# Structure of Nuclei with Masses $A=\mathbf{3 0 - 3 5}$, as Calculated in the Shell Model 

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#### Abstract

Properties of positive-parity states of nuclei with $A=30-35$ have been calculated in a shellmodel space which encompasses all Pauli-allowed basis vectors of all configurations $(0 s)^{4}(0 p)^{12}\left(0 d_{5 / 2}\right)^{n_{1}}\left(1 s_{1 / 2}\right)^{n_{2}}\left(0 d_{3 / 2}\right)^{n_{3}}$ for which $n_{1} \geq 10$. Two different empirical Hamiltonians, one of a $\delta$-function form, were used. Calculated energies and spectroscopic factors are in good agreement with an extensive body of experimental data. The model wave functions also yield satisfactory agreement with many available experimental data on electric quadrupole observables if effective charges of $0.5 e$ are added to the proton and neutron. The model predictions for magnetic dipole observables are generally in qualitative agreement with experimental observations, but inconsistencies between theory and experiment are more noticeable in this area.


## I. INTRODUCTION

This paper describes the results of a series of shell-model calculations which have been carried out in an attempt to understand the structure of energy levels in nuclei of $A=30-35$. The properties of these nuclei present an attractive challenge to theoretical interpretation in that experimental information ${ }^{1}$ about spins and parities of observed levels in this mass region is extensive and there are also many data on lifetimes, spectroscopic factors, and the like. Thus, any theory for these nuclei can be critically examined with unusual thoroughness.
Despite all this experimental activity, there have been relatively few theoretical investigations of these nuclei. This probably results from the fact that experimental phenomena typical of the region do not readily yield to analysis by the simplest forms of any of the popular models for describing nuclear structure. The level structures of $A=30-$ 35 do not exhibit obvious rotational features to the extent found, for example, in $A=20-25$. Thus, the most straightforward Nilsson-type calculations for nuclei with $A \geqslant 30$ are not as successful as for the lighter nuclei of the $s d$ shell. ${ }^{2}$ However, more involved calculations of this kind have successfully accounted for some properties of individual nuclei. ${ }^{3}$ Similarly the simplest kind of weakcoupling vibrational calculation meets with little success in this region, ${ }^{4}$ although recent, more
sophisticated intermediate-coupling calculations ${ }^{5}$ appear promising.

A detailed shell-model calculation of the structure of these nuclei was first made by Glaudemans and his co-workers ${ }^{6}$ in a study which assumed an inert ${ }^{28} \mathrm{Si}$ core and active $1 s_{1 / 2}$ and $0 d_{3 / 2}$ orbits. They treated all nuclei from $A=29-40$, but the results obtained for the lighter nuclei were obviously impaired in some respects by the omission of an active $d_{5 / 2}$ orbit. This region was also studied via shell-model methods as part of a general survey of the $s d$ shell by Bouten, Elliot, and Pullen. ${ }^{7}$ By making simplifying assumptions about the Hamiltonians and the wave functions, they were able to include effects of some excitations out of the $d_{5 / 2}$ orbit. However, they presented and discussed only energy-level spectra.

The results of previous calculations, and examination of the experimental data, strongly suggest that any conventional shell-model calculation for the $A=30-35$ region should allow some excitation out of the $d_{5 / 2}$ orbit. As we shall discuss further below, there are no obvious indications from experiment that the orbits of the $0 f-1 p$ shell are necessary for an adequate description of the properties of the low-lying positive-parity states in the $A=30-35$ region. We have accordingly studied the properties of these nuclei by means of a conventional shell-model calculation in a vector space which includes basis states with particles in the $d_{5 / 2}, s_{1 / 2}$, and $d_{3 / 2}$ orbits. We shall show that a

TABLE I. Values of Hamiltonian parameters obtained by iterative fits of shell-model level energies to experimental level energies and the average deviations of the theoretical predictions from the experimental values. Here abs means average absolute deviation, while rms means root-mean-square deviation. Units are MeV.

| Calculations | $A_{0}$ | Parameters |  |  | Single-particle energies |  |  | Average deviations |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $A_{1}$ | $B_{0}$ | $B_{1}$ | $\epsilon_{5 / 2}$ | $\epsilon_{1 / 2}$ | $\epsilon_{3 / 2}$ | abs/rms | abs/rms |
| $A=30-34 \mathrm{MSDI}$ | 0.646 | 0.906 | -1.470 | 0.770 | $-7.56$ | -6.03 | -3.96 | 0.170/0.220 | 0.320/0.410 |
| $A=30-34$ FPSDI | 0.939 | 0.711 | -1.598 | 0.866 | -7.74 | -4.78 | -2.82 | 0.190/0.220 | 0.200/0.250 |
| $A=30-33 \mathrm{MSDI}$ | 0.892 | 0.873 | -1.071 | 0.708 | $-7.55$ | -5.73 | -3.32 |  |  |
| $A=17-22 \mathrm{MSDI}$ | 0.774 | 0.954 | -2.503 | 0.370 | -4.49 | -3.16 | +1.04 |  |  |
| $A=35-39 \mathrm{MSDI}$ | 0.379 | 0.960 | -1.818 | 0.461 | -6.82 | -2.00 | +0.65 |  |  |

TABLE II. Energies $E$ and spectroscopic factors $S$ for states of $A=31, T=\frac{1}{2}\left({ }^{31} \mathrm{P}-{ }^{31} \mathrm{~S}\right)$. The nuclear binding energies, relative to ${ }^{16} \mathrm{O}$, are listed for the ground state, and excitations relative to these ground-state energies are listed for the excited states. Asterisks indicate experimental energies used in fixing the parameters of the Hamiltonians. Footnotes $a, b$, etc., apply to the entire column of entries under the footnoted entry. Experimental energies enclosed in parentheses indicate the lack of a firm experimental spin-parity assignment. The target state for the stripping transfers $\left(S_{\text {st }}\right)$ is $A=30, T=1, J^{\pi}=0^{+}\left({ }^{30} \mathrm{Si}\right)$, and the target state for the pickup transfers $\left(S_{\mathrm{pk}}\right)$ is $A=32, T=0, J^{\pi}=0^{+}\left({ }^{32} \mathrm{~S}\right)$.

| $A J T$ | Energy $E$ ( MeV ) |  |  |  | $\left(100 \times S_{\text {st }}\right) /\left(100 \times S_{\text {pk }}\right)$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Expt. | FPSDI | MSDI |  |  | Expt. |  |  | FPSDI | MSDI |
| $31 \frac{1}{2} \frac{1}{2}$ | 168.50* | 168.63 | 168.68 | $70^{\text {a }}$ | $68^{\text {b }} / 174{ }^{\text {c }}$ | $208{ }^{\text {d }}$ | $180^{\text {e }}$ | $220{ }^{\text {f }}$ | 109/257 | 97/226 |
| $31 \frac{1}{2} \frac{1}{2}$ | 3.13* | 3.40 | 3.63 | 3 | 2/ |  |  |  | 4/3 | 8/2 |
| $31 \frac{1}{2} \frac{1}{2}$ | 5.25 | 4.30 | 3.97 | 5 | 5/ |  |  | 40 | 1/0 | 1/0 |
| $31 \frac{1}{2} \frac{1}{2}$ | 6.41 | 5.42 | 5.01 |  |  |  |  | 64 | 3/30 | 2/9 |
| $31 \frac{3}{2} \frac{1}{2}$ | 1.27* | 1.24 | 1.23 | 72 | 68/110 | 188 | 220 | 136 | 76/120 | 78/162 |
| $31 \quad \frac{3}{2} \quad \frac{3}{2}$ | 3.51* | 3.81 | 3.93 | $\leq 1$ | <1/ |  |  |  | 18/3 | 12/1 |
| $31 \frac{3}{2} \quad \frac{1}{2}$ | 4.26* | 4.18 | 4.55 | 1 | 4/ |  |  |  | 0/0 | 0/1 |
| $31 \frac{3}{2} \frac{1}{2}$ | 4.59 | 4.77 | 4.69 | 3 | $2 /$ |  |  |  | 1/8 | 6/5 |
| $31 \frac{5}{2} \frac{1}{2}$ | 2.23* | 2.41 | 2.48 | 7 | 6/480 | 554 | 580 | 304 | 4/490 | 6/455 |
| $31 \frac{5}{2} \frac{1}{2}$ | 3.29* | 2.96 | 2.84 | $\leq 1$ | $\leq 1 /$ | 146 | 160 | 106 | $0 / 5$ | $0 / 1$ |
| $31 \frac{5}{2} \frac{1}{2}$ | 4.19* | 4.35 | 4.70 | 3 | / | 172 |  | 106 | $0 / 1$ | 1/33 |
| $31 \frac{5}{2} \frac{1}{2}$ | 4.78 | 4.61 | 4.91 | $<1$ | 1 |  |  | 46 | 0/29 | 0/11 |
| $31 \frac{7}{2} \frac{1}{2}$ | 3.41* | 3.63 | 3.65 |  |  |  |  |  |  |  |
| $31 \frac{7}{2} \frac{1}{2}$ |  | 3.91 | 4.09 |  |  |  |  |  |  |  |
| $31 \frac{7}{2} \frac{1}{2}$ |  | 4.83 | 5.53 |  |  |  |  |  |  |  |
| $31 \frac{7}{2} \frac{1}{2}$ |  | 5.32 | 5.73 |  |  |  |  |  |  |  |
| $31 \frac{9}{2} \frac{1}{2}$ |  | 4.85 | 5.02 |  |  |  |  |  |  |  |
| $31 \frac{9}{2} \frac{1}{2}$ |  | 5.26 | 5.64 |  |  |  |  |  |  |  |
| $31 \frac{11}{2} \frac{1}{2}$ |  | 5.94 | 6.42 |  |  |  |  |  |  |  |
| $31 \frac{13}{2} \frac{1}{2}$ |  | 7.63 | 7.66 |  |  |  |  |  |  |  |


| a Reference 27. | ${ }^{\text {c Reference 29. }}$ | e Reference 31. |
| :--- | :--- | :--- |

[^0]great many details of the experimental properties of low-lying levels in all the nuclei from $A=30-$ 35 are reproduced successfully by this one model. In addition, the over-all qualitative features of the spectra, which are not easily describable in terms of any simple rotational or vibrational model, readily appear. We wish to emphasize the consistency and comprehensiveness of the results we will present. We treat eleven different mass-isospin systems simultaneously with mass-independent Hamiltonians. We calculate for these nuclei a variety of physical observables under assumptions for the various effective operators that are also held constant for the entire study and are completely state independent.

## II. DESCRIPTION OF THE CALCULATION

## A. Model Space

We wished to use a fixed truncation scheme for all of the nuclei $A=30-35$. The vector space we chose is spanned by all Pauli-allowed $(0 s)^{4}(0 p)^{12}\left(0 d_{5 / 2}\right)^{n_{1}}\left(1 s_{1 / 2}\right)^{n_{2}}\left(0 d_{3 / 2}\right)^{n_{3}}$ states which satisfy the condition that $n_{1}$, the number of particles occupying the $0 d_{5 / 2}$ orbit, is greater than or equal to 10. The choice of this particular truncation scheme was founded in part on physical considerations (two-particle excitations out of the $0 d_{5 / 2}$ orbit were considered a minimal expansion of the $1 s_{1 / 2}-0 d_{3 / 2}$ space towards a satisfactory basis), and in part on technical feasibility (a general inclusion of three or more particle excitations out of the $d_{5 / 2}$ orbit would have led to energy matrices too large for present capabilities).

It is possible that there are states mainly characterized by four-particle excitations of the $0 d_{5 / 2}$ orbit which might appear at the same excitation energy as some states formed mainly with the mod-el-allowed two-particle excitations. We think that the most serious limitation of our vector space is this omission of $s d$-shell states having fewer than 10 nucleons in the $d_{5 / 2}$ orbit.
The exclusion of $0 p$ and $1 p, 0 f$ excitations from the model space appears to be a good approximation. The evidence for this comes from pickup and stripping experiments. Single-nucleon stripping data ${ }^{1}$ indicate that the lowest $0 f_{7 / 2}-$ and $1 p_{3 / 2}$-type states in the $A=31-35$ even-odd nuclei occur at excitation energies higher than 3 MeV . Pickup experiments do not reveal any $0 p$ holes in levels below 6 MeV . For the nuclei we consider here, the situation with respect to states which appear to have their origins in configurations outside the $s d$-shell space is quite favorable in comparison to what occurs in the lighter and heavier nuclei of the $s d$ shell. In the $A=18$ and $A=38$ systems, for example, positive-parity states coming as low as 3 MeV in excitation energy appear to have their origins entirely in two-particle, four-particle, etc., configurations of the adjacent negative-parity orbits.

## B. Form of Effective Hamiltonian

Because of the model space we use, it is difficult to obtain the one- and two-body matrix elements of the effective Hamiltonian operator by either of two popular current methods. One of


FIG. 1. Plots of the 63 two-body matrix elements of each of the $s d$-shell interactions used in the present study (MSDI and FPSDI), together with an interaction calculated by Kuo's techniques from the Hamada-Johnston nucleon-nucleon potential. The matrix elements $\left\langle\left(d_{5 / 2} s_{1 / 2}\right)_{J, T}\right| V\left|\left(d_{5 / 2} d_{3 / 2}\right)_{J, T}\right\rangle$ are labeled by the symbol 5153 , and so on. Within each such group, the order follows the sequence of increasing values of the $J, T$ pair: For example, the 5555 matrix elements are ordered $(J=0, T=1),(J=1, T=0),(J=2, T=1),(J=3, T=0),(J=4, T=1),(J=5, T=0)$.
these methods involves the calculation of the twobody matrix elements from free nucleon-nucleon interactions by reaction-matrix methods. Most such calculations in the literature ${ }^{8,9}$ are aimed at obtaining the effective interaction for two particles (or two holes) outside an inert core. It is well known that renormalization effects arising from relatively low-lying core excitations are extremely important in these calculations. There are problems in using such interactions in treating systems of many particles or holes, or in a truncated space such as we use here. First, the calculated interactions are most applicable for nuclei at the beginning (or the end) of the shell, where, indeed, they have been used with quite some success. ${ }^{10,11}$ Some of the approximations of the technique (e.g., neglect of effective three-body forces) may be applicable in few-nucleon or fewhole calculations, but not applicable in calculations for the middle of the shell. In addition, while these reaction-matrix interactions are re-
normalized to compensate for the omission of excitations into (or out of) orbits absent from the active model space, this is not the case for the omission of the configurations involving orbits that can be partially occupied for some basis states within the model space. Since we restrict ourselves to configurations with at least 10 particles in the $d_{5 / 2}$ shell, we omit many configurations involving a less completely filled $d_{5 / 2}$ shell. There is very little experience which suggests how to calculate renormalizations to make up for such omissions.
A second popular method of obtaining effective interactions is to treat the two-body matrix elements as free parameters, ${ }^{6,12}$ these being determined so as to give a best fit between calculated eigenvalues and observed level energies. In this way, the interaction is empirically renormalized to compensate for the omission of configurations not included in the active model space. In the three-shell model space we use, there are 63 twobody matrix elements. For the nuclei we treat,

| $\frac{5.25}{5.12}$ | $1 / 2+$ |
| :--- | :--- |
| $\frac{4.78}{-4.59-4.64}$ | $5 / 2+$ |
| $\frac{3 / 2+}{4.43}$ | $7 / 2-$ |
| $-4.19 \underline{4.26}$ | $5 / 2+3 / 2+$ |


| $\frac{3.51}{-3.29}-3.41$ | $3 / 2+$ |
| :--- | :--- |
| $-3 / 2+$ |  |
| 3.13 |  |
|  | $1 / 2+$ | 2.23 5/2+



FIG. 2. Observed and calculated spectra for $A=31, T=\frac{1}{2}$ (see Table II). The experimental spectrum is taken from data on ${ }^{31} \mathrm{P}$. All observed levels are plotted up to the maximum energy indicated, with the exception that all known nega-tive-parity levels are omitted except for the lowest one. Only four levels for each spin have been calculated. All of these that fall within the indicated range are plotted. The states of the model spectra which correspond to the observed ground state have been aligned with it in energy.
there are not enough energy levels with known spins and parities to determine accurately such a large number of parameters. Even if there were sufficient data, the search procedure would be difficult.
Because of these difficulties with the above described procedures, we decided to use, as matrix elements for the two-body part of the effective interaction, the matrix elements $\left\langle j_{a} j_{b} J T\right| V\left|j_{c} j_{d} J T\right\rangle$ of the surface $\delta$ interaction (SDI), modified with isospin-dependent monopole terms in order to fit binding energies. We shall refer to this interaction as the modified surface $\delta$ interaction (MSDI). The use of this effective interaction in finite-nucleus calculations has been discussed by various authors. ${ }^{13-16}$ Our main reason for the use of the MSDI in the present study is that it has the very desirable feature of providing a parametrization of the model Hamiltonian which has a small number of degrees of freedom.
The MSDI is expressed as,

$$
V_{T}(i j)=-4 \pi A_{T} \delta\left(r_{i}-r_{j}\right) f_{i j}+B_{T},
$$

where $T$ indicates the isospin ( 0 or 1 ) of the interacting nucleon pair, $A_{T}$ and $B_{T}$ are strengths depending only on $T$, and $f_{i j}$ is an operator which has the following effect on the radial part of

$$
\begin{aligned}
& \left\langle j_{a} j_{b} J T\right| \\
& \left.\int_{0}^{\infty} \int_{0}^{\infty} j_{c} j_{d} J T\right\rangle: \\
& r_{i}^{2} d r_{i} r_{j}^{2} d r_{j} R_{a}\left(r_{i}\right) R_{b}\left(r_{j}\right) \\
& \quad \times \delta\left(r_{i}-r_{j}\right) f_{i j} R_{c}\left(r_{i}\right) R_{d}\left(r_{j}\right)=(-1)^{n_{a}+n_{b}+n_{c}+n_{d}} .
\end{aligned}
$$

Here $R_{a}, R_{b}, R_{c}$, and $R_{d}$ are the single-particle radial wave functions involved in

$$
\left\langle j_{a} j_{b} J T\right| V\left|j_{c} j_{d} J T\right\rangle ;
$$

and $n_{a}$, etc., refer to the principal quantum numbers of the orbits associated with $j_{a}$, etc. Our conventions are that the lowest orbit of given $l$ has principal quantum number zero, and that the radial functions $R_{a}$ are positive at the origin. With the assumption of the MSDI and with the parametrization technique we used, the 63 two-body matrix elements of the $s d$-shell Hamiltonian are specified by only four parameters. Together with the three single-particle energies, this leads to a total of seven parameters to specify the complete effective Hamiltonian, which we shall also refer to as MSDI.

Optimized values of these seven parameters were obtained by iteratively adjusting them with the code SMIT ${ }^{17}$ so as to produce a least-squares fit to 66 data on levels with experimentally as signed $J, \pi$, and $T$ in nuclei with $A=30-34$. These data comprized 11 ground-state binding energies (of systems with specific $A, T$ ) and 55 excitation energies (relative to the lowest energy state of the same $A, T$ ). The shell-model matrices were constructed with the aid of the Oak Ridge-Rochester computer code. ${ }^{18}$ The energy-level data used in this procedure are indicated in the tables of results by asterisks. The experimental ground-state binding energies are given relative to ${ }^{16} \mathrm{O}$, with the Coulomb energies removed by the technique described by Glaudemans, Wiechers, and Brussaard. ${ }^{6}$


FIG. 3. Diagrams of observed and calculated spectroscopic factors for single-nucleon transfer to ${ }^{31} \mathrm{P}$ or ${ }^{31} \mathrm{~S}$. The information for these drawings is taken from Table II (Refs. 27 and 32). States with negligible particle or hole strength are not shown. The horizontal scale is such that a bar for $S=1.0$ has the length equivalent to 1 MeV on the vertical scale. $S_{\mathrm{pk}}$ denotes pickup $S$ factors and $S_{\mathrm{st}}$ denotes stripping $S$ factors.

