Homework 1

The Periodic chart of Elements is a useful graphical presentation of the elements of nature. The elements are identified via the number of electrons in the neutral atom. (Z) which corresponds to the element name (e.g. Hydrogen [H] has Z=1, Helium [He] has Z=2 and Iron [Fe] has Z=26). The atoms of an element consist of Z-electrons orbiting a nucleus with charge +Z. The nucleus is made of protons and neutrons. Since neutrons have no charge and protons +1 charge, there are Z-protons. A single element can have more than one isotope; when the number of protons is fixed, but the number of neutrons varies from isotope to isotope. The natural abundance of the element hydrogen is dominated by the isotope ^{1}H, with smaller quantities of heavy hydrogen (or deuterium) ^{2}H. The number in this notation is the associated mass number, it is the sum of the number of protons and neutrons. The element hydrogen has 1 proton, thus ^{1}H has 0 neutrons and ^{2}H has 1 neutron. One can see that the Periodic chart of Elements is not adequate to graphically show this richness of nuclei, thus we create what is known as a chart of nuclides, a graphical representation of nuclei. The chart is organized with the number of neutrons (N) situated on the horizontal axis and the number of protons (Z) on the vertical axis. Please visit the National Nuclear Data Center's chart of nuclides to get familiar with the chart (`http://www.nndc.bnl.gov/chart/`).

1. Mass Excesses and Binding Energies

   You may recall that in atomic physics, we use carbon to define a mass scale. The so-called atomic mass unit (m_u). This provides a convenient mass scale that we can use to estimate the mass of an atom.

   $$M(Z,N) \approx (Z+N)m_u \tag{1}$$

   Since the mass of the atom is dominated by the nucleus (i.e. $m_u \sim m_p \sim m_n$), a better guess of the mass is simply the sum of the constituent masses; the sum of the electron, proton and neutron masses:

   $$M(Z,N) \approx Z(m_p + m_e) + Nm_n \tag{2}$$

   The masses listed in the periodic table are in fact in terms of the atomic mass unit (e.g. A(Z,N) = M(Z,N)/m_u). One can use the values in the periodic table to define the "excess" mass in the nuclide.

   $$\Delta(Z,N) \equiv M(Z,N) - (Z+N)m_u \tag{3}$$

   One can also define a nuclear binding energy, the difference between the actual mass of a atom and the total mass of the electrons, protons and neutrons in the system (assumes atomic binding is negligible).

   $$B(Z,N) \equiv Z(m_p + m_e) + Nm_n - M(Z,N) \tag{4}$$

   (a) Find Atomic masses (A) for the nuclides listed below, from a source that you cite. Use this information to calculate the mass excess, Δ, and the binding energies (B) of the nuclides. If Atomic masses prove difficult to find, you may use the mass excess as input. Please be clear about what your input is. Also note that atomic masses on the periodic table of elements are generally mean atomic masses. This means they are averages of the naturally occurring isotopes (i.e. A(^{1}H)=1.0078, A(^{2}H)=2.0141, A(H)=1.0079). We want to use the isotopic atomic masses, not the average over all isotopes.

   (b) Create a table of *atomic* masses, mass excesses and binding energies (i.e. columns=nuclide, #protons, #neutrons, atomic mass (A), the mass excess, Δ, and the binding energy, B. Using $m_u = 931.494013$ MeV, $m_p = 938.27200$ MeV and $m_n = 939.56533$ MeV, $m_e = 0.51100$ MeV (where I have used Einstein's $E = mc^2$, to convert the mass into energy units).

   (c) Create this table for the following nuclides: (n, p, ^{1}H, ^{2}H, ^{3}H, ^{3}He, ^{4}He, ^{12}C, ^{13}C, ^{14}C, ^{16}O, ^{40}Ca)
   You do not need to calculate the binding energies for n or p. Note the difference between the nuclear mass (p) and atomic mass (^{1}H).

   (d) Please cite where you obtained your input data (e.g. atomic masses in m_u from the Periodic Table of Elements).

   (e) Compare your results to the 2003 Atomic Mass Evaluation (`http://www.nndc.bnl.gov/masses/`).
   Another useful source is the NIST website (`http://physics.nist.gov/cuu/Constants/`).