{BB})}{2}$$

$$x_{AA} - x_{BB}$$

$$x(\text{calc}) = \delta$$

- Symmetry energy drives system towards equilibrium.
  - stiff EOS $\rightarrow$ small diffusion; $|R_i| >> 0$
  - soft EOS $\rightarrow$ fast equilibrium; $R_i \rightarrow 0$
Isospin Diffusion

No diffusion

Complete mixing

Degree of Asymmetry

from isoscaling

Sn+Sn data

from Y(\(^7\)Li)/Y(\(^7\)Be)

Projectile \(^{124}\)Sn

Target \(^{112}\)Sn
Inconsistencies in the constraints on symmetry energy at high and low density?

Wed afternoon discussions
Laboratory experiments to study properties of neutron stars

<table>
<thead>
<tr>
<th></th>
<th>n-star</th>
<th>HI collisions</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho/\rho_0$</td>
<td>~0.1 - 10</td>
<td>~0.1-5</td>
</tr>
<tr>
<td>$y_e$</td>
<td>~0.1</td>
<td>~0.38-0.5</td>
</tr>
<tr>
<td>$T$(MeV)</td>
<td>~1</td>
<td>~4-50</td>
</tr>
</tbody>
</table>

Extrapolate information from limited asymmetry and temperature to neutron stars!
Laboratory experiments to study properties of neutron stars

extrapolation from $^{208}\text{Pb}$ radius to n-star radius
Isotope Distribution Experiment

*MSU, IUCF, WU collaboration*

Sn+Sn collisions involving $^{124}$Sn, $^{112}$Sn at E/A=50 MeV

*Miniball + Miniwall*

4 $\pi$ multiplicity array
Z identification, A<4

*LASSA*

Si strip +CsI array
Good E, position, isotope resolutions

*Xu et al, PRL, 85, 716 (2000)*
N/Z ratios from bound fragments (Z=3-8) → complementary to n/p ratios

\[ E_{\text{sym}} = 12.7 (\rho/\rho_o)^{2/3} + 19 (\rho/\rho_o)^{\gamma_i} \]

Effects are small
Hot fragments produced in calculations.
Sequential decay effects are important.

Data consistent with soft EOS.

\[ E_{\text{sym}} = 12.7 \left( \frac{\rho}{\rho_0} \right)^{2/3} + 19 \left( \frac{\rho}{\rho_0} \right)^{\gamma_i} \]

N/Z ratios from bound fragments (Z=3-8) \( \rightarrow \) complementary to n/p ratios.
ImQMD, $b=6\text{fm}$

$R_1(\delta)$

$v>0.7v_{\text{c.m.}}$

- $X=\delta(v_{n,\text{frag}}>0.7v)_{\text{beam}}$
- $X=\ln(n/p)$
- $X=\delta, v_i>0.7v_{\text{beam}}$
- $X=\delta, Z_{\text{max}}>20$

$\gamma_i$
Experimental Observables to probe the symmetry energy

\[
E/A(\rho,\delta) = E/A(\rho,0) + \delta^2 \cdot S(\rho) \quad ; \quad \delta = (\rho_n - \rho_p)/(\rho_n + \rho_p) = (N-Z)/A
\]

• Collision systems: \(^{124}\text{Sn}+^{124}\text{Sn}, \quad ^{124}\text{Sn}+^{112}\text{Sn}, \quad ^{112}\text{Sn}+^{124}\text{Sn}, \quad ^{112}\text{Sn}+^{112}\text{Sn}\)
• \(E/A=50\text{ MeV}\)
• Low densities \((\rho<\rho_0)\):
  – n/p spectra and flows; \(Y(n)/Y(p), \quad Y(t)/Y(^3\text{He})\),
  – Fragment isotopic distributions,
    • Isoscaling: interpretation with statistical model is incorrect
    • \(<N>/<Z>\) of \(Z=3-8\) fragments
    • Isospin diffusion
  – Correlation function, \(C(q)\)
  – Neutron, proton radii, E1 collective modes.
• High densities \((\rho \approx 2\rho_0)\)
  – Neutron/proton spectra and flows; \(C(q)\)
  – \(\pi^+\) vs. \(\pi^-\) production, \(k\), hyperon production.
Exploring Bulk properties of Nuclear Matter with Heavy Ion Collisions