It should be emphasized that the two monopole constants $B_{T}$ of the MSDI, and one of the three singleparticle energies, contribute only to the binding energy of one $A, T$ system relative to another. The excitation spectrum within a given system is determined only by the two SDI strengths $A_{T}$ and the two single-particle energy splittings.

The values which were obtained for the MSDI Hamiltonian parameters in the iterative search, and the final average deviations of the calculated eigenvalues from experimental energies, are listed in Table I. A plot of the individual two-body matrix elements is shown in Fig. 1. The MSDI parameters we obtain here differ somewhat from the values (see Table I) obtained in a fit to data restricted to $A=30-33$, as published ${ }^{19}$ in a preliminary report of this calculation. However, the eigenvalue-eigenvector spectra are generally quite insensitive to the change from the older set of parameters to the present set. The new $A=30-34$ parameters happen to more closely resemble the values obtained in fitting the MSDI parameters to
energy-level data at the lower and upper ends of the $s d$ shell $(A=17-22$ and $A=35-39)$ in shellmodel calculations ${ }^{10,11}$ with unrestricted $s d$-shell bases.

The structure predictions based upon the MSDI Hamiltonian turned out to yield remarkably good agreement with the large majority of data in the region of interest. Still, the MSDI is a very constrained and simplified effective interaction, and there are examples (e.g., in $A=38$ and also in $A$ $=18-22$ ) in which its use ${ }^{10,11,20}$ leads to disagreements with experiments for specific aspects of nuclear structure. Hence, we have developed an alternative effective Hamiltonian for this same series of nuclei. This second Hamiltonian was derived by taking the final MSDI Hamiltonian as a starting point and then treating the two-body matrix elements which do not involve the $d_{5 / 2}$ orbit ( 15 in all) as independent free parameters in a least-squares search to fit experimental level energies. The remaining two-body matrix elements were readjusted in the four-parameter MSDI for-


FIG. 4. Observed and calculated spectra for $A=31, T=\frac{3}{2}$ (see Table III). The conventions of the presentation are explained in the caption to Fig. 2.
mat, and the three single-particle energies were also readjusted. We label the resulting 22 -parameter Hamiltonian FPSDI, indicating both its freeparameter aspects and its heritage from the strict MSDI form. The underlying idea in using the FPSDI form is that the $s_{1 / 2}$ and $d_{3 / 2}$ orbits are more immediately involved than is $d_{5 / 2}$ in determining the level structure of nuclei in this mass region, and that it is most important to have the best possible values for the two-body matrix elements involving the $d_{3 / 2}$ and $s_{1 / 2}$ orbits.

To determine the FPSDI Hamiltonian matrix elements, the same data were used in the iterative search as for the case of the MSDI. The increase in the number of free parameters from 7 to 22 (resulting in a ratio of data points to parameters of 3 to 1 ) significantly slowed the rate of convergence toward a stable interaction. The one- and twobody matrix elements from which the present results are computed are not the result of a completely converged search, since the iterative chain was cut off after three cycles. From an examination of the changes in matrix elements and
calculated results after each of the three iterative steps, we estimate that further iterations would introduce changes in excitation energies and wave function amplitudes of less than $5 \%$.

The MSDI part of the FPSDI interaction is quite similar to that obtained for the pure MSDI fit (see Table I). The FPSDI single-particle energies differ from those of the MSDI such that the $1 s_{1 / 2}$ and $0 d_{3 / 2}$ orbits are less bound, relative to $0 d_{5 / 2}$, by approximately 1 MeV . The average agreement (see Table I) of the FPSDI-calculated ground-state binding energies with the experimental values is no better than that obtained with the MSDI. This indicates that the degrees of freedom allowed by the MSDI suffice to give quite good agreement for binding energies. ${ }^{21}$ However, the FPSDI average deviations from experimental excited-state energies are about half of those obtained from the MSDI. Even after account is taken of the increased number of parameters in the fit, the improvement is still appreciable. The FPSDI matrix elements are compared with the MSDI values in Fig. 1.
We have plotted, for further comparison, a third

TABLE III. Energies $E$ and spectroscopic factors $S$ for states of $A=31, T=\frac{3}{2}\left({ }^{31} \mathrm{Si}\right)$. The conventions of the presentation are explained in the caption to Table II. The target state for the stripping transfers $\left(S_{\mathrm{st}}\right)$ is $A=30, T=1, J^{\pi}=0\left({ }^{30} \mathrm{Si}\right)$.

| $A J T$ | Energy E (MeV) |  |  | $100 \times S_{\text {st }}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Expt. | FPSDI | MSDI | Expt. | FPSDI | MSDI |
| $31 \frac{1}{2} \frac{3}{2}$ | 0.75* | 0.92 | 0.80 | $25^{\text {a }} 23^{\text {b }}$ | 14 | 19 |
| $31 \frac{1}{2} \frac{3}{2}$ | 4.72* | 4.14 | 3.94 |  | 5 | 1 |
| $31 \frac{1}{2} \frac{3}{2}$ | 5.26* | 5.00 | 4.44 |  | 0 | 3 |
| $31 \frac{1}{2} \frac{3}{2}$ |  | 5.50 | 5.26 |  | 0 | 0 |
| $31 \frac{3}{2} \frac{3}{2}$ | 162.16* | 162.32 | 162.28 | 8652 | 72 | 71 |
| $31 \quad \frac{3}{2} \quad \frac{3}{2}$ | 2.32* | 2.27 | 2.42 | 5 | 2 | 1 |
| $31 \frac{3}{2} \frac{3}{2}$ | 4.26* | 3.81 | 3.33 |  | 6 | 6 |
| $31 \frac{3}{2} \frac{3}{2}$ |  | 4.72 | 3.92 |  | 0 | 1 |
| $31 \frac{5}{2} \frac{3}{2}$ | 1.69* | 1.89 | 1.82 | 2 | 0 | 0 |
| $31 \frac{5}{2} \frac{3}{2}$ | 2.79* | 2.54 | 3.10 | 4 | 2 | 1 |
| $31 \frac{5}{2} \frac{3}{2}$ |  | 4.02 | 3.39 |  | 0 | 0 |
| $31 \frac{5}{2} \frac{3}{2}$ |  | 4.64 | 4.33 |  | 0 | 0 |
| $31 \quad \frac{7}{2} \quad \frac{3}{2}$ |  | 3.71 | 3.53 |  |  |  |
| $31 \frac{7}{2} \frac{3}{2}$ |  | 4.32 | 3.74 |  |  |  |
| $31 \frac{7}{2} \frac{3}{2}$ |  | 5.13 | 4.63 |  |  |  |
| $31 \frac{7}{2} \frac{3}{2}$ |  | 5.45 | 5.25 |  |  |  |
| $31 \frac{9}{2} \frac{3}{2}$ |  | 4.19 | 4.52 |  |  |  |
| $31 \frac{9}{2} \frac{3}{2}$ |  | 5.30 | 5.03 |  |  |  |
| $31 \frac{11}{2} \frac{3}{2}$ |  | 6.92 | 6.12 |  |  |  |
| $31 \frac{13}{2} \frac{3}{2}$ |  | 8.02 | 7.92 |  |  |  |

[^1]TABLE IV. Values of $B(E 2)$ and $B(M 1)$ for transitions between states of $A=31$. The experimental values are calculated from the lifetimes, branching ratios, and mixing ratios tabulated by Glaudemans, Endt, and Dieperink (Ref. 26). In cases of unkown $E 2 / M 1$ mixing, we have assumed that the $B(E 2)$ value is 20 Weisskopf units (W.u.) and so extracted a lower limit on the $B(M 1)$. The column of calculated $B(E 2)$ values headed FPSDI results from the FPSDI wave functions with the assumptions of $\tilde{e}_{p}=1.5 e, \tilde{e}_{n}=0.5 e$, and harmonic oscillator wave functions for $\hbar \omega=41 A^{-1 / 3} \mathrm{MeV}$. The column headed MSDI results from the MSDI wave functions and the same assumptions for effective charge and radial wave functions. The column entitled MSDI-A is taken from Ref. 26; it results from the MSDI wave functions and the assumption of $\tilde{e}_{p}=1.44 e, \tilde{e}_{n}=0.68 e$. The $B(M 1)$ columns entitled FPSDI and MSDI result from using these sets of wave functions and free-nucleon $g$ factors. The MSDI-A column is taken from Ref. 26 ; it results from changing the values of the isovector single-particle matrix elements $\left\langle s_{1 / 2}\left\|\overrightarrow{\mathrm{M}}^{1}\right\| s_{1 / 2}\right\rangle,\left\langle d_{3 / 2}\left\|\overrightarrow{\mathrm{M}}^{1}\right\| d_{3 / 2}\right\rangle$, and $\left\langle s_{1 / 2}\left\|\overrightarrow{\mathrm{M}}^{1}\right\| d_{3 / 2}\right\rangle$ from their free-nucleon values of $(6.90,-1.58$, and 0$) \mu_{N}$ to $(4.4,0.5$, and 0.4$) \mu_{N}$ respectively for $A=30-32$, and to (4.1, -0.3 , and -0.2$) \mu_{N}$, respectively, for $A=33$ and 34 . The transitions are identified by the $J, T$, and the experimentally measured excitation energy (in units of MeV ) of the initial and final states. The model states which are to be matched with these experimental energies are noted in the tables which list energies and spectroscopic factors. Zeroes in the tables indicate $J$-forbidden transition; three dots indicate no calculation was made for the transition in question.

| Nucleus | Initial state |  | Final state |  |  | $B(E 2)\left(e^{2} \mathrm{~F}^{4}\right)$ |  |  |  |  | $100 \times B(M 1)\left(\mu_{N}{ }^{2}\right)$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $J T$ | $E$ | $J$ | $T$ | E | Expt. ${ }^{\text {a }}$ | FPSDI | MSDI | MSDI- ${ }^{\text {a }}$ | Expt. ${ }^{\text {a }}$ | FPSDI | MSDI | MSDI-A ${ }^{\text {a }}$ |
| ${ }^{31} \mathrm{P}$ | $\frac{3}{2} \quad \frac{1}{2}$ | 1.27 | $\frac{1}{2}$ | $\frac{1}{2}$ | 0.00 | $23 \pm 4$ | 27 | 27 | 31 | $3.3 \pm 0.2$ | 2.2 | 2.5 | 2.7 |
| ${ }^{31} \mathrm{P}$ | $\frac{5}{2} \frac{1}{2}$ | 2.23 | $\frac{1}{2}$ |  | 0.00 | $36 \pm 2$ | 21 | 12 | 13 | 0 | 0 | 0 | 0 |
| ${ }^{31} \mathrm{P}$ | $\frac{5}{2} \frac{1}{2}$ | 2.23 | $\frac{3}{2}$ |  | 1.27 | $<30$ | 1.7 | 2.4 | 2.3 | <0.19 | 0.6 | 0.0 | 0.4 |
| ${ }^{31} \mathrm{P}$ | $\frac{1}{2} \frac{1}{2}$ | 3.13 | $\frac{1}{2}$ |  | 0.00 | 0 | 0 | 0 | 0 | $18 \pm 1$ | 47 | 44 | 29 |
| ${ }^{31} \mathrm{P}$ | $\frac{1}{2} \frac{1}{2}$ | 3.13 | $\frac{3}{2}$ | $\frac{1}{2}$ | 1.27 | <68 | 9.4 | 10 | 13 | <1.6 | 20 | 6.4 | 0.5 |
| ${ }^{31} \mathrm{P}$ | $\frac{1}{2} \frac{1}{2}$ | 3.13 | $\frac{5}{2}$ |  | 2.23 | -•• | 2.0 | -•• | -•• | 0 | 0 | 0 | 0 |
| ${ }^{31} \mathrm{P}$ | $\frac{5}{2} \quad \frac{1}{2}$ | 3.29 | $\frac{1}{2}$ | $\frac{1}{2}$ | 0.00 | <0.2 | 6.0 | 11 | 14 | 0 | 0 | 0 | 0 |
| ${ }^{31} \mathrm{P}$ | $\frac{5}{2} \frac{1}{2}$ | 3.29 | $\frac{3}{2}$ | $\frac{1}{2}$ | 1.27 | $22 \pm 4$ | 11 | 9.4 | 10 | $3.7 \pm 0.4$ | 15 | 17 | 2.9 |
| ${ }^{31} \mathrm{P}$ | $\frac{5}{2} \frac{1}{2}$ | 3.29 | $\frac{5}{2}$ |  | 2.23 | <20 | 24 | - $\cdot$ | 20 | $5.4 \pm 0.9$ | 2.9 | 5.5 | 13 |
| ${ }^{31} \mathrm{P}$ | $\frac{7}{2} \frac{1}{2}$ | 3.41 | $\frac{3}{2}$ |  | 1.27 | $58 \pm 12$ | 34 | -•• | 15 | 0 | 0 | 0 | 0 |
| ${ }^{31} \mathrm{P}$ | $\frac{7}{2} \frac{1}{2}$ | 3.41 | $\frac{5}{2}$ | $\frac{1}{2}$ | 2.23 | <29 | 36 | - $\cdot$ | 81 | $<0.27$ | 1.9 | 20 | 2.1 |
| ${ }^{31} \mathrm{P}$ | $\frac{3}{2} \frac{3}{2}$ | 6.38 | $\frac{1}{2}$ |  | 0.00 | -•• | -•• | , ${ }^{\text {- }}$ | 0.6 | - | 7.4 | 4.9 | 7.0 |
| ${ }^{31} \mathrm{P}$ | $\frac{3}{2} \quad \frac{3}{2}$ | 6.38 | $\frac{3}{2}$ | $\frac{1}{2}$ | 1.27 | . $\cdot$ | -•• | -•• | 0.6 | -•• | 4.0 | 1.8 | 9.5 |
| ${ }^{31} \mathrm{P}$ | $\frac{1}{2} \frac{3}{2}$ | 7.14 | $\frac{1}{2}$ | $\frac{1}{2}$ | 0.00 | 0 | 0 | 0 | 0 | $34 \pm 4$ | 62 | 69 | 39 |
| ${ }^{31} \mathrm{P}$ | $\frac{1}{2} \frac{3}{2}$ | 7.14 | $\frac{3}{2}$ | $\frac{1}{2}$ | 1.27 | $<13$ | -•• | -•• | 0.0 | <3 | 6.3 | 5.2 | 0.7 |
| ${ }^{31} \mathrm{Si}$ | $\frac{1}{2} \frac{3}{2}$ | 0.76 | $\frac{3}{2}$ | $\frac{3}{2}$ | 0.00 | . | 3.5 | - | 6.4 | $16 \pm 4$ | 11 | 5.2 | 9.1 |
| ${ }^{31} \mathrm{Si}$ | $\frac{5}{2} \frac{3}{2}$ | 1.69 | $\frac{3}{2}$ | $\frac{3}{2}$ | 0.00 | $64 \pm 15$ | 48 | -•• | 58 | $0.06 \pm 0.03$ | 0.7 | 0.5 | 0.5 |
| ${ }^{31} \mathrm{Si}$ | $\frac{5}{2}$ | 1.69 | $\frac{1}{2}$ | $\frac{3}{2}$ | 0.76 | <80 | 7.4 | -•• | 12 | 0 | 0 | 0 | 0 |
| ${ }^{31} \mathrm{Si}$ | $\frac{3}{2} \frac{3}{2}$ | 2.32 | $\frac{3}{2}$ | $\frac{3}{2}$ | 0.00 | -•• | 3.5 | $\cdots$ | 5.2 | $4.6 \pm 3.4$ | 5.2 | 2.2 | 5.4 |
| ${ }^{31} \mathrm{Si}$ | $\frac{3}{2} \quad \frac{3}{2}$ | 2.32 | $\frac{1}{2}$ | $\frac{3}{2}$ | 0.76 | . $\cdot$ | 30 | - . | 41 | $4.8 \pm 3.2$ | 3.1 | 2.5 | 1.4 |
| ${ }^{31} \mathrm{Si}$ | $\frac{3}{2} \quad \frac{3}{2}$ | 2.32 | $\frac{5}{2}$ | $\frac{3}{2}$ | 1.69 | - | 4.2 | . $\cdot$ | 12 | $63 \pm 27$ | 54 | 72 | 38 |
| ${ }^{31} \mathrm{Si}$ | $\frac{5}{2} \quad \frac{3}{2}$ | 2.79 | $\frac{3}{2}$ | $\frac{3}{2}$ | 0.00 | >4.6 | 6.9 | -•• | 6.4 | >6.2 | 16 | 13 | 14 |
| ${ }^{31} \mathrm{Si}$ | $\frac{5}{2} \frac{3}{2}$ | 2.79 | $\frac{1}{2}$ | $\frac{3}{2}$ | 0.76 | <86 | 23 | - | 25 | 0 | 0 | 0 | 0 |

${ }^{\text {a }}$ Reference 26.


FIG. 5. Observed and calculated spectra for $A=33, T=\frac{1}{2}$ (See Table V). The conventions of the presentation are explained in the caption to Fig. 2.


FIG. 6. Diagrams of observed and calculated spectroscopic factors for transfer to ${ }^{33} \mathrm{~S}-{ }^{33} \mathrm{Cl}$. The information for these drawings was taken from Table V. Conventions of the presentation are discussed in the caption to Fig. 3.
set of matrix elements in this figure. This is a set of realistic matrix elements calculated from the Hamada-Johnston potential according to the techniques of $\mathrm{Kuo}^{9}$ with $\hbar \omega=12.5 \mathrm{MeV}$. The qualitative similarity which can be observed between the realistic interaction and the MSDI and FPSDI suggest that calculations employing the former interaction might account for the observed experimental phenomena as successfully as the latter two do. Our first studies of these nuclei were, indeed, carried out with such realistic interactions. We found that, while the calculated energy-level spectra looked "respectably" like experiment, the model wave functions often completely failed to account for the qualitative features of the spectroscopic factor data. We are presently investigat$\mathrm{ing}^{22,23}$ whether "small" and simple empirical alterations in the realistic matrix elements can remove the problems which initially led us to abandon realistic interactions and turn to SDI types of interactions for these nuclei.

