Isospin fractionation and isoscaling in dynamical nuclear collisions

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Abstract

Isoscaling is found to hold for fragment yields in the antisymmetrized molecular dynamics (AMD) simulations for collisions of calcium isotopes at 35 MeV/nucleon. This suggests the applicability of statistical considerations to the dynamical fragment emission. The observed linear relationship between the isoscaling parameters and the isospin asymmetry of fragments supports the above suggestion. The slope of this linear function yields information about the symmetry energy in low density region where multifragmentation occurs.

In typical intermediate energy nuclear collisions, numerous fragments of intermediate size are produced in addition to light particles [1]. The multifragmentation phenomenon is believed to be related to the liquid-gas phase-coexistence in low density expanding nuclear matter. In a two-component system with more neutrons than protons ($N^{\text{tot}} > Z^{\text{tot}}$) in equilibrium, the gas phase becomes more neutron-rich than the liquid phase [2]. This fractionation phenomenon should reflect the features of the symmetry energy in nuclear matter.

Recently, a scaling relation

$$Y_2(N, Z)/Y_1(N, Z) \propto e^{\alpha N + \beta Z} \tag{1}$$

has been observed [3] in the measured fragment yields $Y_i(N, Z)$ for two similar systems $i = 1, 2$ with different neutron to proton ratios. This phenomenon is called isoscaling. If one assumes thermal and chemical equilibrium, the isoscaling parameters α and β are related to the neutron-proton content of the emitting source. In fact, statistical models have successfully explained the isoscaling data [4]. However, as fragments are formed during a dynamical evolution of the collision system, multifragmentation should be understood in the dynamical models as well. In fact, a stochastic mean field model has predicted very large scaling parameters for the dynamically produced fragments in the model [5]. Such result is difficult to understand without any dynamical effects. It is also important to determine if the scaling parameters in the data can be directly related to the asymmetry term of the equation of state (EOS) of nuclear matter in equilibrium, for a dynamic production.

To explore whether any kind of equilibrium is achieved regarding the isospin fractionation and the fragmentation in dynamical nuclear collisions, we compare the result of the antisymmetrized molecular dynamics (AMD) simulation with what is expected under a statistical assumption. We first derive, under an equilibrium assumption, a linear relation between the isoscaling parameter α and the fragment isospin asymmetry $(Z/A)^2$. We then test such a relationship using results from the AMD simulations. By studying the dependence on the asymmetry term of the effective force, we will explore whether the relation is useful for assessing the asymmetry term of the EOS.

AMD is a microscopic model for following the time evolution of nuclear collisions [6, 7, 8]. It represents the colliding system in terms of a fully antisymmetrized product of Gaussian wave packets. Through the time evolution, the wave packet centroids move according to

an equation of motion. Besides, the followed state of the simulation branches stochastically and successively into a huge number of reaction channels. The branching is caused by the two-nucleon collisions and by the splittings of the wave packet. The interactions are parametrized in the AMD model in terms of the effective force between nucleons and the two-nucleon collision cross sections.

We perform reaction simulations employing two different effective forces in order to study effects of the asymmetry term. One is the usual Gogny force [9], consistent with the saturation of symmetric nuclear matter at the incompressibility $K = 228$ MeV. The force is composed of finite-range two-body terms and of a density-dependent term of the form $t_3\rho^{1/3}(1 + P_\sigma)\delta(\mathbf{r}_1 - \mathbf{r}_2)$, where P_σ is the spin exchange operator and t_3 is a coefficient. The second force (called Gogny-AS force) is obtained by modifying the Gogny force with

$$V_{\text{Gogny-AS}} = V_{\text{Gogny}} - (1 - x)t_3\Big(\rho(\mathbf{r}_1)^{1/3} - \rho_0^{1/3}\Big)P_\sigma\delta(\mathbf{r}_1 - \mathbf{r}_2), \tag{2}$$