High density/energy
- differential flow
- n/p, LIF ratios
- pions ratios
- kaon ratios
- neutron stars

Low density/energy
- fragments, ratios
- isospin diffusion
- isoscaling
- migration/fractionation.
- collective excitations
- surface phenomena
- phase transitions

QF Li, Di Toro
Di Toro, Lukasic
DiToro, Reisdorf, QF Li
Prassa, QF Li
BA Li, Kubis
Hermann Wolter

Tsang, HW
BA Li, HW
Tsang
Di Toro
Aumann, Ducoin
Danielewicz
Lehaut
Cluster effects are important for low energy nucleons but cannot explain the large discrepancy between data and IBUU04 calculations.
Constraints from Isospin Diffusion Data

Data:
- Collisions of Sn+Sn isotopes at E/A=50 MeV
  - $b/b_{\text{max}} > 0.8$
  - $<b> \sim 7.2$ fm
- Projectile-like residue determined from multiplicity gates.
  - $y/y_b > 0.7$
- Results obtained with clusters

IBUU04:
- Collisions of Sn+Sn isotopes at E/A=50 MeV
  - $b = 6$ fm
  - No clusters
  - $y/y_b > 0.5$
Isospin diffusion in the projectile-like region

**Basic ideas:**

- **Peripheral reactions**
- **Asymmetric collisions** $^{124}\text{Sn} + ^{112}\text{Sn}, ^{112}\text{Sn} + ^{124}\text{Sn}$
  
  -- diffusion

- **Symmetric Collisions** $^{124}\text{Sn} + ^{124}\text{Sn}, ^{112}\text{Sn} + ^{112}\text{Sn}$
  
  -- no diffusion

- **Relative change between target and projectile is the diffusion effect**
Density region sampled depends on collision observable & beam energy
• $\rho > \rho_0$ examples:
  – Pion energy spectra
  – Pion production ratios
  – Isotopic spectra
  – Isotopic flow
  – With NSCL beams, densities up to $1.7 \rho_0$ are accessible
  – Beams: 50-150 MeV, 50,000pps
    106Sn-126Sn, 37Ca-49Ca
The density dependence of symmetry energy is largely unconstrained.

Pressure, i.e. EOS is rather uncertain even at $\rho_0$. 

$$E/A (\rho, \delta) = E/A (\rho, 0) + \delta^2 \cdot S(\rho)$$

$$\delta = (\rho_n - \rho_p) / (\rho_n + \rho_p) = (N - P) \rho^2 \frac{\partial (E/A)}{\partial \rho}_{s/a}$$

**Equation of State for Asymmetric Matter**

- Soft ($\beta=0$, $K=200$ MeV)
- Asy-soft, $\beta=1/3$ (Colonna)
- Asy-stiff, $\beta=1/3$ (PAL)

Extra effort to understand symmetry energy at high densities

Observables: n/p spectra & flows; np, pp correlations, $\pi^+/\pi^-$ spectra & flows, kaons, hyperon production.

Require: New Detectors, travel money, theory support
• $\rho < \rho_0$ – created in multifragmentation process

Observables:
– n/p ratios
– Isotopic spectra
– Beams: 50 MeV, $^{112}\text{Sn-124}\text{Sn}$
Transport description of heavy ion collisions:

non-relativistic: BUU

\[
\frac{\partial f}{\partial t} + \frac{\vec{p}}{m} \nabla f - \nabla U \nabla \rho f = I_{\text{coll}}[f, \sigma] = \frac{1}{(2\pi)^{3/2}} \int \int \int dp_2 dp_3 dp_4 \, v_{12} \frac{d\sigma}{d\Omega_{12 \rightarrow 4}} \delta(p_1 + p_2 - p_3 - p_4) \\
[(1-f)(1-f_2)f_3f_4 - f_2(1-f_3)(1-f_4)]
\]

2-body hard collisions

Vlasov eq.; mean field

Relativistic BUU

\[
\left[ p^*_\mu \frac{\partial}{\partial x^\mu} + (p^*_\nu F^{\mu\nu} + m^* \partial^\mu m^*) \right] f(x, p^*) = I_{\text{coll}}(f, \sigma)
\]

Simulation with Test Particles:

Transport model comparison (tw2003 homework)