## III. RESULTS OF CALCULATIONS

## A. General Remarks

In evaluating the success of our model calculations, we direct our primary attention to excitation energies and to spectroscopic factors for sin-gle-nucleon transfer. Agreement between model and experimental energy-level spectra is usually the first criterion of theoretical success in a calculation such as this. In these calculations some success in this direction is assured, since we vary the parametrized effective interactions to optimize such agreement. We think it is essential therefore to establish, wherever possible, that the dominant components of the model wave functions are consistent with the experimentally observed properties of the levels with which they have been correlated. The spectroscopic factors lend themselves to this end most readily. They provide very direct and clear information about the relationships between different wave functions, and in the ag-

TABLE V. Energies $E$ and spectroscopic factors $S$ for the states of $A=33, T=\frac{1}{2}\left({ }^{33} \mathrm{~S}-{ }^{33} \mathrm{Cl}\right)$. The conventions of the presentation are explained in the caption of Table II. The target state for the stripping transfers ( $S_{\mathrm{st}}$ ) is $A=32, T=0$, $J^{\pi}=0^{+}\left({ }^{32} \mathrm{~S}\right)$, and the target state for the pickup transfers $\left(S_{\mathrm{pk}}\right)$ is $A=34, T=1, J^{\pi}=0\left({ }^{34} \mathrm{~S}\right)$.

| A | $J$ |  | Energy $E$ ( MeV ) |  |  | $\left(100 \times S_{\text {st }}\right) /\left(100 \times S_{\text {pk }}\right)$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Expt. | FPSDI | MSDI | Expt. | FPSDI | MSDI |
| 33 | $\frac{1}{2}$ | $\frac{1}{2}$ | 0.84* | 1.00 | 0.52 | $27^{\text {a } / 65 ~}{ }^{\text {b }} 50^{\text {c }}$ | 17/85 | 27/98 |
| 33 | $\frac{1}{2}$ | $\frac{1}{2}$ |  | 3.78 | 3.49 |  | 3/10 | $0 / 6$ |
| 33 | $\frac{1}{2}$ | $\frac{1}{2}$ |  | 4.47 | 4.33 |  | 0/15 | 0/0 |
| 33 | $\frac{1}{2}$ | $\frac{1}{2}$ |  | 4.63 | 4.56 |  | $0 / 1$ | 0/0 |
| 33 | $\frac{3}{2}$ | $\frac{1}{2}$ | 192.19* | 192.24 | 192.36 | $69 / 190185$ | 70/187 | 64/176 |
| 33 | $\frac{3}{2}$ | $\frac{1}{2}$ | 2.31* | 2.31 | 2.02 | 5/24 16 | 5/25 | 7/15 |
| 33 | $\frac{3}{2}$ | $\frac{1}{2}$ |  | 3.61 | 3.47 |  | 0/6 | 0/8 |
| 33 | $\frac{3}{2}$ | $\frac{1}{2}$ |  | 4.39 | 3.91 |  | 0/5 | $0 / 4$ |
| 33 | $\frac{5}{2}$ | $\frac{1}{2}$ | 1.97* | 2.02 | 1.81 | $/ 5 \quad 3$ | $0 / 17$ | $0 / 5$ |
| 33 | $\frac{5}{2}$ | $\frac{1}{2}$ | 2.87* | 2.74 | 3.41 | /90 129 | 3/158 | 2/90 |
| 33 | $\frac{5}{2}$ | $\frac{1}{2}$ | 3.83 | 3.61 | 3.64 | $/ 50 \quad 54$ | 0/0 | 0/40 |
| 33 | $\frac{5}{2}$ | $\frac{1}{2}$ |  | 3.88 | 3.95 |  | 0/13 | 0/22 |
| 33 | $\frac{7}{2}$ | $\frac{1}{2}$ |  | 2.71 | 2.87 |  |  |  |
| 33 | $\frac{7}{2}$ | $\frac{1}{2}$ |  | 3.65 | 4.42 |  |  |  |
| 33 | $\frac{7}{2}$ | $\frac{1}{2}$ |  | 4.39 | 4.75 |  |  |  |
| 33 | $\frac{7}{2}$ | $\frac{1}{2}$ |  | 4.84 | 5.03 |  |  |  |
| 33 | $\frac{9}{2}$ | $\frac{1}{2}$ |  | 3.81 | 4.80 |  |  |  |
| 33 | $\frac{9}{2}$ | $\frac{1}{2}$ |  | 4.69 | 5.25 |  |  |  |
| 33 | $\frac{11}{2}$ | $\frac{1}{2}$ |  | 5.50 | 6.57 |  |  |  |

${ }^{\mathrm{a}}$ Reference 34. ${ }^{\mathrm{b}}$ Reference 35. ${ }^{\mathrm{c}}$ Reference 36.
gregate they yield information about the relative occupations of the various orbits included in the model space. ${ }^{24}$ The spectroscopic factors are proportional to the square of the reduced matrix elements of a creation operator between given initial and final states. It is worth noting that the truncation of the model space calls for some renormalization of this operator, just as it does for the effective two-body operator and various one-body operators. But we use the simple creation operator without any such renormalization.

Experimental values for spectroscopic factors are extracted from data on direct-reaction transfers of single nucleons via such processes as $(d, p),\left({ }^{3} \mathrm{He}, d\right),(d, t)$, etc., by comparison of the data with the predictions ${ }^{25}$ of the distorted-wave Born approximation (DWBA) theory for these reactions. There are numerous, well-documented, sources of uncertainty in this procedure. Because of these uncertainties, we believe that the relative predictions of the DWBA cross sections for "strong" transitions are accurate to no better than 20-30\%. In weak transitions, a significant amount of the observed cross section may result from processes not considered in the DWBA picture, and
so the relationship between experiment and DWBApredicted cross sections is even more uncertain.
Despite these cautions, we give the agreement between model spectroscopic factors and those extracted from experimental data a central place in our analysis. These observables are directly related to specific orbits of the model space in a way in which other measurable quantities, such as lifetimes, are not. (It must be remembered, of course, that the quantities we are inspecting are essentially overlaps of two different wave functions, and not solely dependent on a single wave function. This is true for $M 1$ and $E 2$ transition rates as well as for spectroscopic factors.)
In addition to studying the relationships between model-calculated and measured spectroscopic factors, we have used the wave functions generated with the MSDI and FPSDI interactions to calculate observables involving the magnetic dipole and electric quadrupole operators. In calculating electric quadrupole expectation values, we use the effective operator

$$
\overrightarrow{\mathrm{Q}}^{2}=\sum_{k=1}^{A} \tilde{e}_{k} \boldsymbol{\gamma}_{k}^{2} \overrightarrow{\mathrm{Y}}^{2}\left(\Omega_{k}\right) .
$$



FIG. 7. Observed and calculated spectra for $A=33, T=\frac{3}{2}$ (see Table VI). The conventions of the presentation are explained in the caption to Fig. 2.

Here $\tilde{e}_{k}$ is the total effective charge of the $k$ th nucleon, $r_{k}^{2}$ is the square of its radial coordinate, and $\overrightarrow{\mathrm{Y}}^{2}$ is the usual spherical harmonic operator of rank 2. The single-particle radial wave functions from which we obtain the matrix elements of $r^{2}$ are of the harmonic-oscillator form, with $\hbar \omega$ $=41 A^{-1 / 3} \mathrm{MeV}$. We have employed effective charges of $\tilde{e}_{p}=1.5 e$ and $\tilde{e}_{n}=0.5 e$. This choice was made because we have made the same assumption in other $s d$-shell calculations with some success, ${ }^{10}$ and there is no obvious empirical or theoretical reason for any other choice. Effective charges are introduced in an attempt to renormalize the $E 2$ operator in a simple way, so as to compensate for the exclusion of part of the $s d$-shell configuration space and the exclusion of all other oscillator shells. Glaudemans and co-workers ${ }^{26}$ have used our MSDI wave functions to search empirically for effective charges in this region. They arrive at values $\tilde{X}_{p}=1.44 e$ and $\tilde{e}_{n}=0.68 e$. The differences in $B(E 2)$ 's and quadrupole moments resulting from the two different choices are generally insignificant. (See Tables IV, VII, XI, XV, XVIII, and XIX for comparisons.)

Magnetic dipole moments and transition strengths have been calculated using the operator

$$
\begin{aligned}
\overrightarrow{\mathbf{M}}^{1}= & \sum_{k=1}^{A} \frac{1}{2}\left\{\overrightarrow{\mathbf{l}}(k)+\left(g_{p}+g_{n}\right) \overrightarrow{\mathbf{s}}(k)\right\} \\
& +\sum_{k=1}^{A} t_{z}(k)\left\{\overrightarrow{\mathbf{l}}(k)+\left(g_{p}-g_{n}\right) \overrightarrow{\mathbf{s}}(k)\right\}
\end{aligned}
$$

where we have used the free-nucleon values of the spin-gyromagnetic ratios,

$$
g_{p}=5.58 \text { and } g_{n}=-3.82 .
$$

That is, we have assumed negligible renormalization for the $M 1$ operator. In the same study of our MSDI wave functions just mentioned, Glaudemans, Endt, and Dieperink ${ }^{26}$ conclude that agreement between model predictions and experiment could be improved in some cases by changing the values of three of the isovector single-particle matrix elements,
$\left\langle s_{1 / 2}\left\|\overrightarrow{\mathrm{M}}^{1}\right\| s_{1 / 2}\right\rangle,\left\langle d_{3 / 2}\left\|\overrightarrow{\mathrm{M}}^{1}\right\| d_{3 / 2}\right\rangle$, and $\left\langle s_{1 / 2}\left\|\overrightarrow{\mathrm{M}}^{1}\right\| d_{3 / 2}\right\rangle$
from their free-nucleon values of (6.90, -1.58 , and $0) \mu_{N}$, respectively, to (4.4, 0.5, and 0.4) $\mu_{N}$ for

TABLE VI. Energies $E$ and spectroscopic factors $S$ for states of $A=33, T=\frac{3}{2}\left({ }^{33} \mathrm{P}\right)$. The conventions of the presentation are explained in the caption to Table II. The target nucleus in the pickup transfers $\left(S_{\mathrm{pk}}\right)$ is $A=34, T=1, J^{\pi}=0^{+}$ $\left({ }^{34}\right.$ S) .

| $\boldsymbol{A} \boldsymbol{J}$ T | Energy E (MeV) |  |  | $100 \times S_{\text {pk }}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $33 \quad \frac{1}{2} \frac{3}{2}$ | 186.58* | 186.74 | 186.66 | 180 | 157 | 140 |
| $33 \quad \frac{1}{2} \frac{3}{2}$ |  | 3.64 | 3.43 |  | 58 | 45 |
| $33 \frac{1}{2} \frac{3}{2}$ |  | 5.14 | 4.56 |  | 4 | 1 |
| $33 \quad \frac{1}{2} \frac{3}{2}$ |  | 5.51 | 5.73 |  | 0 | 1 |
| $33 \quad \frac{3}{2} \frac{3}{2}$ | 1.43* | 1.18 | 1.01 |  | 73 | 112 |
| $33 \quad \frac{3}{2} \frac{3}{2}$ | 2.53* | 2.34 | 2.08 |  | 1 | 2 |
| $33 \quad \frac{3}{2} \frac{3}{2}$ |  | 3.96 | 3.82 |  | 4 | 2 |
| $33 \frac{3}{2} \frac{3}{2}$ |  | 4.51 | 4.41 |  | 0 | 0 |
| $33 \quad \frac{5}{2} \frac{3}{2}$ | 1.84* | 1.72 | 1.78 | 340 | 160 | 153 |
| $33 \quad \frac{5}{2} \frac{3}{2}$ |  | 3.35 | 2.73 |  | 0 | 0 |
| $33 \quad \frac{5}{2} \frac{3}{2}$ |  | 3.75 | 3.37 |  | 115 | 234 |
| $33 \quad \frac{5}{2} \frac{3}{2}$ |  | 4.78 | 4.44 |  | 108 | 28 |
| $33 \frac{7}{2} \frac{3}{2}$ |  | 3.30 | 3.33 |  |  |  |
| $33 \frac{7}{2} \frac{3}{2}$ |  | 5.09 | 4.82 |  |  |  |
| $33 \frac{7}{2} \frac{3}{2}$ |  | 5.33 | 5.48 |  |  |  |
| $33 \frac{7}{2} \frac{3}{2}$ |  | 5.81 | 5.56 |  |  |  |
| $33 \frac{9}{2} \frac{3}{2}$ |  | 4.13 | 4.32 |  |  |  |
| $33 \frac{9}{2} \frac{3}{2}$ |  | 5.43 | 5.49 |  |  |  |
| $33 \frac{11}{2} \frac{3}{2}$ |  | 6.36 | 6.24 |  |  |  |

${ }^{\text {a References } 37 \text { and } 38 . ~}$
${ }^{\mathrm{b}}$ Reference 39.
$A=30-32$ and (4.1, -0.3 , and -0.2 ) $\mu_{N}$ for $A=33-34$. We shall compare the results calculated with these different effective $M 1$ operators below.

## B. Method of Presentation

We present the calculated and experimentally obtained values of energies and spectroscopic factors in tabular form. The results for each different $A, T$ system are presented in a separate table. In each table the states are grouped according to spin, and the spectroscopic factors associated with a given residual level are presented together with its energy. For the ground states of each system we list the nuclear binding energies relative to ${ }^{16} \mathrm{O}$, and for the other states we list excitations relative to these ground-state energies. Experimental energies and spectroscopic factors are entered in the tables only when there is an assumed or suggested correspondence with a model state.
In addition to the tables, energy-level diagrams, and for some nuclei spectroscopic-factor diagrams, are also presented to simplify theory-experiment comparisons. In these figures the energies of calculated and experimental ground states are set equal, so that only the excitation energies in each spectrum are shown. The only experimentally observed levels which are omitted from the energylevel figures below the maximum excitation indi-
cated are those which have firm negative-parity assignments. The lowest energy negative-parity state in each nucleus is indicated.

Calculated results for electromagnetic ( $E 2$ and $M 1$ ) observables are presented in tabular form. We discuss most completely the results obtained with the FPSDI wave functions. Also, in many instances we present comparisons with results obtained with the MSDI wave functions, as calculated both with our assumptions for the $M 1$ and $E 2$ operators and with the empirically modified operators of Glaudemans, Endt, and Dieperink. ${ }^{26}$

Finally, we present in an Appendix the major components of selected FPSDI wave functions, in order to convey a picture of how states are constructed in our model.

Experimental results which are quoted without reference in the energy-level spectroscopic-factor tables have been taken from the compilation of Endt and van der Leun. ${ }^{1}$ The data presented in the $B(E 2)$ and $B(M 1)$ tables are calculated from the lifetimes, mixing ratios, etc., compiled in Ref. 26.
We shall discuss the results in the order $A=31$, $33,35,30,32$, and 34 . We shall usually confine discussion to the lowest-lying levels in each system, since it is for these levels that the present model approach is expected to be most accurate. Information about higher excited states not discussed can be gleaned from the tables.

TABLE VII. Values of $B(E 2)$ and $B(M 1)$ for transitions between states of $A=33$. The notation is explained in the caption to Table IV.

| Nucleus | Initial state |  |  | Final state |  |  | $B(E 2)\left(e^{2} \mathrm{~F}^{4}\right)$ |  |  |  | $100 \times B(M 1)\left(\mu_{N}{ }^{2}\right)$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $J$ | $T$ | $E$ | $J$ | $T$ | $E$ | Expt. ${ }^{\text {a }}$ | FPSDI | MSDI | MSDI- ${ }^{\text {a }}$ | Expt. ${ }^{\text {a }}$ | FPSDI | MSDI | MSDI- ${ }^{\text {a }}$ |
| ${ }^{33} \mathrm{~S}$ |  | $\frac{1}{2}$ | 0.84 | $\frac{3}{2}$ | $\frac{1}{2}$ | 0.00 | $38 \pm 19$ | 23 | 32 | 31 | $5.9 \pm 1.1$ | 12 | 8.8 | 5.5 |
| ${ }^{33} \mathrm{~S}$ |  |  | 1.97 | $\frac{3}{2}$ | $\frac{1}{2}$ | 0.00 | $50 \pm 25$ | 37 | 50 | 55 | $3.8 \pm 0.7$ | 12 | 12 | 3.0 |
| ${ }^{33} \mathrm{~S}$ |  | $\frac{1}{2}$ | 1.97 | $\frac{1}{2}$ | $\frac{1}{2}$ | 0.84 | $<14$ | 31 | 31 | 34 | 0 | 0 | 0 | 0 |
| ${ }^{33} \mathrm{~S}$ |  | $\frac{1}{2}$ |  | $\frac{3}{2}$ | $\frac{1}{2}$ | 0.00 | $<38$ | 16 | 17 | 16 | <1.3 | 1.7 | 0.2 | 0.4 |
| ${ }^{33} \mathrm{~S}$ |  | $\frac{1}{2}$ | 2.31 | $\frac{1}{2}$ | $\frac{1}{2}$ | 0.84 | -•• | 38 | 37 | 41 | $6.8 \pm 1.2$ | 4.3 | 5.0 | 7.7 |
| ${ }^{33} \mathrm{~S}$ |  | $\frac{1}{2}$ | 2.87 | $\frac{3}{2}$ | $\frac{1}{2}$ | 0.00 | - | 18 | 13 | 14 | $15 \pm 7$ | 13 | 46 | 36. |
| ${ }^{33} \mathrm{~S}$ |  | $\frac{1}{2}$ | 2.87 | $\frac{1}{2}$ | $\frac{1}{2}$ | 0.84 | $<56$ | 8 | 22 | 22 | 0 | 0 | 0 | 0 |
| ${ }^{33} \mathrm{~S}$ |  | $\frac{1}{2}$ | 2.87 | $\frac{3}{2}$ | $\frac{1}{2}$ | 0.00 | -•• | 27 | 32 | 30 | 0 | 0 | 0 | 0 |
| ${ }^{33} \mathrm{P}$ |  | $\frac{3}{2}$ | (1.43) | $\frac{1}{2}$ | $\frac{3}{2}$ | 0.00 | $46 \pm 19$ | 40 | 27 | 28 | $1.8 \pm 0.5$ | 0.2 | 3.4 | 2.0 |
| ${ }^{33} \mathrm{P}$ |  | $\frac{3}{2}$ | (1.85) | $\frac{1}{2}$ | $\frac{3}{2}$ | 0.00 | $26 \pm 3$ | 29 | 25 | 27 | 0 | 0 | 0 | 0 |
| ${ }^{33} \mathrm{P}$ |  | $\frac{3}{2}$ | (1.85) | $\frac{3}{2}$ | $\frac{3}{2}$ | 1.43 | -•• | 9 | 14 | 16 | $3.3 \pm 1.4$ | 5.2 | 0.2 | 1.1 |
| ${ }^{33} \mathrm{P}$ |  | $\frac{3}{2}$ | (2.54) | $\frac{1}{2}$ | $\frac{3}{2}$ | 0.00 | . $\cdot$ | 0.1 | 13 | 16 | -•• | 15 | 10 | 8.1 |
| ${ }^{33} \mathbf{P}$ |  | $\frac{3}{2}$ | (2.54) | $\frac{3}{2}$ | $\frac{3}{2}$ | 1.43 | * | 51 | 28 | 31 | . $\cdot$ | 2.9 | 11 | 10 |

[^2]
## C. Details of Results

$$
\text { 1. } A=31, T=\frac{1}{2}\left({ }^{31} P-{ }^{31} S\right)
$$

(See Tables II and IV, Figs. 2 and 3)
The nucleus ${ }^{31} \mathrm{P}$ has been extensively explored both in particle-transfer and in $\gamma$-decay experiments ${ }^{27-32}$ The level structures calculated from the FPSDI and the MSDI are very similar to each other both as regards energies and spectroscopic factors. In these areas, agreement of the model results with experimental data is quite good as can be seen from Figs. 2 and 3. The magnetic dipole moment calculated for the $\frac{1}{2}^{+}$ground state of ${ }^{31} \mathrm{P}$ is somewhat too large [(1.6 to 1.1) $\mu_{N}$, see Table XIX]. The calculated $M 1$ and $E 2$ rates for transitions connecting the three lowest states are in what we
consider acceptable over-all agreement with experimental data. These values are shown in Table IV.

