Homework 5

1. Reaction Rates

   To help make the leap from cross-sections to reaction rates, we will need the help of a couple intermediate definitions. First of all, it is useful to restrict ourselves to binary reactions again. We can relax this later, but we want to build up our intuition. In binary reactions, it is useful to talk in terms of a target nuclide and a projectile nuclide, as this is generally how experiments are done.

   The number of target nuclides in a given volume, is set by the number density (n_t). Together with the cross-section, we can define what is called the mean-free-path of the reaction.

   $$\ell_{mfp} = (n_t \sigma)^{-1} \tag{1}$$

   Now imagine being able to travel inside the target volume. As we look around, we'll see target nuclides that have reacted. The mean-free-path, as defined here, is the average distance between these reacted nuclides. We can see that reactions with a large cross-section, will have a smaller mean-free-path. Thus the reacted target nuclides will be more closely spaced. Another way of "viewing" the quantity $n_t\sigma$ is as the number of reactions in the target material per unit length.

   (a) Assuming a reaction cross-section of 1 barn (1.0×10^{-24} cm^2), and a target material with 3 times Avogadro's number of nuclides per cm^3 of volume, find the mean-free-path. Also find the number of reactions per centimeter. What is the number of reactions per millimeter?

   (b) If Avogadro's number has units of moles per gram, or number of particles per gram, then its inverse (N_A^{-1}) has units of grams per particle (i.e. mass). Convert this mass into MeV. Does this mass make sense?

   As one expects from our vocabulary, the target is stationary, while the projectile travels to the target to do the reaction. The projectile nuclides also have a particular number density (n_p). The quantity of interest here is what is called the flux of projectile nuclides, given by:

   $$Flux = n_p v_p \tag{2}$$

   The flux tells us not only how many projectiles we are shooting at the target, but also how often.

   (c) For a particle of mass, m_u and energy, $E = 1$ MeV, find the speed of the particle in units of cm-per-sec. If our experimental set-up allows us to keep, on average, a single projectile in the target volume of 1 cm^3, what is the incident flux?

   Putting this information together we can create a reaction rate.

   $$reaction\ rate = n_p n_t \sigma v_p \tag{3}$$

   This is the number of reactions per second per unit volume of material. Waving our hands (saying something about Galilean invariance), we can convince ourselves that it is actually the relative speed between the target and projectile that matters, we can generalize this by replacing v_p with $v = |\vec{v}_p - \vec{v}_t|$.

   (d) Adopting the target density of 3 times Avogadro's number of nuclides per cm^3 reacting with a cross-section of 1 barn, with the incident flux just calculated, what is the reaction rate?