where $x = -\frac{1}{2}$ and $\rho_0 = 0.16$ fm^{-3}. The two forces coincide at $\rho = \rho_0$. Furthermore, they produce the same EOS of symmetric nuclear matter at all density. However, the two forces produce different density dependences of the symmetry energy, as shown in Fig. 1. The choice of $x = -\frac{1}{2}$ has been made to ensure that the part of the symmetry energy from the direct term is proportional to the density [10]. At densities below ρ_0, the Gogny force has somewhat higher symmetry energy than the Gogny-AS force. At densities above ρ_0, the Gogny-AS symmetry energy continues to rise while the Gogny symmetry energy begins to fall, so that significant differences develop.

The AMD simulations were performed for ^{40}Ca + ^{40}Ca, ^{48}Ca + ^{48}Ca and ^{60}Ca + ^{60}Ca collisions at the incident energy $E/A = 35$ MeV/nucleon and zero impact parameter. The version of AMD of Ref. [8] was utilized. It has been demonstrated that an equivalent version of AMD, for the present purposes, reproduces the experimental data of various fragment observables in ^{40}Ca + ^{40}Ca at the same energy of 35 MeV/nucleon with the Gogny force [7, 11]. Each studied collision event was started by boosting two nuclei with centers separated by 9 fm. The dynamical simulation was continued until $t = 300$ fm/c. About 1000 events were generated for each system.

In central collisions, as shown in a previous paper [7, 11], two nuclei basically penetrate each other and many fragments are formed not only from the projectile-like and target-like parts but also from within the neck region between the two residues. The nuclear matter
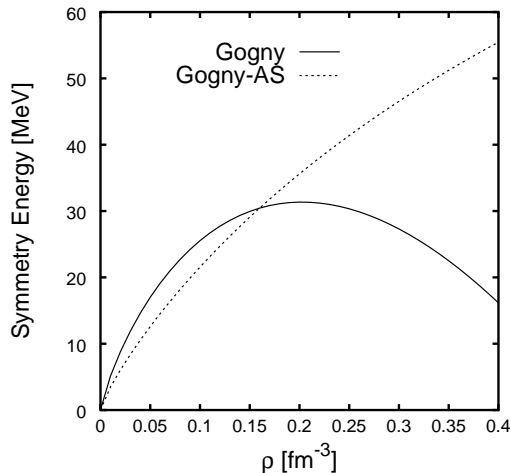
FIG. 1: Density dependence of the symmetry energy of nuclear matter for the Gogny force (solid line) and for the Gogny-AS force (dashed line).

seems to be strongly expanding, one-dimensionally, in the beam direction.