If we pursue the correspondence between model states and the observed states to higher excitations, we find that agreement deteriorates. The higher $\frac{5}{2}^{+}$model states do not exhibit as much $d_{5 / 2}$ hole strength as the pickup experiments indicate they should. Also, the second model $\frac{3^{+}}{}{ }^{+}$state has a larger stripping spectroscopic factor than is found experimentally for the second observed $\frac{3}{2}^{+}$level. Both of our models put the third $\frac{1}{2}^{+}$state at $\approx 4$ MeV , but the third known experimental $\frac{1}{2}^{+}$level occurs at 5.25 MeV . There is, however, a possibility ${ }^{27}$ that the experimentally observed $\frac{3}{2}^{+}$state at 3.51 MeV is actually a $\frac{1}{2}^{+}-\frac{3}{2}^{+}$doublet, which

TABLE VIII. Energies $E$ and spectroscopic factors $S$ for states of $A=35, T=\frac{1}{2}\left({ }^{35} \mathrm{C} 1-{ }^{35} \mathrm{Ar}\right)$ and $A=35, T=\frac{3}{2}\left({ }^{35} \mathrm{~S}\right)$. The conventions of the presentation are explained in the caption to Table II. The target state for the stripping transfer is $A=34, T=1, J^{\pi}=0^{+}\left({ }^{34} \mathrm{~S}\right)$ 。

| $A J T$ | Energy E (MeV) |  |  | $100 \times S_{\text {st }}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Expt. ${ }^{\text {a, b }}$ | FPSDI | MSDI |  |  | FPSDI | MSDI |
| $35 \quad \frac{1}{2} \frac{1}{2}$ | 1.22 | 1.45 | 0.67 | 19 | $28^{\text {c }}$ | 9 | 22 |
| $35 \quad \frac{1}{2} \quad \frac{1}{2}$ | 3.96 | 3.92 | 3.38 | 2 | 6 | 6 | 9 |
| $35 \quad \frac{1}{2} \quad \frac{1}{2}$ |  | 5.22 | 4.40 |  |  | 2 | 0 |
| $35 \quad \frac{1}{2} \quad \frac{1}{2}$ |  | 5.72 | 5.27 |  |  | 1 | 2 |
| $35 \quad \frac{3}{2} \quad \frac{1}{2}$ | 216.32 | 216.44 | 216.02 | 108 | 130 | 94 | 82 |
| $35 \quad \frac{3}{2} \quad \frac{1}{2}$ | 2.69 | 2.18 | 1.85 | 2 |  | 0 | 2 |
| $35 \quad \frac{3}{2} \quad \frac{1}{2}$ |  | 4.64 | 4.41 |  |  | 9 | 7 |
| $35 \quad \frac{3}{2} \quad \frac{1}{2}$ |  | 5.22 | 5.32 |  |  | 1 | 0 |
| $35 \quad \frac{5}{2} \quad \frac{1}{2}$ | 1.76 | 1.48 | 1.44 |  |  | 0 | 0 |
| $35 \quad \frac{5}{2} \quad \frac{1}{2}$ | 3.00 | 2.86 | 2.45 | 4 | 4 | 3 | 6 |
| $35 \quad \frac{5}{2} \quad \frac{1}{2}$ |  | 4.93 | 4.52 |  |  | 0 | 0 |
| $35 \quad \frac{5}{2} \quad \frac{1}{2}$ |  | 4.97 | 5.06 |  |  | 0 | 0 |
| $35 \quad \frac{7}{2} \frac{1}{2}$ | 2.64 | 2.42 | 2.96 |  |  |  |  |
| $35 \quad \frac{7}{2} \frac{1}{2}$ |  | 4.08 | 4.38 |  |  |  |  |
| $35 \quad \frac{7}{2} \quad \frac{1}{2}$ |  | 4.64 | 4.98 |  |  |  |  |
| $35 \quad \frac{7}{2} \quad \frac{1}{2}$ |  | 5.76 | 6.44 |  |  |  |  |
| $35 \quad \frac{9}{2} \frac{1}{2}$ | 3.94 | 3.54 | 4.30 |  |  |  |  |
| $35 \frac{9}{2} \frac{1}{2}$ |  | 4.95 | 5.31 |  |  |  |  |
| $35 \frac{11}{2} \frac{1}{2}$ |  | 6.13 | 7.13 |  |  |  |  |
| $35 \quad \frac{1}{2} \quad \frac{3}{2}$ |  | 1.46 | 1.36 |  |  | 11 | 13 |
| $35 \quad \frac{1}{2} \quad \frac{3}{2}$ |  | 3.64 | 3.24 |  |  | 0 | 0 |
| $35 \quad \frac{3}{2} \quad \frac{3}{2}$ | 210.32 | 210.17 | 209.46 |  |  | 35 | 38 |
| $35 \quad \frac{3}{2} \quad \frac{3}{2}$ |  | 2.93 | 2.55 |  |  | 0 | 0 |
| $35 \quad \frac{5}{2} \quad \frac{3}{2}$ |  | 1.59 | 2.24 |  |  | 4 | 0 |
| $35 \quad \frac{5}{2} \quad \frac{3}{2}$ |  | 2.96 | 3.46 |  |  | 0 | 2 |

[^3]${ }^{\mathrm{b}}$ Reference 41.
${ }^{c}$ Reference 42.
could account for this last discrepancy. Comparison of measured and calculated $B(E 2)$ 's and $B(M 1)$ 's involving the higher excited states shows two instances in which the model numbers are smaller than the lower limit on the measured values, and several instances in which the $B(M 1)$ values obtained with the bare-nucleon operators and either the FPSDI or the MSDI wave functions are at least twice too large. The use of the empirically modified $M 1$ operator ${ }^{26}$ sometimes cures these latter troubles, but at the cost of impairing some previous agreements. The $B(E 2)$ experimental results seem to be in slightly better over-all agreement with the FPSDI wave functions than with those of the MSDI.
Thus, for $A=31, T=\frac{1}{2}$ we find that the properties of the lowest states of each $J$ value are stable from one calculation to the other and in rather good correspondence with experiment, while for the higher excited states the predictions become less stable and less accurate. These results are typical of what we find for the other nuclei of the region. There is however an above-average amount of ex-
perimental information about ${ }^{31} \mathrm{P}$ available from which to draw these conclusions.
$$
\text { 2. } A=31, T=\frac{3}{2}\left({ }^{31} S i\right)
$$
(See Tables III and IV, Fig. 4)
Below an excitation energy of 4 MeV in ${ }^{31} \mathrm{Si}$ the agreement between experimental and calculated values for energies and stripping spectroscopic factors is good. ${ }^{27,33}$ The main question of detail involves the relative strengths of stripping to the first $\frac{3}{2}^{+}$and $\frac{1^{+}}{}{ }^{+}$states.

The predictions for both $M 1$ and $E 2$ transition rates between the low-lying states of ${ }^{31} \mathrm{Si}$ are in excellent agreement with all available experimental data. Results are shown in Table IV. The values for transitions between the lowest $\frac{1}{2}^{+}, \frac{3}{2}^{+}$, and $\frac{5{ }^{+}}{}{ }^{+}$ states are distinctly different from those we obtained for the lowest states of ${ }^{31} \mathrm{P}$, but the good agreement with experiment holds. This good agreement extends throughout the presently available data. For this nucleus we can establish no preference for any single variant of the calculations of $B(M 1)$ and $B(E 2)$ values.



FIG. 8. Observed and calculated spectra for $A=35, T=\frac{1}{2}$ (see Table VIII). The conventions of the presentation are explained in the caption to Fig. 2.

$$
\text { 3. } A=33, T=\frac{1}{2}\left({ }^{33} S-{ }^{33} \mathrm{Cl}\right)
$$

(See Tables V and VII, Figs. 5 and 6)
The agreement between the calculated energies and spectroscopic factors for ${ }^{33} \mathrm{~S}-{ }^{33} \mathrm{Cl}$ and the results of experiment ${ }^{34-36}$ for this nucleus is again very good, as can be seen in Table V and Figs. 5 and 6 . We see here some slight distinction between the FPSDI and MSDI results, the former giving better agreement with level energies and the latter perhaps better agreement with respect to relative $s_{1 / 2}-d_{3 / 2}$ spectroscopic factors. A preference between FPSDI and MSDI results in the case of the higher $\frac{5}{2}^{+}$states must await further experimental investigation of exactly how many such states occur in the region of $4-\mathrm{MeV}$ excitation.
The predicted values of the static electric quadrupole and magnetic dipole moments of the $\frac{3}{2}^{+}$ ground state of ${ }^{33} \mathrm{~S}$ are in acceptable agreement with the experimental values (see Table XIX). The predicted $M 1$ and $E 2$ strengths for transitions among the lowest five levels of ${ }^{33}$ S generally follow the patterns of the existing data. The empiri-
cal $M 1$ operator improves quantitative agreement with the data in two cases. In a third example, the FPSDI value is in better agreement with experiment than is either of the MSDI values. All in all the model results for this system are as satisfactory as any we obtain in this work.

$$
\text { 4. } A=33, \quad T=\frac{3}{2}\left({ }^{33} P\right)
$$

(See Tables VI and VII, Fig. 7)
The experimental picture for ${ }^{33} \mathrm{P}$ is relatively sketchy for this region. Stripping data are unobtainable and there has not been a thorough study made of the ${ }^{34} \mathrm{~S}\left(d,{ }^{3} \mathrm{He}\right){ }^{33} \mathrm{P}$ reaction. Spins of the lowest four levels have recently been assigned on the basis of ( $t, p$ ) and particle- $\gamma$ correlation experiments. ${ }^{37,38}$ The agreement between the model spectra and the existing experimental structure as presented in Table VI is typical of the present series of calculations. Lifetime measurements on the first two excited states have yielded $E 2$ and $M 1$ strengths for these decays and the results are in qualitative accord with our predictions. Addi-


FIG. 9. Observed and calculated spectra for $A=30, T=0$ (see Table IX). The conventions of the presentation are explained in the caption to Fig. 2.

TABLE IX. Energies $E$ and spectroscopic factors $S$ for the states of $A=30, T=0\left({ }^{30} \mathrm{P}\right)$. The energy results are presented in the convention of Table II. The experimental $S$ values are given in terms of the $l=0$ and $l=2$ components, while the theoretical numbers are listed in terms of $j=\frac{1}{2}$, $\frac{3}{2}$, and $\frac{5}{2}$, with the $j=\frac{3}{2}$ and $\frac{5}{2}$ numbers $(l=2)$ set off from the $l=0$ numbers. The target state for the pickup transfers $\left(S_{\mathrm{pk}}\right)$ is $A=31, T=\frac{1}{2}, J^{\pi}=\frac{1+}{2}\left({ }^{31} \mathrm{P}\right)$.

| $A J T$ | Energy $E(\mathrm{MeV})$ |  |  | $\begin{gathered} 100 \times\left(S_{0} ; S_{2}\right)_{\mathrm{pk}} \\ \text { Expt. }{ }^{\text {b }} \end{gathered}$ | $100 \times\left(S_{1 / 2} ; S_{3 / 2} ; S_{5 / 2}\right)_{\mathrm{pk}}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Expt. ${ }^{\text {a }}$ | FPSDI | MSDI |  | FPSDI | MSDI |
| 3000 |  | 5.73 | 5.88 |  |  |  |
| $30 \quad 0$ |  | 7.10 | 7.36 |  |  |  |
| 3010 | 156.22* | 156.49 | 156.21 | 71; <44 | 79; 3, | 93; 0, |
| 3010 | 0.71* | 1.02 | 1.00 | <31; <53 | 13; 19, | 1; 26, |
| 3010 | 3.02* | 3.22 | 3.18 |  | 4; 1, | 1; 1, |
| 3010 |  | 3.76 | 3.36 |  | 1; 1, | $0 ; 1$, |
| 3020 | 1.45* | 1.41 | 1.56 | ; 30 | ; 19, 25 | ; 23, 12 |
| 3020 | 2.72* | 2.07 | 2.31 | ; 47 | ; 1, 27 | ; 0, 39 |
| 3020 | 4.43* | 4.33 | 4.54 |  | ; 0, 14 | ; 0, 14 |
| 3020 | (4.94) | 4.94 | 4.86 |  | ; 1, 12 | ; 0, 12 |
| 3030 | 1.97* | 1.71 | 2.16 | ; 74 | ; , 103 | ; , 95 |
| 3030 | 2.54* | 2.63 | 2.86 | ; 61 | ; , 0 | ; , 0 |
| 3030 | 2.84 | 2.96 | 3.30 | ; <31 | ; , 3 | ; , 8 |
| 3030 |  | 4.20 | 4.26 |  | ; , 6 | ; , 13 |
| 3040 |  | 4.08 | 4.02 |  |  |  |
| 3040 |  | 4.47 | 4.83 |  |  |  |
| 3040 |  | 5.53 | 5.84 |  |  |  |
| 3040 |  | 5.93 | 6.05 |  |  |  |
| 3050 |  | 3.57 | 3.53 |  |  |  |
| 3050 |  | 4.83 | 5.01 |  |  |  |
| 3060 |  | 6.13 | 5.72 |  |  |  |

${ }^{\text {a }}$ Reference 43.
${ }^{\mathrm{b}}$ Reference 44.

TABLE X. Energies $E$ and spectroscopic factors $S$ for the states of $A=30, T=1\left({ }^{30} \mathrm{Si}^{30} \mathrm{~S}\right)$. The conventions of the presentation are explained in the caption of Table IX. The target state for the pickup transfers is $A=31, T=\frac{1}{2}, J^{\pi}=\frac{1}{2}$ $\left({ }^{31} \mathrm{P}\right)$.

| A | $J$ | $T$ | Energy E (MeV) |  |  | $\begin{gathered} 100 \times\left(S_{0} ; S_{2}\right)_{\mathrm{pk}} \\ \text { Expt. } \end{gathered}$ | $100 \times\left(S_{1 / 2} ; S_{3 / 2} ; S_{5 / 2}\right)_{\mathrm{pk}}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Expt. ${ }^{\text {a, b }}$ | FPSDI | MSDI |  | FPSDI |  | MSDI |  |
| 30 | 0 | 1 | 155.60* | 155.52 | 155.13 | 100; | 110; | , | 97; | , |
| 30 | 0 | 1 | 3.79* | 3.52 | 3.69 | 4; | 0 ; | , | 12; | , |
| 30 | 0 | 1 | 5.37 | 4.67 | 3.78 | 18; | 9; | , | 3; | , |
| 30 | 0 | 1 | 6.64 | 7.13 | 6.81 |  | 1; | , | $5 ;$ | , |
| 30 | 1 | 1 | 3.77* | 3.81 | 3.64 | 4; 0 | 0; | 1 , | 0; | 1, |
| 30 | 1 | 1 |  | 7.20 | 7.12 |  | 0 ; | 0, | 0 ; | 2 , |
| 30 | 1 | 1 |  | 7.71 | 7.19 |  | 1; | 0 , | 0; | 4 , |
| 30 | 1 | 1 |  | 7.83 | 7.47 |  | 5; | 6 , | 1; | 3 , |
| 30 | 2 | 1 | 2.23* | 2.48 | 2.42 | ; 160 |  | 21, 31 | ; | 22, 35 |
| 30 | 2 | 1 | 3.51* | 3.84 | 3.61 | ; 67 |  | 1, 131 | ; | 0, 141 |
| 30 | 2 | 1 | 4.81* | 4.65 | 4.70 | ; 29 |  | 0, 1 | ; | 1, 2 |
| 30 | 2 | 1 | 5.61* | 5.77 | 5.05 | ; 57 |  | 1, 15 | ; | 3, 14 |
| 30 | 3 | 1 | 5.23 | 4.64 | 4.12 | ; 220 |  | , 229 | ; | , 220 |
| 30 | 3 | 1 | 4.83 | 5.01 | 5.08 |  |  | , 0 | ; | , 5 |
| 30 | 3 | 1 |  | 6.18 | 6.42 |  |  | , 33 | , | , 31 |
| 30 | 3 | 1 |  | 7.28 | 6.78 |  |  | , 59 | ; | , 57 |
| 30 | 4 | 1 | 5.28* | 5.43 | 5.43 |  |  |  |  |  |
| 30 | 4 | 1 | 5.95 | 6.21 | 5.82 |  |  |  |  |  |
| 30 | 4 | 1 |  | 6.38 | 6.48 |  |  |  |  |  |
| 30 | 4 | 1 |  | 7.27 | 7.23 |  |  |  |  |  |
| 30 | 5 | 1 |  | 7.11 | 7.02 |  |  |  |  |  |
| 30 | 5 | 1 |  | 8.13 | 7.82 |  |  |  |  |  |
| 30 | 6 | 1 |  | 8.90 | 8.76 |  |  |  |  |  |

${ }^{\mathrm{a}}$ Reference 45.
${ }^{\mathrm{b}}$ Reference 46.
${ }^{c}$ Reference 47.
tional experimental information will be very helpful in definitively gauging the quality of our results for this system. One specific question that needs to be answered concerns the distribution of the $d_{5 / 2}$ hole strength. ${ }^{39}$

$$
\text { 5. } \quad A=35, \quad T=\frac{1}{2} \text { and } \frac{3}{2}\left({ }^{35} \mathrm{Cl}-{ }^{35} \mathrm{Ar} \text { and }{ }^{35} \mathrm{~S}\right)
$$

(See Table VIII and Fig. 8)

The calculated energies and stripping spectroscopic factors for the states of ${ }^{35} \mathrm{Cl}$ and ${ }^{35} \mathrm{~S}$ represent predictions based upon the Hamiltonians which were adjusted to fit energies in the $A=30$ 34 range. The energies obtained for $A=35, T=\frac{1}{2}$ with the two Hamiltonians have noticeable differences. The lowest few $\frac{1}{2}^{+}, \frac{3}{2}^{+}$, and $\frac{5}{2}^{+}$states are higher in the FPSDI spectrum than they are in the MSDI spectrum, and the FPSDI results seem in better agreement with experiment. On the other hand, the excitations of the $\frac{7}{2}^{+}$and $\frac{9}{2}^{+}$states in the FPSDI spectrum are lower than those of corre-
sponding states in the MSDI spectrum, and in this case the experimental values fall in between the model differences. ${ }^{40}$ At this point we can reexamine the results for $A=33$ (Tables V and VI) and see this same effect, although there it is weaker. For $A=31$, no such differences are perceptible.
The two predictions for the stripping spectroscopic factors of the lowest $\frac{1^{+}}{}$state in ${ }^{35} \mathrm{Cl}$ are quite different, culminating a trend which can be followed from ${ }^{31} \mathrm{Si}$ on. The experimental values for the spectroscopic factors ${ }^{41,42}$ seem to favor the MSDI result. All in all, however, we believe that the $A=35$ results indicate that the FPSDI Hamiltonian is to be slightly preferred.

$$
\begin{gathered}
\text { 6. A }=30, T=0\left({ }^{30} \mathrm{P}\right) \\
\text { (See Tables IX and XI, Fig. 9) }
\end{gathered}
$$

The observed energy levels of ${ }^{30} \mathrm{P}$ have been extensively studied, and three states each of $J^{\pi}=1^{+}$, $2^{+}$, and $3^{+}$are known. There is available consid-


$$
2.48 \text { 2+ }
$$

$$
2.42 \quad 2+
$$

$\qquad$
FPSDI
$\frac{0.00}{\text { MSDI }} 0+$ ${ }^{30} \mathrm{~S}_{\mathrm{I}}$

FIG. 10. Observed and calculated spectra for $A=30, T=1$ (see Table X). The conventions of the presentation are explained in the caption to Fig. 2.
erable information about the electromagnetic decays of these levels, ${ }^{43}$ in addition to the results of the neutron-pickup transfers to them from a ${ }^{31} \mathrm{P}$ target. ${ }^{44}$ The agreement between model energies and the measured values is fairly good, and considering the volume of data, rather impressive. These results are plotted in Fig. 9.
The experimental determinations of spectroscopic factors are not definitive in some cases because of the difficulty in disentangling the mixtures of $l=0$ and $l=2$ transfers which can both populate the $1^{+}$states. The model predictions of spectroscopic factors are at least roughly consistent with the experimentally determined numbers, with the exception of the second $3^{+}$state. This level is populated experimentally with almost as much strength as is the first $3^{+}$, but the models predict no strength for it at all.

The two different Hamiltonians yield results for energies and spectroscopic factors that are essentially equivalent and in general qualitative agreement with the experimental picture. However, comparison of the calculated $B(M 1)$ 's and $B(E 2)$ 's with the corresponding experimental values shows some distinct disagreements. The calculated patterns of $B(E 2)$ strength do not correlate well at all with the experimental patterns. The serious disagreements for transitions involving the $3^{+}$ states recalls the failure of the $S$-factor predictions for these same states. The strong $\Delta T=1 M 1$ transition from the $0^{+}$to the $1^{+}$ground state is predicted to be even stronger than measured, but the empirically adjusted $M 1$ operator corrects this problem.