Au(100 AMeV)+Au soft eos b=3fm

same ini, \( \sigma=40\text{mb} \)
Analysis of rapidity dependence of Ri with ImQMD model

\[ E_{\text{sym}} = 12.5 \left( \frac{\rho}{\rho_o} \right)^{2/3} + 17.6 \left( \frac{\rho}{\rho_o} \gamma_i \right) \]

New analysis on rapidity dependence of isospin diffusion ratios – not possible with BUU type of simulations due to lack of fragments.
HIC provides a range of density determined from incident energy and impact parameter.
Learning about the Density dependence of Symmetry Energy in Heavy Ion Reactions

Sn+Sn, E/A=50 MeV

RIBF

Au+Au

Preliminary analysis

ρ (fm^-3)

S(ρ) (MeV)

Betty Tsang
The National Superconducting Cyclotron Laboratory
Michigan State University

RIBF Mini-workshop on Nuclear Collisions and Nuclear Matter
Dec 16-17, 2008
Riken
n/p Double Ratios (central collisions)

Double Ratio  $\frac{^{124}_{\text{Sn}} + ^{124}_{\text{Sn}}; Y(n)/Y(p)}{^{112}_{\text{Sn}} + ^{112}_{\text{Sn}}; Y(n)/Y(p)}$  

• Effect is much larger than IBUU04 predictions  \( \rightarrow \) inconsistent with conclusions from isospin diffusion data.

Famiano et al. RPL 97 (2006) 052701
What are the densities created in Heavy Ion Collisions

Highest density reached by central collisions depends on incident beam energy.

E/A=1600 MeV

200 MeV

50 MeV

Types of particles formed depend on emission times and density.

Pions, n, p fragments
• Experiment: measure collective flow (emission patterns) of particles emitted in Au+Au collisions from (E/A~1-8 GeV).

• Transport model (BUU) relates the measurements to pressure and density.
Equation of State of Nuclear Matter

\[ \frac{E}{A}(\rho, \delta) = \frac{E}{A}(\rho, 0) + \delta^2 \cdot S(\rho) ; \quad \delta = \frac{\rho_n - \rho_p}{\rho_n + \rho_p} = \frac{N - Z}{A} \]

Equation of State of Nuclear Matter

\[ E/A(\rho, \delta) = E/A(\rho, 0) + \delta^2 \cdot S(\rho); \quad \delta = \frac{(\rho_n - \rho_p)}{(\rho_n + \rho_p)} = \frac{(N-Z)}{A} \]


- Newer calculation and experiment are consistent with the constraints.
- Transport model includes constraints in momentum dependence of the mean field and NN cross-sections
EOS: symmetric matter and neutron matter

\[ \frac{E}{A} (\rho, \delta) = \frac{E}{A} (\rho, 0) + \delta^2 \cdot S(\rho) \]

\[ \delta = \frac{(\rho_n - \rho_p)}{\rho_n + \rho_p} = \frac{(N-Z)}{A} \]

- The density dependence of symmetry energy is largely unconstrained.
N-detection – neutron wall
p-detection: Scattering Chamber

WU MicroBall (b determination)

3 particle telescopes (p, d, t, $^3\text{He}$, ...)

$\hat{b} < 0.2$

# of charged particles
n/p Double Ratios (central collisions)

Double Ratio \( \frac{^{124}Sn+^{124}Sn; Y(n)/Y(p)}{^{112}Sn+^{112}Sn; Y(n)/Y(p)} \)

minimize systematic errors

Famiano et al. RPL 97 (2006) 052701
Nuclear Collisions simulations with Transport Models
– Nuclear EOS included from beginning of collisions

**BUU models:**
*Semiclassical solution of one-body distribution function.*

**Pros**
Derivable, approximations better understood.

**Cons**
Mean field → no fluctuations
BUU does not predict cluster formation

**QMD:**
*Molecular dynamics with Pauli blocking.*

**Pros**
Predicts cluster production

**Cons**
Cluster properties (masses, level densities) approximate
Need sequential decay codes to de-excite the hot fragments
Code used: ImQMD

At high incident energies, cluster production is weak → the two models yield the same results.
Clusters are important in low energy collisions.
\[ \frac{R_i(\text{^7Li}, \text{^7Be})}{R_i(\text{a})} \approx R_i(\delta) \]
Heavy Ion collision: $^{124}$Sn+$^{124}$Sn, E/A=50 MeV