For the intermediate states, we define the liquid part as the part of the system to be composed of the fragments with $A > 4$ and any two wave packets whose spatial separation is less than 3 fm are treated as belonging to the same fragment. In the context of the results of Ref. [4], Fig. 2 shows the time evolution of the isospin asymmetry $(Z/A)^2$ of the liquid part for the three reaction systems. At the initial value $(t \sim 0)$, $(Z/A)^2_{\text{liq}}$ is $(Z/A)^2$ of the initial nuclei. For the neutron-rich systems, $(Z/A)^2_{\text{liq}}$ increases rapidly before $t \sim 100$ fm/c, and then it continues to increase gradually. This effect can be regarded as the isospin fractionation because the liquid part is getting less neutron-rich and the gas part is getting more neutron-rich. Similar fractionation effects are found in other dynamical model simulations [10, 13]. The diamond points in Fig. 2 are the results obtained from the Boltzmann-Uehling-Uhlenbeck calculations [12] with an interaction symmetry energy of $12.125(\rho/\rho_0)$ MeV. The isospin fractionation has a clear dependence on the asymmetry term of the effective force. The Gogny force (solid lines) always yields a system with larger $(Z/A)^2_{\text{liq}}$ than the Gogny-AS force (dashed lines). The complementary effect of fractionation can also be found in the gas-phase information, such as the neutron and proton emission rates in Fig. 3. While for the symmetric system $(^{40}\text{Ca} + {}^{40}\text{Ca})$ more protons are emitted than neutrons, because of the Coulomb force, for the very neutron-rich system $(^{60}\text{Ca} + {}^{60}\text{Ca})$
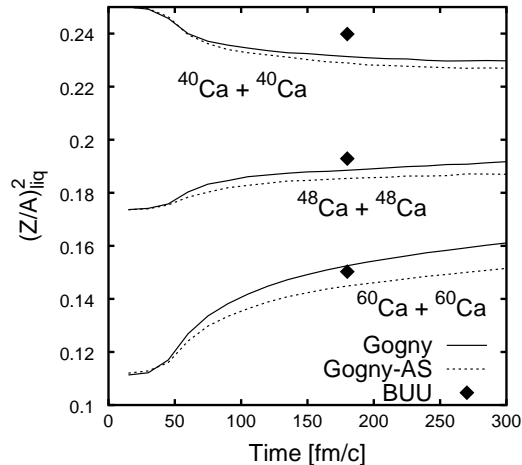
FIG. 2: $(Z/A)^2$ of the liquid part of the system as a function of time for the three reactions systems. The AMD results are represented by the solid and dashed lines, respectively, for the Gogny and Gogny-AS forces. Late-time BUU results are represented by filled diamonds.

much more neutrons are emitted.

Since fragments are produced in a rapidly evolving system in the AMD simulation, it is not evident *a priori* whether the isoscaling [Eq. (1)] is expected in the fragment yield ratio. Nevertheless, when we plot the fragment yield ratio, $Y_2(N,Z)/Y_1(N,Z)$, between two reaction systems, we observe a clear isoscaling relation, as shown in Figs. 4 and 5 obtained for the fragments present at $t = 300$ fm/c. The extracted scaling parameters α and β are provided in the figure captions and indicated in individual panels. The isoscaling parameters depend on the asymmetry term of the effective force. The fitting parameters, α and β, are of larger magnitude when the Gogny force is used. Moreover, α increases with increased differences in the asymmetry of the two systems.

Given that neutron emission costs less energy in a more neutron-rich system and proton emission costs more, it is not surprising that the isospin fractionation is observed in the dynamical simulations. However, the isoscaling is a nontrivial result, difficult to explain outside of statistical considerations. To explore the aspect of equilibrium in fragment emission in AMD simulations, we further explore the relation between the isoscaling and the fragment isospin asymmetry in equilibrium and in simulations.

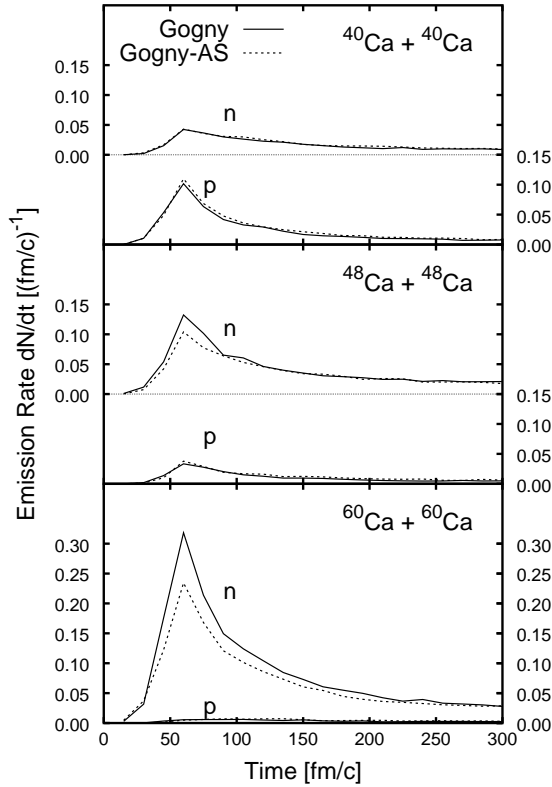In the context of the expanding emitting source model [14], it has been pointed out [4]