The restrictions imposed upon these calculations by the assumption of a truncated $d_{5 / 2}$ configuration space are expected to be felt most strongly in $A=30$, of course. At the same time we would not be inclined to make too much of this. We have found that in all our $s d$-shell investigations, whether in a complete or truncated $s d$-shell space, the results for odd-odd nuclei are consistently the least satisfactory. In summation, the agreement for ${ }^{30} \mathrm{P}$ between model predictions and experimental results is less impressive than that achieved for the odd-even nuclei. But over all, these results represent a great improvement over results obtained in previous calculations.

$$
\text { 7. } A=30, T=1\left({ }^{30} S i-{ }^{30} S\right)
$$

(See Tables $X$ and XI, Fig. 10)
The model energies for a dozen states in ${ }^{30} \mathrm{Si}$ are in good agreement with measured values., ${ }^{1,45}$ The two Hamiltonians again yield quite similar results. The clearest mismatch between the model spectra and experiment involves the third $0^{+}$state, which is predicted about 1 MeV too low.

The pickup spectroscopic factors predicted for the $T=1$ states of $A=30$ are in good agreement with the results of experiment, with the exception that the second $2^{+}$state is predicted to be more strongly excited via $l=2$ transfer than the first $2^{+}$ state, while the opposite relationship is experimentally observed. Examination of the calculated $l=2$ strengths indicates that it is the $d_{5 / 2}$ hole strength which is misplaced. The dominant component of the wave function of the lowest model $2^{+}$

TABLE XI. Values of $B(E 2)$ and $B(M 1)$ for transitions between states of $A=30$. The notation is explained in the caption to Table IV.

| Nucleus | Initial state |  |  | Final state |  |  | $B(E 2)\left(e^{2} \mathrm{~F}^{4}\right)$ |  |  |  | $100 \times B(M 1)\left(\mu_{N}{ }^{2}\right)$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $J$ | $T$ | $E$ | $J$ | $T$ | $E$ | Expt. ${ }^{\text {a }}$ | FPSDI | MSDI | MSDI-A ${ }^{\text {a }}$ | Expt. ${ }^{\text {a }}$ | FPSDI | MSDI | MSDI-A ${ }^{\text {a }}$ |
| ${ }^{30} \mathrm{P}$ | 0 | 1 | 0.68 | 1 | 0 | 0.00 | 0 | 0 | 0 | 0 | $134 \pm 34$ | 503 | 512 | 136 |
| ${ }^{30} \mathrm{P}$ | 1 | 0 | 0.71 | 1 | 0 | 0.00 | $9.5 \pm 2.7$ | 2.4 | 16 | 16 | $0.7 \pm 0.2$ | 1.0 | 0.18 | 0.18 |
| ${ }^{30} \mathrm{P}$ | 1 | 0 | 0.71 | 0 | 1 | 0.68 | 0 | 0 | 0 | 0 | ... | ... | . | . . |
| ${ }^{30} \mathrm{P}$ | 2 | 0 | 1.45 | 1 | 0 | 0.00 | $2.0 \pm 0.5$ | 28 | 27 | 28 | $0.6 \pm 0.2$ | 0.02 | 0.00 | 0.00 |
| ${ }^{30} \mathrm{P}$ | 2 | 0 | 1.45 | 0 | 1 | 0.68 | <45 | 0.1 | 0.0 | 0.0 | 0 | 0 | 0 | 0 |
| ${ }^{30} \mathrm{P}$ | 2 | 0 | 1.45 | 1 | 0 | 0.71 | -•• | 0.3 | 6 | . . | <0.5 | . . | . . | . . |
| ${ }^{30} \mathrm{P}$ | 3 | 0 | 1.97 | 1 | 0 | 0.00 | $6.4 \pm 1.7$ | 19 | 13 | 13 | 0 | 0 | 0 | 0 |
| ${ }^{30} \mathrm{p}$ | 3 | 0 | 1.97 | 1 | 0 | 0.71 | $94 \pm 25$ | 2.2 | 0.03 | 0.0 | 0 | 0 | 0 | 0 |
| ${ }^{30} \mathrm{P}$ | 3 | 0 | 2.54 | 1 | 0 | 0.00 | $47 \pm 16$ | 4.3 | 0.9 | 1.3 | 0 | 0 | 0 | 0 |
| ${ }^{30} \mathrm{P}$ | 3 | 0 | 2.54 | 1 | 0 | 0.71 | . . | 32 | 32 | 37 | 0 | 0 | 0 | 0 |
| ${ }^{30} \mathrm{Si}$ | 2 | 1 | 2.24 | 0 | 1 | 0.00 | $40 \pm 2$ | 17 | 22 | 25 | 0 | 0 | 0 | 0 |
| ${ }^{30} \mathrm{Si}$ | 2 | 1 | 3.51 | 0 | 1 | 0.00 | $8.0 \pm 2.4$ | 14 | 12 | 10 | 0 | 0 | 0 | 0 |
| ${ }^{30} \mathrm{Si}$ | 2 | 1 | 3.51 | 2 | 1 | 2.24 | $46 \pm 28$ | 28 | 31 | 30 | $16 \pm 5$ | ... | 20 | 29 |
| ${ }^{30} \mathrm{Si}$ | 1 | 1 | 3.77 | 0 | 1 | 0.00 | 0 | 0 | 0 | 0 | $0.50 \pm 0.16$ | 1.0 | 2.5 | 3.4 |
| ${ }^{30} \mathrm{Si}$ | 1 | 1 | 3.77 | 2 | 1 | 2.24 | $4.4 \pm 3.4$ | 2.4 | . $\cdot$ | 4.4 | $11 \pm 3$ | 22 | 24 | 36 |
| ${ }^{30} \mathrm{Si}$ | 0 | 1 | 3.79 | 2 | 1 | 2.24 | <17 | 5.1 | 21 | 20 | 0 | 0 | 0 | 0 |

${ }^{\text {a }}$ Reference 26.
is $\left(d_{5 / 2}\right)^{12}\left(s_{1 / 2}\right)^{1}\left(d_{3 / 2}\right)^{1}$, while the lowest observed $2^{+}$state must have significantly more $d_{5 / 2}$ hole intensity. The predicted static electric quadrupole moments of the lowest two $2^{+}$state have opposite signs, and a measurement of this value for the first $2^{+}$would be valuable.
Calculations of spectroscopic factors and $B(E 2)$ values for the lowest two $3^{+}$states suggest that the calculated $3^{+}$states are inverted in energy with respect to their experimental partners. The lower model $3^{+}$is predicted to have a large $d_{5 / 2}$ spectroscopic factor for pickup from ${ }^{31} \mathrm{P}$ and very weak $E(2)$ decays to the first and second excited $2^{+}$states. The second model $3^{+}$is predicted to have zero pickup spectroscopic factors and strong $E(2)$ decays to the $2^{+}$states. These characteristics are similar to the properties observed ${ }^{46,47}$ for
the $5.23-$ and $4.83-\mathrm{MeV}$ levels, respectively. The first two $4^{+}$levels in the model also have qualitatively different wave functions; the first one is dominated by $\left(d_{5 / 2}\right)^{-1}\left(s_{1 / 2}\right)^{2}\left(d_{3 / 2}\right)^{1}$, while the second has no dominating component and is highly fragmented. In this case the apparent experimental ordering of the counterparts of these states is consistent with the model ordering. ${ }^{46}$ The calculated $E 2$ rates for both states are considerably weaker than experiment indicates. This may possibly be due to the $d_{5 / 2}$ truncation, since $4^{+}$states in ${ }^{30}$ Si necessarily involve $d_{5 / 2}$ holes for their primary description.
We would summarize the model results for ${ }^{30} \mathrm{Si}$ as being generally satisfactory, but indicating some discrepancies in detail. The situation is similar to that obtained for ${ }^{30} \mathrm{P}$. However, the dis-

TABLE XII. Energies $E$ and spectroscopic factors $S$ for the states of $A=32, T=0\left({ }^{32} \mathrm{~S}\right)$. The conventions of the notation are explained in the caption to Fig. 9. The target state for the stripping transfers ( $S_{\mathrm{st}}$ ) is $A=31, T=\frac{1}{2}, J^{\pi}=\frac{1+}{2}\left({ }^{31} \mathrm{P}\right)$. The target for the pickup transfers $\left(S_{\mathrm{pk}}\right)$ is $A=33, T=\frac{1}{2}, J^{\pi}=\frac{3+}{2}\left({ }^{33} \mathrm{~S}\right)$.

|  | $J$ | $T$ | Energy E (MeV) |  |  |  | $\begin{gathered} 100 \times\left(S_{0} ; S_{2}\right) \\ \text { Expt. } . \end{gathered}$ | $100 \times\left(S_{1 / 2} ; S_{3 / 2}, S_{5 / 2}\right)$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Expt. ${ }^{\text {a, b }}$ | FPSDI | MSDI |  |  | FPSDI | MSD |  |
| 32 | 0 | 0 | 183.56* | 183.29 | 183.56 | st | 240; | 257; | 226; | , |
|  |  |  |  |  |  | pk |  | ; 70. | ; 64 | 4 , |
| 32 | 0 | 0 | 3.78* | 3.68 | 3.81 | st | 40; | 44; | 68; |  |
|  |  |  |  |  |  | pk |  | ; 6, | ; 7 | 7, |
| 32 | 0 | 0 |  | 7.35 | 7.30 | st |  | 0; , | 0; |  |
|  |  |  |  |  |  | pk |  | ; 0 , |  | 0 , |
| 32 | 0 | 0 |  | 7.99 | 8.02 | st |  | 0; , |  |  |
|  |  |  |  |  |  | pk |  | ; 0 , |  | 0 , |
| 32 | 1 | 0 | 4.70* | 4.81 | 4.50 | st | ; 49 | 0; 78, | 0; 78 |  |
|  |  |  |  |  |  | pk |  | 24; 3, 1 | 25; 3 | 3, 0 |
| 32 | 1 | 0 |  | 7.12 | 6.64 | st |  | 3 ; 4, | 5; 1 | 1, |
|  |  |  |  |  |  | pk |  | 2; 0, 12 | 0; 0 | 0, 1 |
| 32 | 1 | 0 |  | 7.63 | 8.04 | st |  | 1; 1, | 0; 0 | 0 , |
|  |  |  |  |  |  | pk |  | 2; 0, 4 | 0; 0 | 0, 1 |
| 32 | 1 | 0 |  | 8.18 | 8.24 | st |  | 0 ; 0, | 0; | 4, |
|  |  |  |  |  |  | pk |  | 0; 0, 0 | 3; 0 | 0, 8 |
| 32 | 2 | 0 | 2.24* | 2.20 | 2.00 | st | ; 120 | ; 83, 3 | ; 85 | 5, 3 |
|  |  |  |  |  |  | pk |  | 49; 0,0 | 54; 0 | 0, 0 |
| 32 | 2 | 0 | 4.29* | 4.55 | 4.97 | st | ; 5 | ; 0, 1 | ; 0 | 0, 0 |
|  |  |  |  |  |  | pk |  | 0; 2, 26 | 0; 2 | 2, 20 |
| 32 | 2 | 0 | 5.55* | 5.44 | 5.41 | st | ; 13 | ; 15, 0 | ; 15 | 5, 0 |
|  |  |  |  |  |  | pk |  | 3; 17, 0 | 2; 17 | 7, 0 |
| 32 | 2 | 0 |  | 6.64 | 6.78 | st |  | ; 3, 3 | ; 3 | 3, 0 |
|  |  |  |  |  |  | pk |  | 2; 1, 10 | 0 ; 0 | 0, 3 |
| 32 | 3 | 0 | 5.41 | 5.20 | 5.87 | pk |  | ; , 38 | ; 2 | 2, 34 |
| 32 | 3 | 0 |  | 6.60 | 7.07 | pk |  | ; , 4 | ; 3 | 3, 3 |
| 32 | 3 | 0 |  | 7.34 | 7.59 | pk |  | ; , 4 | ; 0 | 0, 6 |
| 32 | 3 | 0 |  | 7.78 | 8.10 | pk |  | ; , 4 | ; 0 | 0, 3 |
| 32 | 4 | 0 | 4.47* | 4.92 | 5.40 | pk |  | ; , 46 | ; | , 21 |
|  | 4 | 0 |  | 5.60 | 6.26 | pk |  | ; , 9 | ; | , 22 |
| 32 | 4 | 0 |  | 6.15 | 6.91 | pk |  | ; , 1 | ; | , 8 |
| 32 | 4 | 0 |  | 7.52 | 8.07 | pk |  | ; , 2 | ; | , 3 |
| 32 | 5 | 0 |  | 6.84 | 7.71 |  |  |  |  |  |
| 32 | 5 | 0 |  | 7.84 | 8.42 |  |  |  |  |  |
| 32 | 6 | 0 |  | 7.91 | 9.08 |  |  |  |  |  |

${ }^{\text {a }}$ Reference 48. $\quad{ }^{\mathrm{b}}$ Reference 50.
crepancies between model and experiment for ${ }^{30} \mathrm{Si}$ seem more specifically to involve the $d_{5 / 2}$ orbit.

$$
\text { 8. } \quad A=32, \quad T=0\left({ }^{32} S\right)
$$

(See Tables XII and XV, Figs. 11 and 12)
The eight positive-parity levels which are definitely assigned in the spectrum of ${ }^{32}$ S appear with essentially the correct ordering and spacing in the model spectra. The only level for which there is a notable discrepancy in energy is the $4^{+}$. The MSDI excitation energy is 1 MeV higher than the experimental value; the FPSDI adjustment reduces this gap somewhat, to 0.5 MeV . More detailed experimental investigation of the $6-8-\mathrm{MeV}$ region of excitation should determine the accuracy of the model predictions for the energies of the higher states.

The spectroscopic factors experimentally obtained ${ }^{48}$ with the ${ }^{31} \mathrm{P}\left({ }^{3} \mathrm{He}, d\right)^{32} \mathrm{~S}$ reaction for $J=0,1$, and 2 states of ${ }^{32} \mathrm{~S}$ are satisfactorily matched by the model predictions. The differences between the results calculated from our two different Hamiltonians are slight. The study of neutron pickup from ${ }^{33} \mathrm{~S}$ would complement the stripping data and give a very complete picture of the single-particle aspects of the ${ }^{32}$ S level structure.

The electromagnetic observable data on this nucleus predominantly involve the $E 2$ operator. The static electric quadrupole moment of the first $2^{+}$ state has recently been measured. ${ }^{49}$ The calculated moments for this state, which indicate a prolate deformation, are smaller in magnitude than the experimental value of $-20 \pm 6 e \mathrm{~F}^{2}$, but are almost within the quoted experimental errors (see Table XIX). The $B(E 2)$ values calculated for the

TABLE XIII. Energies $E$ and spectroscopic factors $S$ for the states of $A=32, T=1\left({ }^{32} \mathrm{P}\right)$. The conventions of the presentation are explained in the caption to Table IX. The target state for the stripping transfers $\left(S_{\mathrm{st}}\right)$ is $A=31, T=\frac{1}{2}, J^{\pi}$ $=\frac{1}{2}+\left({ }^{31} \mathrm{P}\right)$. The target for the pickup transfers $\left(S_{\mathrm{pk}}\right)$ is $A=33, T=\frac{1}{2}, J^{\pi}=\frac{3+}{2}\left({ }^{33} \mathrm{~S}\right)$.

| A | $T$ | Energy E (MeV) |  |  |  | $\begin{gathered} 100 \times\left(S_{0} ; S_{2}\right) \\ \text { Expt. }^{\text {a }} \end{gathered}$ | $100 \times\left(S_{1 / 2} ; S_{3 / 2}, S_{5 / 2}\right)$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Expt. | FPSDI | MSDI |  |  | FPSDI | MSDI |
| 320 | 1 | 0.51* | 0.42 | 0.35 | st | 32; | 21; | 26; |
|  |  |  |  |  | pk |  | ; 7, | ; 14, |
| 320 | 1 |  | 3.29 | 3.55 | st |  | 3; , | 3; , |
|  |  |  |  |  | pk |  | ; 0, | ; 1, |
| 320 | 1 |  | 4.70 | 5.13 | st |  | 2; , | 3; , |
|  |  |  |  |  | pk |  | ; 0, | ; 1, |
| 320 | 1 |  | 6.16 | 5.67 | st |  | 0 ; , | 0 ; , |
|  |  |  |  |  | pk |  | ; 0, | ; 0, |
| 321 | 1 | 176.47* | 176.76 | 176.84 | st | ; 59 | $0 ; 68 \text {, }$ | $0 ; 65$ |
|  |  |  |  |  | pk |  | $73 ; 10,3$ | $69 ; 14,1$ |
| 321 | 1 | 1.15* | 1.11 | 0.94 | st | 19; | 10; 0, | 14; 3, |
|  |  |  |  |  | pk |  | 0; 5, 1 | 0; 8, 2 |
| 321 | 1 |  | 1.93 | 1.72 | st |  | 1; 9, | 2; 9, |
|  |  |  |  |  | pk |  | 2; 1, 1 | 1; 4, 1 |
| 321 | 1 |  | 3.12 | 2.53 | st |  | 3 ; 2, | 0; 0, |
|  |  |  |  |  | pk |  | 6; 0,28 | 0; 0, 0 |
| 322 | 1 | 0.08* | 0.18 | 0.16 | st | ; 70 | ; 66, 0 | ; 67, 0 |
|  |  |  |  |  | pk |  | 105; 2, 11 | 94; 1, 8 |
| 322 | 1 | (1.32)* | 1.17 | 1.14 | st |  | ; 0, 0 | ; 0, 0 |
|  |  |  |  |  | pk |  | 1; 28, 10 | 3; 31, 12 |
| 322 | 1 |  | 2.25 | 2.08 | st |  | ; 0,1 | ; 0, 0 |
|  |  |  |  |  | pk |  | 1; 0, 2 | 3; 1, 18 |
| 322 | 1 |  | 2.65 | 2.51 | st |  | ; 0,1 | ; 3, 1 |
|  |  |  |  |  | pk |  | 2; 2,0 | 5; 4, 14 |
| 323 | 1 | (1.76) | 1.50 | 1.64 | pk |  | ; 14, 80 | ; 33, 33 |
| 323 | 1 |  | 1.79 | 2.10 | pk |  | ; 10, 22 | ; 5,97 |
| 323 | 1 |  | 2.39 | 2.53 | pk |  | ; , 23 | ; 1, 2 |
| 323 | 1 |  | 3.30 | 3.15 | pk |  | ; 1, 0 | ; 1, 5 |
| 324 | 1 |  | 2.89 | 2.96 | pk |  | ; , 75 | ; , 129 |
| 324 | 1 |  | 3.30 | 3.35 | pk |  | ; , 64 | ; , 13 |
| 324 | 1 |  | 4.17 | 4.30 | pk |  | ; , 1 | ; , 24 |
| 325 | 1 |  | 3.51 | 3.32 |  |  |  |  |
| 325 | 1 |  | 5.02 | 4.88 |  |  |  |  |
| 326 | 1 | - | 6.32 | 6.12 |  |  |  |  |

${ }^{\mathrm{a}}$ Reference 48.
transitions from the first and second $2^{+}$states to the ground $0^{+}$state agree with experimental measurements, but the predictions for the $B(E 2)$ 's of the transitions from the second $2^{+}$, the second $0^{+}$, and the $4^{+}$states to the $2^{+}$first excited state are weaker by a factor of $\approx 2$ than the present experimental values. ${ }^{50}$

$$
\text { 9. } A=32, T=1\left({ }^{32} P\right)
$$

(See Tables XIII and XV, Fig. 13)
The level structure of ${ }^{32} \mathrm{P}$ is yet to be thoroughly worked out experimentally. The energy levels with spin-parity assignments are in good accord with both model spectra. The ${ }^{31} \mathrm{P}\left({ }^{3} \mathrm{He}, d\right)^{32} \mathrm{~S}$ reaction populates the $T=1$ states in ${ }^{32} \mathrm{~S}$, which are isobaric analogs of the low-lying states of ${ }^{32} \mathrm{P}$, as well as the lower-lying $T=0$ states. Experimental results for this reaction yield spectroscopic factors for these $T=1$ states simultaneously with
those for the $T=0$ states. ${ }^{48}$ The model predictions for $l=2$ and $l=0$ stripping strengths to the $T=1$ states are again in good agreement with the values extracted from experiment. We note, for instance, the correct predictions of $l=2$ population of the first $1^{+}$and $l=0$ population of the second $1^{+}$, with essentially the right magnitudes. Pickup experiments on ${ }^{33}$ S would yield information which would greatly clarify the situation regarding both the presently assigned levels of ${ }^{32} \mathrm{P}$ and those unassigned in the $1-3-\mathrm{MeV}$ region of excitation.