Reaction mechanism of fragment productions depends on impact parameters

Charged fragments (Z=3-20) are formed at subnormal density

\[
S(\rho) = 12.5(\rho/\rho_o)^{2/3} + 19(\rho/\rho_o)
\]
Learning about the **Equation of State of Asymmetric Nuclear Matter with Heavy Ion Collisions**

**Examples of possible research areas in FRIB**

- **Nuclear Structure** – What is the nature of the nuclear force that binds protons and neutrons into stable nuclei and rare isotopes?
- **Nuclear Astrophysics** – What is the nature of neutron stars and dense nuclear matter? What is the origin of elements heavier than iron in the cosmos? What are the nuclear reactions that drive stars and stellar explosions?
- **Tests of Fundamental Symmetries** – Why is there now more matter than antimatter in the universe?
Nuclear Equation of State

\[
\frac{E}{A}(\rho, \delta) = \frac{E}{A}(\rho,0) + \delta^2 \cdot S(\rho)
\]

\[
\delta = \frac{(\rho_n - \rho_p)}{(\rho_n + \rho_p)} = \frac{(N-Z)}{A}
\]

Examples of possible research areas in FRIB

✓ **Nuclear Structure** – What is the nature of the nuclear force that binds protons and neutrons into stable nuclei and rare isotopes?

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What is the role of JUSEIPEN?
To foster US scientists to do experiments in RIKEN

Funds requested: ~$100k/year 2k/trip  
50 person trips a year to Japan funding of whole experimental program is not practical  
How to best use the resources?  
Facilitate funding applications by US collaborators  
Fund Proposal presentations at RIKEN PAC?  
Fund Collaboration workshops?  
Summer schools for experimental students?  
JUSEIPEN user organization -- streamline the collaborations, exchange ideas and consolidation of resources  
(e.g. video presentations to RIKEN PAC)
Trautmann et al

best fits with UrQMD of
Qingfeng Li JGP 31(2005)
in comparison with
FOPI/LAND v2 ratios

e_{sym} = e_{kin} + 22.0 * u^{0.95}
e_{sym} = e_{kin} + 18.0 * u^{1.02}
e_{kin} = 12.0 * u^{(2/3)}
Model Assumptions
Describe longer time scale decays from single source.
• nuclear mass,
• level densities (isospin?)
• decay rates

Uncertainties
Source parameters: $A_o$, $Z_o$, $E_o$, $V_o$, $J$
Information obtained is for finite nuclei, not for infinite nuclear matter

Advantages
fast turn around
Production of very rare isotopes possible
provide lots of physics insights
What are the uncertainties in theoretical predictions?

**Transport models:**
Describe dynamical evolution of the collision process
• Self consistent mean field
• $n$-$n$ collisions,
• Pauli exclusion

**Uncertainties**
Semi-classical Approximations needed to make computation feasible.

Symmetry energy included in the nuclear EOS for infinite nuclear matter at various densities from the beginning of collision.
N/Z ratios from bound fragments (Z=3-8) → complementary to n/p ratios

\[ E_{\text{sym}} = 12.7\left(\frac{\rho}{\rho_o}\right)^{2/3} + 19\left(\frac{\rho}{\rho_o}\right)^{\gamma_i} \]
Learning about Symmetry Energy at Low Density with Heavy Ion Reactions

Defining the neutron star crust: Giant flares, superbursts and x-ray bursts

May 17-21, 2009
Santa Fe, NM

Betty Tsang
Density dependence of Symmetry Energy

Questions:
What have we learned about symmetry energy at low density?
What are the unique opportunities offered at high density?
How to connect different symmetry energy representations

Expansion around $\rho_0$:

\[ E_{sym} = a_4 + \frac{L}{3} \left( \frac{\rho_B - \rho_0}{\rho_0} \right) + \frac{K_{sym}}{18} \left( \frac{\rho_B - \rho_0}{\rho_0} \right)^2 \]

\[ L = 3\rho_0 \left. \frac{\partial E_{sym}}{\partial \rho_B} \right|_{\rho_B=\rho_0} = \frac{3}{\rho_0} P_{sym} \]

Value of symmetry energy at saturation

\[ E_{sym}(\rho_0) = a_4 \]
Summary

Constraints from HIC at sub-saturation density:
Sn+Sn collisions at E/A=35 and 50 MeV

Consistent with IAS, PDR & GDR