FIG. 3: Neutron and proton emission rates described by the left- and right-hand scales, respectively, for the three reaction systems as a function of time. The results of AMD simulations with the Gogny and Gogny-AS forces are, respectively, represented by the solid and dashed lines.

that the isoscaling parameter α, obtained from the yield ratio of the emitted fragments, is related to the $(Z_i/A_i)^2$ of an equilibrated emitting source by

$$\alpha = 4C_{\text{sym}}[(Z_1/A_1)^2 - (Z_2/A_2)^2]/T, \tag{3}$$

where C_{sym} is the symmetry energy and T is the source temperature. However, in the AMD simulations of collisions there are no easily identifiable emitting sources. All fragments are emitted on about equal footing. To remedy the situation, we show that, for an equilibrated system, Eq. (3) holds also when Z_i/A_i is replaced by $Z/\bar{A}_i$, where $\bar{A}_i$ is the average mass number for fragment charge number Z in system i, provided that fragment properties change gradually with nucleon content.

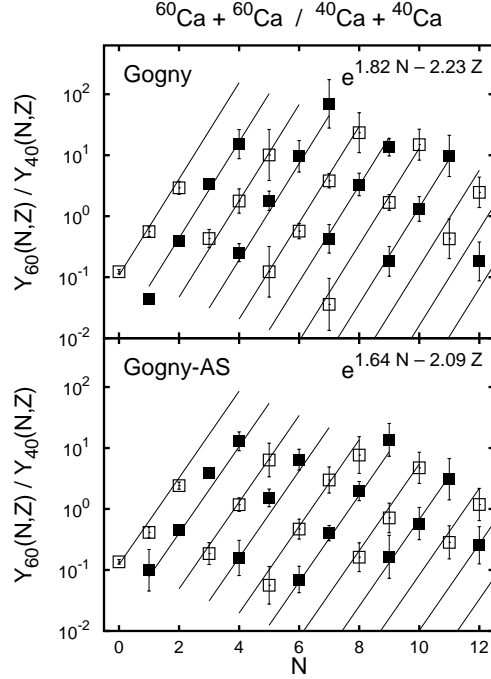When we consider a system in equilibrium at the temperature T and pressure P, the

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FIG. 4: The fragment yield ratio between the AMD simulations of central ^{60}Ca $+$ ^{60}Ca and ^{40}Ca $+$ ^{40}Ca collisions at 35 MeV/nucleon, at time $t = 300$ fm/c. The top and bottom panels show, respectively, the results obtained using the Gogny and Gogny-AS forces. The extracted isoscaling parameters are $\alpha = 1.82 \pm 0.06$ and $\beta = -2.23 \pm 0.08$ for the Gogny force, and $\alpha = 1.64 \pm 0.05$ and $\beta = -2.09 \pm 0.07$ for the Gogny-AS force.

number (or yield) of nucleus composed of N neutrons and Z protons is given by

$$Y_i(N, Z) = Y_{0i} \exp\left[-\left(G_{\text{nuc}}(N, Z) - \mu_{ni}N - \mu_{pi}Z\right)/T\right], \tag{4}$$

where the index i specifies the reaction system, with the total neutron and proton numbers N_i^{tot} and Z_i^{tot}, and $G_{\text{nuc}}(N, Z)$ stands for the internal Gibbs free energy of the (N, Z) nucleus. The net Gibbs free energy G_{tot} for the system is related to the chemical potentials μ_{ni} and μ_{pi} by $G_{\text{tot}} = \mu_{ni}N_i^{\text{tot}} + \mu_{pi}Z_i^{\text{tot}}$. In Eq. (4) and the following equations, we suppress the (T, P) dependence for different quantities including G_{nuc}, μ_{ni} and μ_{pi}. It is clear that isoscaling [Eq. (1)] is satisfied for Eq. (4), with $\alpha = (\mu_{n2} - \mu_{n1})/T$ and $\beta = (\mu_{p2} - \mu_{p1})/T$, as long as the two systems have common temperature and pressure.