The electromagnetic data measured for ${ }^{32} \mathrm{P}$ all depend upon the $M 1$ operator. The model predictions of a small magnetic moment for the ground state are consistent with the small measured value of $0.24 \mu_{N}$. (See Table XIX.) The strong transition from the $2^{+}$first excited state to the $1^{+}$ ground state is satisfactorily predicted, and the general experimental trend of strong $M 1$ transi-

$\qquad$
$\frac{0.00}{} 0+\quad 0+$
$\frac{0.00}{\text { MSDI }} 0+$
${ }^{32} \mathrm{~S}$
FIG. 11. Observed and calculated spectra for $A=32, T=0$ (see Table XII). The conventions of the presentation are explained in the caption to Fig. 2.


FIG. 12. Diagrams of observed and calculated spectroscopic factors for transfer to states of $A=32, T=0$. The information for these drawings was taken from Table XII. The conventions of the presentation are explained in the caption to Fig. 3.
tions is also evident in the model results. However, several of the predicted transition strengths involving higher excited states appear to be too large. It would be of interest to test the model predictions for $E 2$ strengths, but the strong competing $M 1$ transitions make this very difficult.
10. $A=32, T=2\left({ }^{32} S i\right)$
(See Table XIV)
The energy levels predicted for the $A=32, T=2$ system, which could be studied experimentally via the ${ }^{30} \mathrm{Si}(t, p)^{32} \mathrm{Si}$ reaction, are presented in Table XIV.

TABLE XIV. Energies $E$ of the states of $A=32, T=2$ $\left({ }^{32} \mathrm{Si}-{ }^{32} \mathrm{Ar}\right)$. The conventions of the presentation are explained in the caption to Table II.

|  | $J$ | $T$ | Energy E (MeV) |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Expt. | FPSDI | MSDI |
|  | 0 | 2 | 171.48* | 171.35 | 171.41 |
| 32 | 0 | 2 |  | 4.30 | 4.02 |
| 32 | 0 | 2 |  | 5.30 | 5.06 |
| 32 | 0 | 2 |  | 7.02 | 6.43 |
| 32 | 1 | 2 |  | 5.15 | 5.05 |
| 32 | 1 | 2 |  | 5.69 | 5.28 |
| 32 | 2 | 2 | 1.94* | 1.86 | 1.92 |
| 32 | 2 | 2 |  | 4.21 | 3.96 |
| 32 | 2 | 2 |  | 5.06 | 4.74 |
| 32 | 2 | 2 |  | 5.76 | 5.31 |
| 32 | 3 | 2 |  | 5.11 | 4.40 |
| 32 | 3 | 2 |  | 6.62 | 5.92 |
| 32 | 4 | 2 |  | 4.88 | 4.87 |
| 32 | 4. | 2 |  | 5.98 | 5.68 |

## 11. $A=34, T=0\left({ }^{34} C l\right)$

(See Tables XVI and XVIII, Fig. 14)
The energy-level spectrum of ${ }^{34} \mathrm{Cl}$ is the most complex of those we study and it is not yet well understood experimentally. When these calculations were performed, only the first two energy levels were definitely assigned. Since then, several $\gamma$-ray decay studies have yielded spin assignments to the next three levels ${ }^{51,52}$ and recent sin-gle-nucleon transfer experiments ${ }^{36,53}$ have provided information about the parities and wave functions.

The MSDI results are in distinct disagreement with experiment. This is apparent from the energy spectra alone but becomes even clearer when the details of the spectroscopic factor results are examined. The FPSDI adjustment of the Hamiltonian, even though it involved only the minimal information then available from the ${ }^{34} \mathrm{Cl}$ spectrum itself, yielded an energy-level spectrum and spectroscopic factors for this system in better agreement with experiment. This distinction between the two Hamiltonians, and these criteria for choosing one over the other, occur only in this instance. The few $B(E 2)$ 's measured experimentally are well reproduced by the models. The $B(M 1)$ results from the FPSDI are qualitatively similar to observation, but the absolute magnitudes are too large, most noticeably in the case of the second-exeited-to-ground-state transition.
Inspection of Tables XVI and XVIII indicates that the most striking differences in the two sets of predicted observables occur for the lowest two $J^{\pi}$ $=1^{+}, T=0$ states. These differences suggest in


FIG. 13. Observed and calculated spectra for $A=32, T=1$ (see Table XIII). The conventions of the presentation are explained in the caption to Fig. 2.
fact that the wave functions of the FPSDI calculation occur in inverted order in the MSDI calculation. From the Appendix we note that the lowest $J^{\pi}=1^{+}, T=0$ FPSDI wave function is "dominated" ( $28 \%$ ) by the $\left(d_{5 / 2}\right)^{12}\left(s_{1 / 2}\right)^{3}{ }_{J=1 / 2, T=1 / 2}\left(d_{3 / 2}\right)^{3}{ }_{J=3 / 2, T=1 / 2}$ component, while in the second $1^{+}$state, the $\left(d_{5 / 2}\right)^{12}\left(s_{1 / 2}\right)^{4}\left(d_{3 / 2}\right)^{2}{ }_{J=1, T=0}$ component is largest $(36 \%)$. In the MSDI wave functions these same two components each dominate a $1^{+}$state, but the energies of the two states are inverted.

The $1^{+}$state showing strong $s_{1 / 2}$ pickup and stripping strength is the one dominated by the $\left(s_{1 / 2}\right)^{3}{ }_{J=1 / 2, T=1 / 2}\left(d_{3 / 2}\right)^{3}{ }_{J=3 / 2, T=1 / 2}$ component which connects to the $\left(d_{5 / 2}\right)^{12}\left(s_{1 / 2}\right)^{2}{ }_{J=0},\left(d_{3 / 2}\right)^{3}{ }_{J=3 / 2, T=1 / 2}$, and $\left(d_{5 / 2}\right)^{12}\left(s_{1 / 2}\right)^{4}\left(d_{3 / 2}\right)^{3}{ }_{J=3 / 2, T=1 / 2}$ components of the ground states of ${ }^{33} \mathrm{~S}$ and ${ }^{35} \mathrm{Cl}$, respectively. Hence the lowest FPSDI $1^{+}$state has the $s_{1 / 2}$ transfer strength in contrast to the second MSDI $1^{+}$state. Similarly, the $\left(d_{5 / 2}\right)^{12}\left(s_{1 / 2}\right)^{4}\left(d_{3 / 2}\right)^{2}{ }_{J=1, T=0}$ component mediates the $d_{3 / 2}$ transfer strength to these levels

TABLE XV. Values of $B(E 2)$ and $B(M 1)$ for transitions between states of $A=32$. The notation is explained in the caption to Table IV.

| Nucleus | Initial state |  |  | Final state |  |  | $B(E 2)\left(e^{2} \mathrm{~F}^{4}\right)$ |  |  |  | $100 \times B(M 1)\left(\mu_{N}{ }^{2}\right)$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $J$ | $T$ | $E$ | $J$ | T | $E$ | Expt. ${ }^{\text {a }}$ | FPSDI | MSDI | MSDI-A ${ }^{\text {a }}$ | Expt. ${ }^{\text {a }}$ | FPSDI | MSDI | MSDI- ${ }^{\text {a }}$ |
| ${ }^{32} \mathrm{~S}$ | 2 | 0 | 2.24 | 0 | 0 | 0.00 | $45 \pm 4$ | 38 | 43 | 49 | 0 | 0 | 0 | 0 |
| ${ }^{32} \mathrm{~S}$ | 0 | 0 | 3.78 | 2 | 0 | 2.24 | $141 \pm 36$ | 46 | 27 | 30 | 0 | 0 | 0 | 0 |
| ${ }^{32} \mathrm{~S}$ | 2 | 0 | 4.29 | 0 | 0 | 0.00 | $14 \pm 6$ | 13 | 14 | 15 | 0 | 0 | 0 | 0 |
| ${ }^{32} \mathrm{~S}$ | 2 | 0 | 4.29 | 2 | 0 | 2.24 | >52 | 16 | 15 | 17 | <0.14 | 0.02 | 0.0 | 0.0 |
| ${ }^{32} \mathrm{~S}$ | 4 | 0 | 4.47 | 2 | 0 | 2.24 | $106 \pm 19$ | 45 | 58 | 66 | 0 | 0 | 0 | 0 |
| ${ }^{32} \mathrm{~S}$ | 1 | 0 | 4.70 | 0 | 0 | 0.00 | 0 | 0 | 0 | 0 | $0.05 \pm 0.01$ | 0.09 | 0.0 | 0.0 |
| ${ }^{32} \mathrm{~S}$ | 1 | 0 | 4.70 | 2 | 0 | 2.24 | -•• | 2.4 | 3 | . . | <0.5 | 0.9 | 1.1 | 1.1 |
| ${ }^{32} \mathrm{~S}$ | 3 | 0 | 5.41 | 2 | 0 | 2.24 | - | 21 | 23 | $\cdots$ | . . | . . | . . | . . |
| ${ }^{32} \mathrm{P}$ | 2 | 1 | 0.08 | 1 | 1 | 0.00 | . . | 6.9 | 11 | 12 | $35 \pm 11$ | 54 | 34 | 12 |
| ${ }^{32} \mathrm{P}$ | 0 | 1 | 0.51 | 1 | 1 | 0.00 | 0 | 0 | 0 | 0 | $14 \pm 4$ | 5.6 | 0.4 | 2.0 |
| ${ }^{32} \mathrm{P}$ | 0 | 1 | 0.51 | 2 | 1 | 0.08 | -•• | 8.5 | . $\cdot$ | 3.0 | 0 | 0 | 0 | 0 |
| ${ }^{32} \mathrm{P}$ | 1 | 1 | 1.15 | 1 | 1 | 0.00 | . . | 9.2 | 10 | 12 | $<2$ | 8.6 | 6.8 | 7.0 |
| ${ }^{32} \mathrm{P}$ | 1 | 1 | 1.15 | 2 | 1 | 0.08 | . $\cdot$ | 6.8 | - | 11 | $<13$ | 24 | 30 | 27. |
| ${ }^{32} \mathrm{P}$ | 2 | 1 | 1.32 | 1 | 1 | 0.00 | . . | 14 | 12 | 13 | $3.3 \pm 0.9$ | 16 | 16 | 4.5 |
| ${ }^{32} \mathrm{P}$ | 2 | 1 | 1.32 | 2 | 1 | 0.08 | . . | 14 | $\cdots$ | 18 | $2.4 \pm 0.7$ | 0.0 | 2.5 | 0.0 |

[^4]and the FPSDI and MSDI states show a reversal of this property also
$$
\text { 12. } A=34, T=1\left({ }^{34} S-{ }^{34} A r\right)
$$
(See Tables XVII and XVIII, Figs. 15 and 16)
The level structure of ${ }^{34} \mathrm{~S}$ has been thoroughly explored up through 5 MeV of excitation energy by sin-gle- and double-nucleon stripping and pickup experiments, ${ }^{53,54}$ and by $\gamma$-ray decay studies. ${ }^{55}$ The model results for energies, spectroscopic factors, and $B(E 2)$ 's and $B(M 1)$ 's are in over-all excellent agreement with the experimental picture. The two calculations differ in their predictions for the properties of the $3^{+}$and $4^{+}$states, the MSDI spectrum yielding more concentration of $d_{5 / 2}$-hole strength in the lowest state of each spin. The experimental data on this point favor the MSDI results.
The pickup properties of the lowest $J^{\pi}=2^{+}, T=1$ states also differ between the FPSDI and MSDI calculations. These differences have their origin in the same sort of wave-function differences discussed for the $1^{+}$states of ${ }^{34} \mathrm{Cl}$. The FPSDI wave function has a larger $\left(d_{5 / 2}\right)^{12}\left(s_{1 / 2}\right)^{4}\left(d_{3 / 2}\right)^{2}{ }_{J=2, T=1}$ component than does the MSDI state ( 28 to $18 \%$ ) and this explains part of the larger value for $d_{3 / 2}$ pick up to the FPSDI state. Similarly, the MSDI state has a larger $\left(d_{5 / 2}\right)^{12}\left(s_{1 / 2}\right)^{3}{ }_{J=1 / 2, T=1 / 2}\left(d_{3 / 2}\right)^{3}{ }_{J=3 / 2, T=1 / 2}$ component than does the FPSDI state (15 to 7\%),
and this term is the dominant influence on the $s_{1 / 2}$ pickup.

The static quadrupole moment predicted for the first excited $2^{+}$state has an oblate value, as compared with a somewhat larger prolate value obtained for ${ }^{32}$ S. (See Table XIX.) The $E 2$ transition strengths which have been measured between the various excited states and from the excited states to the ground state are consistently close to the FPSDI predictions and hence also are in contrast to the ${ }^{32}$ S situation, where predicted strengths to the first $2^{+}$were too weak.

## IV. SUMMARY AND CONCLUSIONS

In this section we summarize the rather detailed results presented above. We shall discuss in turn results for energy levels, spectroscopic factors, $E 2$ observables, and $M 1$ observables.
The most extensive and unambiguous set of experimental data for these nuclei is on the spins and excitation energies of low-lying states. For these quantities, the calculations are in good overall agreement with experiment. There is relatively little qualitative difference in the calculated results obtained with MSDI or FPSDI, insofar as agreement with experiment is concerned. The considerable amount of freedom introduced in going from MSDI to FPSDI does not appear to lead to commensurate improvement in the agreement with experiment. The most marked differences be-


FIG. 14. Observed and calculated spectra for $A=34, T=0$ (see Table XVI). The conventions of the presentation are explained in the caption to Fig. 2.
tween MSDI and FPSDI results are in the calculated spectrum of ${ }^{34} \mathrm{C}$, where the FPSDI spectrum is in better agreement with experiment than is the MSDI spectrum.
There are also many experimental data on sin-gle-nucleon transfer reactions involving the nuclei
studied here. These data can be used in the discussion of three aspects of the nuclei concerned: (1) the occupation numbers in each active shellmodel orbit for the ground states of target nuclei;
(2) the energy centroids of single-particle and sin-gle-hole strengths in the residual nuclei; and

TABLE XVI. Energies $E$ and spectroscopic factors $S$ for the states of $A=34, T=0\left({ }^{34} \mathrm{Cl}\right)$. The conventions of the presentation are explained in the caption to Table VI. The target state for the stripping transfers ( $S_{\mathrm{st}}$ ) is $A=33, T=\frac{1}{2}, J^{\pi}$ $=\frac{3}{2}{ }^{+}\left({ }^{33} \mathrm{~S}\right)$. The target state for the pickup transfers $\left(S_{\mathrm{pk}}\right)$ is $A=35, T=\frac{1}{2}, J^{\pi}=\frac{3+}{2}\left({ }^{35} \mathrm{Cl}\right)$.


[^5](3) the details of the distribution of single-particle and single-hole strengths among individual levels. There are, of course, difficulties in extracting unambiguous information about these three aspects from experimental data. The accuracy of any particular experimentally determined spectroscopic
factor is limited to about $30 \%$ by uncertainties in DWBA theory. Also, in the experimental determination of ground-state occupation numbers and of single-particle (or single-hole) energy centroids, it is assumed that all single-particle (or singlehole) strengths have been observed. It is impossi-

TABLE XVII. Energies $E$ and spectroscopic factors $S$ for the states of $A=34, T=1\left({ }^{34} \mathrm{~S}-{ }^{34} \mathrm{Ar}\right)$. The conventions of the presentation are explained in the caption to Table VI. The target state for the stripping transfers $\left(S_{\mathrm{st}}\right)$ is $A=33$, $T=\frac{1}{2}, J^{\pi}=\frac{3+}{2}\left({ }^{33} \mathrm{~S}\right)$. The target state for the pickup transfers $\left(S_{\mathrm{pk}}\right)$ is $A=35, T=\frac{1}{2}, J^{\pi}=\frac{3+}{2}\left({ }^{35} \mathrm{Cl}\right)$.

| A | $J$ |  | Energy E (MeV) |  |  |  | $100 \times\left(S_{0} ; S_{2}\right)$ <br> Expt. ${ }^{\text {b, c }}$ | $100 \times\left(S_{1 / 2} ; S_{3 / 2}, S_{5 / 2}\right)$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Expt. ${ }^{\text {a }}$ | FPSDI | MSDI |  |  |  | PPSDI |  | MSDI |
| 34 | 0 | 1 | 203.64* | 203.50 | 203.32 | st ${ }^{\text {b }}$ | ; 200 | ; | 188, |  | 176, |
|  |  |  |  |  |  | $\mathrm{pk}^{\text {c }}$ | ; 86 | ; | 94, |  | 82, |
| 34 | 0 | 1 | 3.92* | 4.04 | 3.32 | st |  | ; | 0, |  |  |
|  |  |  |  |  |  | pk |  | ; | 4, |  | 14, |
| 34 | 0 | 1 | 5.22* | 5.50 | 5.40 | st |  | ; | 0, |  | 0 , |
|  |  |  |  |  |  | pk |  | ; | 0, |  | 1 , |
| 34 | 0 | 1 | 5.86* | 6.36 | 6.31 | st |  | ; | 0, | ; | 0 , |
|  |  |  |  |  |  | pk |  | ; | 0 , | ; | 0 , |
| 34 | 1 | 1 | 4.08* | 3.83 | 3.46 | st |  | 0; | 0, 0 | 0 ; | 0, 0 |
|  |  |  |  |  |  | pk | $\left(60\right.$; <28) ${ }^{\text {d }}$ | 52; | 11, 2 | 60; | 11, 2 |
| 34 | 1 | 1 |  | 5.60 | 5.04 | st |  | 6; | 8, 0 | 7; | 10, 0 |
|  |  |  |  |  |  | pk |  | 0; | 0, 0 | 0; | 0, 0 |
| 34 | 1 | 1 |  | 6.21 | 6.58 | st |  | $1 ;$ | 1, 0 | 0; | 0, 0 |
|  |  |  |  |  |  | pk |  | 2; | 0, 0 | 26; | 0, 7 |
| 34 | 1 | 1 |  | 6.90 | 7.07 | st |  | 0; | 0, 0 | 0 ; | 0, 0 |
|  |  |  |  |  |  | pk |  | 36; | 0, 7 | 0; | 0, 1 |
| 34 | 2 | 1 | 2.13* | 1.99 | 1.99 | st | ; 106 | 0; | 86, 1 | 1; | 71, 0 |
|  |  |  |  |  |  | pk | 21; <26 | 14; | 33, 1 | 32; | 12, 1 |
| 34 | 2 | 1 | 3.30* | 2.96 | 2.68 | st | ; 50 | 3; | 13, 1 | 5; | 24, 1 |
|  |  |  |  |  |  | pk | 70; <52 | 75; | 28, 7 | 69 ; | 26, 12 |
| 34 | 2 | 1 | 4.12* | 4.19 | 3.87 | st | $(60 ;<28){ }^{\text {d }}$ | 0 ; | 6, 0 | 0 ; | 9, 0 |
|  |  |  |  |  |  | pk |  | 1; | 37, 18 | 0; | 53, 12 |
| 34 | 2 | 1 | 4.89 | 4.48 | 4.30 | st |  | 6 ; | 38, 0 | 7; | 38, 0 |
|  |  |  |  |  |  | pk | (<4; 79) | 3 ; | 2, 3 | 0 ; | 4, 1 |
| 34 | 3 | 1 | (4.69) | 4.28 | 4.18 | st |  | ; | 1, 1 | ; | 2, 1 |
|  |  |  |  |  |  | pk | ( ; 41) | ; | 9, 47 |  | 7, 62 |
| 34 | 3 | 1 |  | 5.58 | 5.73 | st |  | ; | 1, 1 |  | 1, 0 |
|  |  |  |  |  |  | pk |  | ; | 6, 42 |  | 5, 57 |
| 34 | 3 | 1 |  | 6.26 | 6.60 | st |  | ; | 1, 4 |  | 1, 1 |
|  |  |  |  |  |  | pk |  | ; | 0, 0 |  | 4, 34 |
| 34 | 3 | 1 |  | 7.21 | 7.27 | st |  | ; | 0, 0 |  | 0, 1 |
|  |  |  |  |  |  | pk |  | ; | 0, 12 |  | 0, 1 |
| 34 | 4 | 1 | (4.88) | 4.36 | 5.09 | st |  | ; | , 1 |  | , 0 |
|  |  |  |  |  |  | pk | ( ; 79) | ; | , 16 |  | , 74 |
| 34 | 4 | 1 |  | 5.55 | 5.72 | st |  | ; | , 0 |  | , 0 |
|  |  |  |  |  |  | pk |  | ; | , 30 |  | , 20 |
| 34 | 4 | 1 |  | 6.20 | 6.52 | st |  | ; | , 0 |  | , 1 |
|  |  |  |  |  |  | pk |  | ; | , 23 |  | , 40 |
| 34 | 4 | 1 |  | 6.86 | 6.89 | st |  | ; | , 0 |  | , 0 |
|  |  |  |  |  |  | pk |  | ; | , 14 |  | , 20 |
| 34 | 5 | 1 |  | 6.98 | 7.06 |  |  |  |  |  |  |
| 34 | 5 | 1 |  | 8.12 | 8.42 |  |  |  |  |  |  |
| 34 | 6 | 1 |  | 7.79 | 8.17 |  |  |  |  |  |  |
| 34 | 6 | 1 |  | 8.86 | 8.84 |  |  |  |  |  |  |

${ }^{\text {a }}$ Reference 55.
${ }^{\mathrm{b}}$ Reference 53.
${ }^{\mathrm{c}}$ Reference 54.
${ }^{\mathrm{d}} 1^{+}$and $2^{+}$states experimentally unresolved.
ble to determine to what extent this assumption about the experimental data is valid. (Indeed, since we only look at our calculated $S$ factors for transitions to the lowest several states of each $J$, $T$ value, there are similar limitations on the accuracy of our estimates for the MSDI and FPSDI energy centroids of single-particle strengths.)
Within the limitations just mentioned, we find that in $A=30-35$ the observed relative occupations of the $1 s_{1 / 2}$ and $0 d_{3 / 2}$ shell-model orbits in the ground states of target nuclei are reasonably well reproduced by both shell-models.
A general idea of the accuracy with which the energy centroids of the single-particle and singlehole strengths are reproduced by these calculations can be obtained from the information presented in Figs. 3, 6, 11, and 16. In these figures, the calculated and experimentally determined sin-gle-particle and single-hole strengths in low-lying states are plotted. The plots of model results indicate that for each nucleus, most of the calcu-
lated single-particle strength for a given orbit is concentrated in one or two levels. Similarly, for each nucleus most of the calculated single-hole strength is concentrated in one or two levels. The plots of experimental results indicate that, for each nucleus, most of the experimentally observed single-particle strength is concentrated in one or two states, and that these observed strong states are at about the same energy as the calculated states for which there are significant single-particle strengths. For the single-hole strengths, the experimentally determined strengths are generally spread over more low-lying states than is the case in the model results, but it is apparent from the figures that the observed strength is generally concentrated in the same excitation-energy region as is the calculated strength. To this extent, the calculated centroids of single-particle and singlehole strengths are in agreement with experiment.
With regard to details of the distribution of strengths to individual levels, there is reasonably

TABLE XVIII. Values of $B(E 2)$ and $B(M 1)$ for transitions between states of $A=34$. The notation is explained in the caption to Table IV.

| Nuclear | Inital <br> state |  | Final state |  |  | $B(E 2)\left(e^{2} \mathrm{~F}^{4}\right)$ |  |  |  | $100 \times B(M 1)\left(\mu_{N}{ }^{2}\right)$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $J$ | $T E$ |  | T | $T E$ | Expt. ${ }^{\text {a }}$ | FPSDI | MSDI | MSDI- ${ }^{\text {a }}$ | Expt. ${ }^{\text {a }}$ | FPSDI | MSDI | MSDI- ${ }^{\text {a }}$ |
| ${ }^{34} \mathrm{Cl}$ | 1 | $0 \quad 0.46$ | 0 | 1 | 0.00 | 0 | 0 | 0 | 0 | $6.5 \pm 2.9$ | 56 | 11 | 0.0 |
| ${ }^{34} \mathrm{Cl}$ | 1 | $0 \quad 0.46$ | 3 | 0 | 0.15 | -•• | 14 | 0.7 | 0.6 | 0 | 0 | 0 | 0 |
| ${ }^{34} \mathrm{Cl}$ | 1 | 0 , 0.66 | 0 | 1 | 0.00 | 0 | 0 | 0 | 0 | $1.2 \pm 0.0$ | 0.3 | 34 | 32 |
| ${ }^{34} \mathrm{Cl}$ | 1 | 00.66 | 3 | 0 | 0.15 | - | 10 | 24 | 25 | 0 | 0 | 0 | 0 |
| ${ }^{34} \mathrm{Cl}$ | 2 | 01.23 | 0 | 1 | 0.00 | <0.4 | 0.06 | 0.09 | 0.0 | 0 | 0 | 0 | 0 |
| ${ }^{34} \mathrm{Cl}$ | 2 | 01.23 | 3 | 0 | 0.15 | $<13$ | 0.01 | 0.5 | 0.6 | <0.2 | 0.3 | 0.2 | 0.2 |
| ${ }^{34} \mathrm{Cl}$ | 2 | 01.23 | 1 | 0 | 0.46 | <46 | 20 | 2.1 | 2.5 | <0.12 | 0.08 | 0.09 | 0.0 |
| ${ }^{34} \mathrm{Cl}$ | 2 | 01.23 | 1 | 0 | 0.66 | <39 | 12 | -•• | 40 | <0.7 | . . | 0.9 | 1.1 |
| ${ }^{34} \mathrm{Cl}$ | 2 | 12.16 | 0 | 1 | 0.00 | $78 \pm 34$ | 42 | 50 | 54 | 0 | 0 | 0 | 0 |
| ${ }^{34} \mathrm{Cl}$ | 4 | 02.38 | 3 | 0 | 0.15 | $52 \pm 13$ | 55 | 54 | 57 | $0.03 \pm 0.03$ | 0.2 | 0.2 | 0.2 |
| ${ }^{34} \mathrm{~S}$ | 2 | 12.13 | 0 | 1 | 0.00 | $41 \pm 4$ | 35 | 50 | 55 | 0 | 0 | 0 | 0 |
| ${ }^{34} \mathrm{~S}$ | 2 | 13.30 | 0 |  | 0.00 | $6.1 \pm 1.2$ | 5.6 | 0.9 | 0.7 | 0 | 0 | 0 | 0 |
| ${ }^{34} \mathrm{~S}$ | 2 | 13.30 | 2 |  | 2.13 | $22 \pm 5$ | 35 | 70 | 72 | 12.3 | 3.3 | 4.5 | 3.0 |
| ${ }^{34} \mathrm{~S}$ | 0 | 13.91 | 2 |  | 2.13 | $26 \pm 9$ | 12 | $\cdots$ | 14 | 0 | 0 | 0 | 0 |
| ${ }^{34} \mathrm{~S}$ | 1 | 14.07 | 0 | 1 | 0.00 | 0 | 0 | 0 | 0 | >2.1 | $\cdots$ | 4.1 | 2.9 |
| ${ }^{34} \mathrm{~S}$ | 1 | 14.07 | 2 | 1 | 2.13 | $\cdots$ | 4.5 | 2.9 | 3.3 | >5.5 | 30 | 36 | 9.0 |
| ${ }^{34} \mathrm{~S}$ | 2 | 14.13 | 0 | 1 | 0.00 | . . | 2.2 | 6.5 | -•• | 0 | 0 | 0 | 0 |
| ${ }^{34} \mathrm{~S}$ | 2 | 14.13 | 2 |  | 2.13 | $\cdots$ | 0.1 | 0.6 | $\cdots$ | $\cdots$ | 23 | 1.2 | $\cdots$ |
| ${ }^{34} \mathrm{~S}$ | 2 | 14.13 | 2 | 1 | 3.30 | . . | 2.1 | 9.3 | ... | . $\cdot$ | 19 | 3.3 | $\cdots$ |
| ${ }^{34} \mathrm{~S}$ | 3 | 14.13 | 2 |  | 2.13 | . . | 3.3 | 0.08 | $\cdots$ | $\cdots$ | 3.2 | 0.9 | - $\cdot$ |
| ${ }^{34} \mathrm{~S}$ | 3 | 14.13 | 2 |  | 3.30 | ... | 93 | 105 | . . | . | 0.6 | 0.8 | . $\cdot$ |
| ${ }^{34} \mathrm{~S}$ | 4 | 14.13 | 2 |  | 2.13 | . . | 35 | 44 | $\cdots$ | 0 | 0 | 0 | 0 |

${ }^{\text {a }}$ Reference 26.



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FIG. 15. Observed and calculated spectra for $A=34, T=1$ (see Table XVII). The conventions of the presentation are explained in the caption to Fig. 2.

TABLE XIX. Static electric quadrupole and magnetic dipole moments in $A=30-35$.

| Nucleus | $J_{n}^{\pi}$ | $\begin{gathered} Q 2\left(e \mathrm{~F}^{2}\right) \\ \text { Calc. } \end{gathered}$ |  |  |  | $M M\left(\mu_{N}\right)$ Calc. |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Expt. | FPSDI | MSDI | MSDI-A ${ }^{\text {a }}$ | Expt. | FPSDI | MSDI | MSDI-A ${ }^{\text {a }}$ |
| ${ }^{30} \mathrm{P}$ | $1^{+}$ |  | +3.2 | +0.8 | +0.8 |  | +0.64 | +0.71 | +0.70 |
| ${ }^{30} \mathrm{Si}$ | $2^{+}$ |  | -6.6 | -6.4 | ... |  | . . | . | . |
| ${ }^{31} \mathrm{P}$ | $\frac{1}{2}+$ |  | 0 | 0 | 0 | +1.13 | +1.61 | +1.58 | +0.91 |
| ${ }^{31} \mathrm{Si}$ | $\frac{3}{2}+$ |  | -7.4 | . . | -8.0 |  | +0.92 | +1.03 | +0.41 |
| ${ }^{32} \mathrm{~S}$ | $2^{+}$ | $-20 \pm 6^{\text {b }}$ | -13.6 | -13.8 | ... |  | +1.02 | . . | . . . |
| ${ }^{32} \mathrm{P}$ | $1^{+}$ |  | -4.5 | -3.7 | -4.0 | -0.24 | -0.02 | 0.00 | -0.25 |
| ${ }^{33} \mathrm{~S}$ | $\frac{3+}{2+}$ | -5.5 | -7.1 | -7.8 | -8.4 | +0.64 | +0.97 | +1.06 | +0.65 |
| ${ }^{33} \mathrm{P}$ | $\frac{1}{2}+$ |  | 0 | 0 | 0 |  | +1.52 | +1.54 | +0.86 |
| ${ }^{34} \mathrm{Cl}$ | $3^{+}$ |  | -17 | -18 | . . . |  | 1.35 | +1.37 | . . . |
| ${ }^{34} \mathrm{~S}$ | $2^{+}$ |  | +6.7 | +2.5 | $\ldots$ |  | +0.96 | +1.31 | -•• |
| ${ }^{35} \mathrm{Cl}$ | $\frac{3}{2}+$ | $-7.9$ | -9.0 | ... | -.. | +0.82 | +0.45 | +0.51 | - . |
| ${ }^{35} \mathrm{~S}$ | $\frac{3}{2}+$ | $+4$ | +5.8 | -•• | . $\cdot$ |  |  |  |  |

[^6]${ }^{\mathrm{b}}$ Reference 49.
good agreement for the strong transitions between theory and experiment. Both theoretically and experimentally, the lowest $\frac{1}{2}^{+}$and $\frac{3^{+}}{}{ }^{+}$states in each odd-mass system carry the preponderance of $s_{1 / 2}$ and $d_{3 / 2}$ single-particle and single-hole strengths. The lowest $\frac{5}{2}^{+}$states in $A=31$ and in $A=33, T=\frac{3}{2}$ carry most of the $d_{5 / 2}$ hole strength, but in $A=33$, $T=\frac{1}{2}$ and $A=35, T=\frac{1}{2}$ the second $\frac{5}{2}^{+}$states have the major $d_{5 / 2}$ hole strength.
In summary, the spectroscopic factor calculations indicate that the shell-model wave functions generally have single-particle characteristics consistent with those inferred from experimental observations.

The results of the calculations of $B(E 2)$ values (with effective charges, as discussed above) can be summarized as follows:
(1) In almost all cases, observed strong transitions are calculated to be strong, and weak transitions are calculated to be weak.
(2) For strong transitions, there is fair-to-good quantitative agreement of calculation with experiment for the transition strengths.
(3) The relative strengths of weak transitions are not well reproduced by the calculation.
The most obvious exceptions to generalizations (1) and (2) are for the $A=30$ nuclei. The slightly different effective $E 2$ operator used by Glaudemans, Endt, and Dieperink ${ }^{26}$ with the MSDI wave functions leads to results which can be summarized by these same three statements.

The qualitative features of the $M 1$ transition strengths are reproduced by the calculations; i.e., observed strong transitions are calculated to be strong, and similarly for moderately strong tran-
sitions and weak transitions. The quantitative agreement between models and experiment for the strengths of strong and moderate transitions is distinctly poorer for $M 1$ transitions than for $E 2$ transitions. For a number of the strong transitions, the use of the effective $M 1$ operator of Ref. 26 yields superior results to those obtainable with an unrenormalized $M 1$ operator. (However, there are also a number of transitions for which this is not the case.) This general assessment of the $M 1$ transition calculations is very similar to that made for analogous shell-model calculations in the $A=18-22$ region. ${ }^{10}$
Next we consider our energy, spectroscopic factor, and electromagnetic results as functions of mass number $A$. We see that better agreement with experiment is obtained as $A$ increases. The main culprit here is the $A=30$ system. The nuclei with $A=30$ are the closest in mass to the semi-closed-shell nucleus ${ }^{28} \mathrm{Si}$, and it is possible that the restrictions on $d_{5 / 2}$ excitations are most critical for these $A=30$ nuclei. This is obviously only speculation. It suggests the need for a more careful investigation of the significance of the $s d$-shell configurations which have been omitted from our active model space for $A=30$. For the $A=31$ to $A=35$ nuclei, the discrepancies between our model results and experiment are such that they could easily arise from relatively small errors in detail in our effective Hamiltonian and the other effective operators we use.
An important goal of this study was to determine whether straightforward shell-model techniques could give a detailed accounting for experimental data on low-lying states in nuclei which are many


FIG. 16. Diagrams of observed and calculated spectroscopic factors for transfer to states of $A=34, T=1$ (see Table XVII). The conventions of the presentation are explained in the caption to Fig. 3.
particles (or holes) removed from major shell closures. The nuclei of $A=30-34$ have observed characteristics which suggest that configurations of nucleons in the three orbits of the $s d$ shell should be sufficient to allow a good theoretical description of their structure. The general questions which arose about the approach we have used in this work were: (1) Could enough $s d$-shell configurations be removed from the full $s d$-shell space to make calculations practically feasible without obviously limiting the capability of the model to account for experimental phenomena, and (2) could a specification of the model Hamiltonian be found which was both "reasonable" and "successful"?
There are unavoidable ambiguities in evaluating which aspects of a nuclear model calculation depend mainly on assumptions about the vector space and which depend mainly on assumptions about the Hamiltonian. We initially used an inflexible Hamiltonian form (the MSDI) that was definitely not the most sophisticated possible, but one that did embody a basic attribute of any nuclear effective interaction. Investigation showed that most major aspects of the phenomena characteristic of the region of interest could be accounted for with this combination of model space and Hamiltonian as sumptions. Because of the constrained form of the MSDI, we took this success as an implication that the assumed model space was generally adequate for the region studied. We then attempted to find out if an empirically altered form of the Hamiltonian (the FPSDI) could further improve the theoretical situation with respect to observation. Qualitative improvements did not result from this approach, possibly because we started the search which led to the FPSDI Hamiltonian with the MSDI wave functions, possibly because the MSDI results were already close to the best that could be obtained in our limited model space.

In our presentation of this study we have valued over-all consistency in the agreement between theory and experiment for a given kind of observable more than a situation in which some expectation values were in very close agreement and others of the same observable in very poor agreement. Likewise, we valued comprehensive accounting for a variety of observables over close agreement for one type of observable at the expense of failing to even qualitatively account for some other aspect of the region.

Our attitude was and is that a fundamental virtue of the shell model is its potential to account for a variety of nuclear phenomena with a unified and constant set of assumptions. We have tried to see just how far toward an understanding of observed data we could come with this approach. In particular, we treated all nuclei and all levels using ex-
actly the same set of assumptions. The only variation from nucleus to nucleus was the number of particles. Thus, we feel that the totality of our results for the $A=30-35$ nuclei have an import greater than merely the sum of the successes for individual spectra or for specific kinds of observables.

## APPENDIX

In this section we present details of the FPSDImodel wave functions for the lowest energy states of each $A, T$ system. The material is arranged in order of increasing mass and isospin. The conventions of the presentation are as follows: A particular model state is labeled by its mass number (A), twice its total angular momentum ( $2 J^{\pi}$ ), twice its total isospin ( $2 T$ ), its calculated binding energy $(E)$, and an ordinal number, denoting whether it is the 1 st , 2 nd , 3 rd , etc., lowest state for the particular $A, J, T$ combination in question. Also noted on this initial identifying line are the number of core particles in the model ( 16 in all present cases) and the dimensionality (number of basis states) for that $A, J, T$.
Following this first line of information, all components of the wave function of this state whose amplitudes have an absolute values greater than 0.1414 are listed and identified. Under each "amplitude" value is a triplet of columns, one for each of the three $s d$-shell orbits $\left(0 d_{5 / 2}=D 5,1 s_{1 / 2}\right.$ $=S 1$, and $0 d_{3 / 2}=D 3$ ). The information in these columns serves to completely specify the basis vector associated with the particular amplitude in question as follows. The "configuration" triplet gives the occupation number for each of the orbits. The 2(S-shell J's) triplet gives the angular momenta to which the particles in the single orbits (shells) are separately coupled. The 2(coupled $J$ 's) doublet then gives first the angular momentum $J_{12}$ which results from coupling the angular momentum of shell $1\left(0 d_{5 / 2}\right)$ to that of shell $2\left(1 s_{1 / 2}\right)$ and then gives the total angular momentum $J$ which results from coupling $J_{12}$ to the angular momentum of shell $3\left(0 d_{3 / 2}\right)$. The 2 ( S -shell $T^{\prime} \mathrm{s}$ ) triplet and the 2 (coupled $T$ 's) doublet give analogous information about the isospin couplings. The S-S seniorities triplet lists the seniorities of the particle couplings within each orbit.
The concluding information given about the state comes on the last two lines after all components have been listed. The percentage of the total wavefunction structure that is identified in the preceding listing is noted on the first of these lines, and on the second (and last line for the particular state in question) we list the average occupation number for each orbit as calculated from the complete wave function.