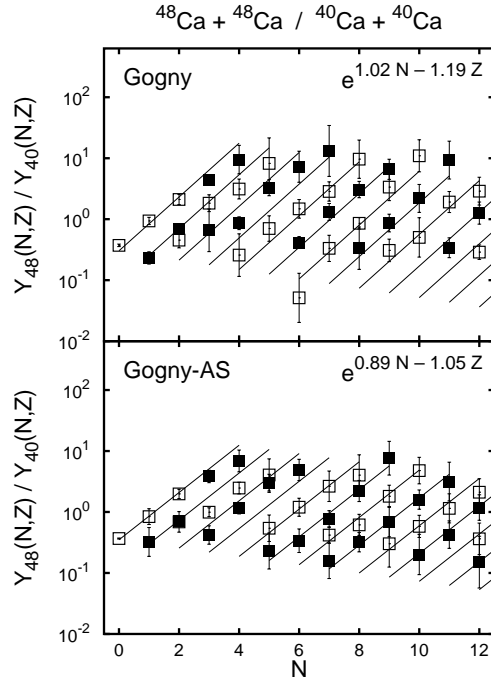For each given Z, the dependence of G_{nuc} on N, assuming gradual changes, takes the

FIG. 5: The same as Fig. 4 but for the ^{48}Ca + ^{48}Ca and ^{40}Ca + ^{40}Ca collisions. The extracted isoscaling parameters are $\alpha = 1.02 \pm 0.05$ and $\beta = -1.19 \pm 0.06$ for the Gogny force, and $\alpha = 0.89 \pm 0.04$ and $\beta = -1.05 \pm 0.05$ for the Gogny-AS force.

form

$$G_{\text{nuc}}(N, Z) = a(Z)N + b(Z) + C(Z)(N - Z)^2/A. \tag{5}$$

Because the important range of N is limited for a given Z, this expansion is practically sufficient even when G_{nuc} contains surface terms, Coulomb terms, and any other terms which are smooth in A as *e.g.* the term $\tau T \ln A$ introduced by Fisher [15]. We can regard $C(Z)$ as the symmetry energy coefficient in the usual sense, because the second order terms in N from the other terms are very small. This fact can be proved by a straightforward analytical calculation if a typical liquid-drop mass-formula is assumed as an example.

Let us consider the average neutron number $\bar{N}_i(Z)$ of each element Z. By identifying the average value with the maximum of the N distribution [Eq. (4)], we get

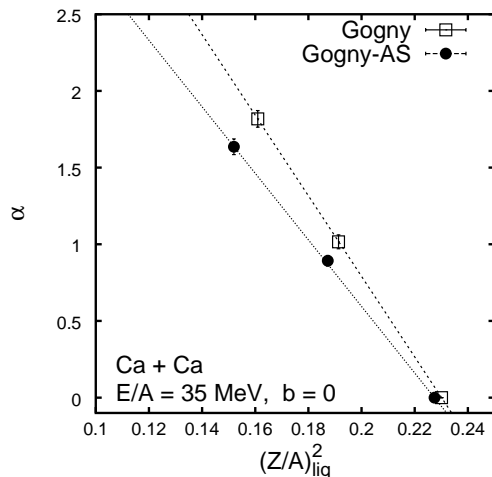$$(\partial/\partial N)[G_{\text{nuc}}(N, Z) - \mu_{ni}N - \mu_{pi}Z]|_{N=\bar{N}_i(Z)} = 0. \tag{6}$$

FIG. 6: Relation between $(Z/A)^2_{\text{liq}}$ and α for the three systems ^{60}Ca $+$ ^{60}Ca, ^{48}Ca $+$ ^{48}Ca and ^{60}Ca $+$ ^{60}Ca (from left to right). Open squares and filled circles show the results of AMD with Gogny force and Gogny-AS force, respectively. Both $(Z/A)^2_{\text{liq}}$ and α are calculated for fragments recognized at $t = 300$ fm/c. The straight lines are drawn so as to connect the points for ^{40}Ca$+$^{40}Ca and ^{60}Ca $+$ ^{60}Ca. Their slopes are -26.27 for Gogny force and -21.70 for Gogny-AS force.

A straightforward calculation, using the specific form of G_{nuc} of Eq. (5), results in

$$C(Z)\big[1 - 4\big(Z/\bar{A}_i(Z)\big)^2\big] = \mu_{ni} - a(Z), \tag{7}$$

with $\bar{A}_i(Z) = Z + \bar{N}_i(Z)$. The equations for the two reaction systems, $i = 1$ and $i = 2$, subtracted side by side then yield

$$\frac{\alpha}{\big(Z/\bar{A}_1(Z)\big)^2 - \big(Z/\bar{A}_2(Z)\big)^2} = 4C(Z)/T, \tag{8}$$

relating the isoscaling parameter α, the $(Z/A)^2$ of fragments and the symmetry energy coefficient $C(Z)$ which is a function of (T, P). An interesting fact is that this relation does not depend on the terms in G_{nuc} other than the symmetry energy term.

Both sides of Eq. (8) depend on the fragment charge Z, in principle, and can provide information of the surface effect in the symmetry energy. This issue will be pursued in a separate paper. In the present letter, we use the fact the Z-dependence of $Z/\bar{A}_i(Z)$ in simulations is weak for $Z \gtrsim 5$, making it meaningful to consider the isospin asymmetry of the liquid part $(Z/A)^2_{\text{liq}} \equiv (Z_{\text{liq}}/A_{\text{liq}})^2$ that has been averaged over all the fragments with

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$A > 4$. For this quantity we expect the relation

$$\frac{\alpha}{\left(Z/A\right)^2_{\text{liq},1} - \left(Z/A\right)^2_{\text{liq},2}} = 4C/T. \tag{9}$$

Let us check whether this equilibrium relation (9) is satisfied by the AMD simulations that do not incorporate any assumption of equilibrium. Figure 6 shows the correlation in the AMD simulations between $(Z/A)^2_{\text{liq}}$ and the isoscaling parameter α from Figs. 4 and 5 for the three reaction systems. A linear relation is observed in accordance to the equilibrium relation (9), suggesting applicability of statistical laws to the isospin composition of fragments. The extracted value of $4C/T$ depends on the time of the fragment recognition, but the dependence is weak as both α and $(Z/A)^2_{\text{liq},1} - (Z/A)^2_{\text{liq},2}$ are decreasing functions of t for $t \gtrsim 200$ fm/c. From the slope of the linear relation, we obtain $4C/T = 26.3$ for Gogny force and $4C/T = 21.7$ for Gogny-AS force. The ratio of $C(\text{Gogny})/C(\text{Gogny-AS}) \approx 5/4$, from Fig. 1, is consistent with the idea that fragmentation occurs at low density, $\rho < \rho_0$. The absolute values of C of 16-20 MeV and 22-26 MeV are reasonable assuming the temperatures $T \sim 3\text{-}4$ MeV.

In conclusion, a linear relation is expected between the isoscaling parameter and the fragment isospin asymmetry $(Z/A)^2$ under statistical assumptions. Isoscaling is observed in the dynamical AMD simulation and the results well comply with the linear relation, suggesting that the fragment isospin composition is subject to the statistical laws even in a dynamical picture of production. The slope of this linear function yields information on the symmetry energy for the fragments.

Acknowledgments

RIKEN as a nuclear theory project.

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