$A=30,2 J^{\pi}=2^{+}, 2 T=0, E=156.492$, eigenvector 1 of this $A, J, T$. Model core $=16$, dimension $=82$.



$\stackrel{9}{7}$

Listed components account for $85.2 \%$ of the wave function.
Occupation $\quad(D 5)=11.38,(S 1)=1.93,(D 3)=0.69$.

$A=30,2 J^{\pi}=2^{+}, 2 T=0, E=155.469$, eigenvector 2 of this $A, J, T$. Model core $=16$, dimension $=82$.
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Listed components account for $83.3 \%$ of the wave function.


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$A=30,2 J^{\pi}=6^{+}, 2 T=0, E=153.860$ ，eigenvector 2 of this $A, J, T$. Model core $=16$ ，dimension $=115$ ．



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（D5）$=10.97,(S 1)=1.29, \quad(D 3)=1.74$ ．

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Model core $=16$, dimension $=50$.
$A=30,2 J^{\pi}=0^{+}, 2 T=2, E=155.521$ ，eigenvector 1 of this $A, J, T$. Model core $=16$ ，dimension $=50$.


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\end{aligned}
$$



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$A=30,2 J^{\pi}=6^{+}, 2 T=2, E=150.883$, eigenvector 1 of this $A, J, T$. Model core $=16$, dimension $=162$



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 Amplitude
S-shell labels
Configuration
2(S-shell $J$ 's)
2(coupled $J$ 's)
2(S-shell $T$ 's)
2(coupled $T$ 's)
S-S seniorities

Listed components account for $77.0 \%$ of the wave function.
Occupation $(D 5)=10.77,(S 1)=2.15,(D 3)=1.08$.


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imension $=162$ ．

| 0.282 |  |  | －0．238 |  |  | 0.692 |  |  | 0.188 |  |  | 0.262 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| D5 | S1 | D3 | D 5 | S1 | D3 | D 5 | S1 | D3 | D5 | S1 | D3 | D5 | S1 | D3 |
| 10 | 3 | 1 | 11 | 1 | 2 | 11 | 2 | 1 | 11 | 2 | 1 | 11 | 2 | 1 |
| 6 | 1 | 3 | 5 | 1 | 4 | 5 | 0 | 3 | 5 | 2 | 3 | 5 | 2 | 3 |
|  | 5 | 6 |  | 4 | 6 |  | 5 | 6 |  | 3 | 6 |  | 5 | 6 |
| 0 | 1 | 1 | 1 | 1 | 2 | 1 | 2 | 1 | 1 | 0 | 1 | 1 | 0 | 1 |
|  | 1. | 2 |  | 0 | 2 |  | 1 | 2 |  | 1 | 2 |  | 1 | 2 |
| 2 | 1 | 1 | 1 | 1 | 2 | 1 | 0 | 1 | 1 | 2 | 1 | 1 | 2 | 1 |

Listed components account for $77.3 \%$ of the wave function． Occupation $(D 5)=10.72,(S 1)=1.96,(D 3)=1.32$ ．
${ }^{+}+8={ }_{\Perp} \Gamma$＇ $08=V$
$\begin{array}{ll}\text { core }=16, & \text { dimension }= \\ & 147 . \\ 0.594 \quad 0.506\end{array}$ －
（ Mod
87，eigenvector 1 of this $A, J, T$ ．Mo






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| $\stackrel{\sim}{\square} \boldsymbol{\sim}$ | $\rightarrow \quad-$ |
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| $\stackrel{\text { ® }}{\square} \mathrm{F}$ | － |

$=150.087$
$D 5$
11
5
1
1

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\begin{aligned}
& \text { ロッ~ロ }
\end{aligned}
$$

-1000
$n_{0}^{2}=1$




${ }^{\prime}+8={ }_{4} \Gamma 6 \quad 0 \varepsilon=V$


Listed components account for $78.3 \%$ of the wave function．
Occupation $(D 5)=10.55,(S 1)=2.03, \quad(D 3)=1.42$ ．

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Listed components account for $72.7 \%$ of the wave function．
Occupation $(D 5)=10.92, \quad(S 1)=2.90, \quad(D 3)=1.17$ ．


$$
\begin{aligned}
& 69 E^{\circ} 0
\end{aligned}
$$

$\begin{aligned} & \text { Listed components account for } 71.1 \% \text { of the wave function．} \\ & \text { Occupation } \\ & (D 5)=10.91,(S 1)=2.23,(D 3)=1.86 \text { ．}\end{aligned}$
$\begin{aligned} & \text { Amplitude } \\ & \text { S－shell labels } \\ & \text { Configuration } \\ & \text { 2（S－shell } J \text {＇s）} \\ & \text { 2（coupled } J \text {＇s）} \\ & \text { 2（S－shell } T \text {＇} \mathrm{s} \text { ）} \\ & \text { 2（coupled } T \text {＇s）} \\ & \text { S－S seniorities }\end{aligned}$

$$
\begin{aligned}
& 1 \text {-1 } \\
& \text { Occupation }(D 5)=10.91,(S 1)=2.23,(D 3)=1.86 \text {. }
\end{aligned}
$$

| $A=31,2 J^{\pi}=1^{+}, 2 T=3, E=161.405$ ，eigenvector 1 of this $A, J, T$. Model core $=16$ ，dimension $=96$ |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Amplitude | －0．142 |  |  | －0．165 |  |  | －0．329 |  |  | 0.242 |  |  | －0．258 |  |  | －0．244 |  |  | －0．170 |  |  |
| S－shell labels | D 5 | S1 | D 3 | D 5 | S1 | D3 | D 5 | S1 | D3 | D 5 | S1 | D3 | D 5 | S1 | D3 | D 5 | S1 | D3 | D5 | S1 | D3 |
| Configuration | 10 | 1 | 4 | 10 | 2 | 3 | 10 | 3 | 2 | 10 | 3 | 2 | 11 | 1 | 3 | 11 | 2 | 2 | 11 | 2 | 2 |
| 2 （S－shell J＇s） | 0 | 1 | 0 | 4 | 0 | 3 | 0 | 1 | 0 | 2 | 1 | 0 | 5 | 1 | 3 | 5 | 0 | 4 | 5 | 2 | 4 |
| 2 （coupled $J$＇s） |  | 1 | 1 |  | 4 | 1 |  | 1 | 1 |  | 1 | 1 |  | 4 | 1 |  | 5 | 1 |  | 3 | 1 |
| 2 （S－shell $T$＇s） | 2 | 1 | 4 | 2 | 2 | 3 | 2 | 1 | 2 | 0 | 1 | 2 | 1 | 1 | 3 | 1 | 2 | 2 | 1 | 0 | 2 |
| 2 （coupled $T$＇s） |  | 3 | 3 |  | 0 | 3 |  | 1 | 3 |  | 1 | 3 |  | 0 | 3 |  | 1 | 3 |  | 1 | 3 |
| S－S seniorities | 0 | 1 | 0 | 2 | 0 | 1 | 0 | 1 | 0 | 2 | 1 | 0 | 1 | 1 | 1 | 1 | 0 | 2 | 1 | 2 | 2 |

Listed components account for $87.5 \%$ of the wave function．
Occupation $(D 5)=11.18,(S 1)=1.57,(D 3)=2.25$ ．

$A=31,2 J^{\pi}=1^{+}, 2 T=3, E=161.405$ ，eigenvector 1 of this $A, J, T$. Model core $=16$ ，dimension $=96$.

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Amplitude Configuration
2 （S－shell J＇s） 2 （coupled $J^{\prime} \mathrm{s}$ ）

2 （coupled $T$＇s）
2（coupled $T$＇s）
S－S seniorities

| -0.724 |  |  |
| ---: | ---: | ---: |
| $D 5$ | $S 1$ | $D 3$ |
| 12 | 2 | 1 |
| 0 | 0 | 3 |
|  | 0 | 3 |
| 0 | 2 | 1 |
|  | 2 | 3 |
| 0 | 0 | 1 |




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[^8]Listed components account for $84.2 \%$ of the wave function． Occupation $(D 5)=11.33,(S 1)=2.13,(D 3)=1.55$ ．


$A=31,2 J^{\pi}=3^{+}, 2 T=3, E=160.048$ ，eigenvector 2 of this $A, J, T$. Model core $=16$, dimension $=168$.

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S－shell labels
Configuration

2（S－shell $T$＇s）
2 （coupled $T$＇$s$ ）
$S-S$ seniorities
Listed components account for $86.9 \%$ of the wave function．
Occupation $\quad(D 5)=11.06,(S 1)=2.05, \quad(D 3)=1.88$ ．



[^9]| Amplitude | -0.276 |  |  | 0.149 |  |  | -0.171 |  |  | -0.162 |  |  | 0.377 |  |  | 0.273 |  |  | -0.156 |  |  | -0.570 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| S-shell labels | D5 | S1 | D3 | D 5 | S1 | D3 | D 5 | S1 | D3 | D5 | S1 | D3 | D5 | S1 | D3 | D5 | S1 | D3 | D5 | S1 | D3 | D5 | S1 | D3 |
| Configuration | 10 | 3 | 3 | 10 | 3 | 3 | 11 | 3 | 2 | 12 | 0 | 4 | 12 | 1 | 3 | 12 | 2 | 2 | 12 | 2 | 2 | 12 | 3 | 1 |
| 2 (S-shell J's) | 0 | 1 | 3 | 0 | 1 | 3 | 5 | 1 | 0 | 0 | 0 | 4 | 0 | 1 | 3 | 0 | 0 | 4 | 0 | 2 | 2 | 0 | 1 | 3 |
| 2 (coupled $J$ 's) |  | 1 | 4 |  | 1 | 4 |  | 4 | 4 |  | 0 | 4 |  | 1 | 4 |  | 0 | 4 |  | 2 | 4 |  | 1 | 4 |
| 2 (S-shell $T$ 's) | 2 | 1 | 1 | 2 | 1 | 3 | 1 | 1 | 2 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 2 | 2 | 0 | 0 | 0 | 0 | 1 | 1 |
| 2 (coupled $T$ 's) |  | 1 | 0 |  | 3 | 0 |  | 2 | 0 |  | 0 | 0 |  | 1 | 0 |  | 2 | 0 |  | 0 | 0 |  | 1 | 0 |
| S-S seniorities | 0 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 2 | 0 | 1 | 1 | 0 | 0 | 2 | 0 | 2 | 2 | 0 | 1 | 1 |
| Listed componen Occupation (D5) | 11.3 | , (S | $\begin{aligned} & \text { or } 72 \\ & \text { 1) }=2 \text {. } \end{aligned}$ | of | = 2 . | $\begin{aligned} & \text { ave } \\ & 32 . \end{aligned}$ |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |




| Amplitude | 0.222 |  |  | -0.270 |  |  | -0.181 |  |  | 0.636 |  |  | 0.230 |  |  | -0.414 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| S-shell labels | D 5 | S1 | D3 | D5 | S1 | D3 | D 5 | S1 | D3 | D 5 | S1 | D3 | D 5 | S1 | D3 | D 5 | S1 | D3 |
| Configuration | 10 | 4 | 2 | 11 | 2 | 3 | 11 | 2 | 3 | 11 | 4 | 1 | 12 | 1 | 3 | 12 | 2 | 2 |
| 2(S-shell J's) | 2 | 0 | 6 | 5 | 0 | 3 | 5 | 0 | 3 | 5 | 0 | 3 | 0 | 1 | 7 | 0 | 2 | 6 |
| 2 (coupled $J$ 's) |  | 2 | 8 |  | 5 | 8 |  | 5 | 8 |  | 5 | 8 |  | 1 | 8 |  | 2 | 8 |
| 2 (S-shell $T$ 's) | 0 | 0 | 0 | 1 | 2 | 1 | 1 | 2 | 3 | 1 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 |
| 2 (coupled $T$ 's) |  | 0 | 0 |  | 1 | 0 |  | 3 | 0 |  | 1 | 0 |  | 1 | 0 |  | 0 | 0 |
| S-S seniorities | 2 | 0 | 2 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 0 | 1 | 3 | 0 | 2 | 2 |















Listed components account for $75.3 \%$ of the wave function.



[^10]

$A=33,2 J^{\pi}=1^{+}, 2 T=1, E=191.244$ ，eigenvector 1 of this $A, J, T$. Model core $=16$ ，dimension $=147$ ．


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\stackrel{\bullet}{\mathrm{Q}} \mathrm{-1}
$$
$$
\ddot{A}^{+}
$$
\[

$$
\begin{aligned}
& \text { II } \\
& 0.0 \\
& 0 \\
& 0 \\
& \stackrel{\rightharpoonup}{0} \\
& 0 \\
& 0
\end{aligned}
$$
\]

$\qquad$ Listed components account for $71.0 \%$ of the wave function． Occupation $(D 5)=11.32$ ，$(S 1)=2.66,(D 3)=3.02$ ．

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\begin{aligned}
& \text { かのサのベか }
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& \text { ロッののールー }
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\begin{aligned}
& \text { ロッのかのート }
\end{aligned}
$$

$$
\begin{aligned}
& \begin{array}{l}
\text { Listed components account for } 78.4 \% \text { of the wave function. } \\
\text { Occupation }(D 5)=11.41,(S 1)=3.39,(D 3)=2.20 \text {. }
\end{array} \\
& \begin{array}{l}
\text { Amplitude } \\
\text { S-shell labels } \\
\text { Configuration } \\
\text { 2(S-shell } J \text { 's) } \\
\text { 2(coupled } J \text { 's) } \\
\text { 2(S-shell } T \text { 's } \\
\text { 2(coupled } T \text { 's) } \\
\text { S-S seniorities } \\
\text { Listed compone } \\
\text { Occupation }(D 5)
\end{array}
\end{aligned}
$$



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$$
\begin{aligned}
& \text { Amplitude } \\
& \text { S-shell labels } \\
& \text { Configuration } \\
& \text { 2(S-shell } J ' s \text { ) } \\
& \text { 2(coupled } J \text { 's) } \\
& \text { 2(S-shell } T \text { 's) } \\
& \text { 2(coupled } T \text { 's) } \\
& \text { S-S seniorities }
\end{aligned}
$$

Listed components account for 65.1 \％of the wave
Occupation $(D 5)=11.32,(S 1)=2.78,(D 3)=2.90$ ．




\[

\]

| 0.169 |  |  |  |
| ---: | ---: | ---: | :---: |
| $D 5$ | $S 1$ | $D 3$ |  |
| 12 | 3 | 2 |  |
| 0 | 1 | 6 |  |
|  | 1 | 5 |  |
| 0 | 1 | 0 |  |
|  | 1 | 1 |  |
| 0 | 1 | 2 |  |



Occupation $(D 5)=11.34,(S 1)=2.83,(D 3)=2.83$.






$A=33,2 J^{\pi}=5^{+}$,

Listed components account for $72.3 \%$ of the wave function.
Occupation $\quad(D 5)=11.14,(S 1)=3.06, \quad(D 3)=2.80$.

[^11]
$A=34,2 J^{\pi}=2^{+}, 2 T=0, E=203.016$, eigenvector 2 of this $A, J, T$. Model core $=16$, dimension $=109$.





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$\vdots$
$i=1$
$i$
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$i$

Listed components account for $73.7 \%$ of the wave function.
Occupation $(D 5)=11.38,(S 1)=3.32,(D 3)=3.30$.


Listed components account for $69.4 \%$ of the wave function.
Occupation $(D 5)=11.18$, (S1) $=3.00$, (D 3$)=3.82$.


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$A=34,2 J^{\pi}=6^{+}, 2 T=0, E=203.914$ ，eigenvector 1 of this $A, J, T$. Model core $=16$, dimension $=149$.
－
星

## －



[^12]$A=34,2 J^{\pi}=2^{+}, 2 T=2, E=199.663$ ，eigenvector 1 of this $A, J, T$ ．Model core $=16$ ，dimension $=148$ ．

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S－shell labels S－shell labels
 2 （S－shled $J$＇s）放
 S－S seniorities Listed components account for $72.5 \%$ of the wave function． Occupation $(D 5)=11.23,(S 1)=2.86,(D 3)=3.91$ ．


$A=34,2 J^{\pi}=4^{+}, 2 T=2, E=201.511$ ，eigenvector 1 of this $A, J, T$. Model core $=16$ ，dimension $=218$ ． $\begin{array}{ccc}T . & \\ -0.157 & -0.152 & 0.274\end{array}$

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| Amplitude | -0.172 |  |  |
| :--- | ---: | ---: | ---: |
| S－shell labels | $D 5$ | $S 1$ | $D 3$ |
| Configuration | 10 | 3 | 5 |
| 2（S－shell $J ' s)$ | 0 | 1 | 3 |
| 2（coupled $J ' s)$ |  | 1 | 4 |
| 2（S－shell $T$＇s） | 2 | 1 | 3 |
| 2（coupled $T$＇s） |  | 1 | 2 |
| S－S seniorities | 0 | 1 | 1 | Listed components account for $75.7 \%$ of the wave function． Occupation $\quad(D 5)=11.40,(S 1)=3.25, \quad(D 3)=3.35$ ．



Model core $=16$ ， dimension $=218$ ．

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T•0－
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$\begin{array}{llllll}\boldsymbol{-} & +1 & 0 & 0 & -1 & 0 \\ 10 & -1 & 0 & -1 & -1\end{array}$

$\circ$


がロサウNNの Listed components account for $62.2 \%$ of the wave function．

Occupation $(D 5)=11.25$ ，（S1）$=2.77$ ，（D 3$)=3.98$ ，


OCPMion（D5） $11.25,(\mathrm{~S})=2.77$, （D3）$=3.98$
$A=34,2 J^{\pi}=6^{+}, 2 T=2, E=199.220$ ，eigenvector 1 of this $A, J, T$. Model core $=16$ ，dimension $=208$ ．








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|  |
| $i$ |
| $i$ |

$\begin{array}{llllll}\boldsymbol{H} & +0 & \text { 上 } & 0 & -1 & 0 \\ \text { 上 } & -1 & \circ & -1 & -1\end{array}$



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Amplitude
S－shell labels
号 $\widehat{n}$
0
0
0
0
0
1
1
$n$
0

2 （coupled $J ' s$ ）
2 （S－shell $T$＇s）
2 （coupled $T$＇s）
S－S seniorities
Listed components account for $67.7 \%$ of the wave function．
Occupation $\quad(D 5)=11.11, \quad(S 1)=2.92, \quad(D 3)=3.96$ ．
*Research supported in part by the National Science Foundation.
$\dagger$ Research supported by the U. S. Atomic Energy Commission under contract with Union Carbide Corporation.
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[^0]:    ${ }^{\mathrm{b}}$ Reference 28.
    ${ }^{\mathrm{d}}$ Reference 30.
    ${ }^{\mathrm{f}}$ Reference 32.

[^1]:    ${ }^{\mathrm{b}}$ Reference 27.

[^2]:    ${ }^{\text {a }}$ Reference 26.

[^3]:    ${ }^{\mathrm{a}}$ Reference 40 .

[^4]:    ${ }^{\text {a }}$ Reference 26.

[^5]:    ${ }^{\mathrm{a}}$ Reference 51. $\quad{ }^{\mathrm{b}}$ Reference 52. ${ }^{\mathrm{c}}$ Reference 53. ${ }^{\mathrm{d}}$ Reference 36.

[^6]:    ${ }^{\text {a }}$ Reference 26.

[^7]:    Listed components account for $88.4 \%$ of the wave function．
    $\begin{aligned} & \text { Occupation } \\ & (D 5)=11.23,(S 1)=1.46, \quad(D 3)=1.31 .\end{aligned}$

[^8]:    Amplitude
    S－shell labels Configuration 2 （S－shell $J$＇s）
    2 （coupled $J$＇s） 2（S－shell $T$＇s）
    2 （coupled $T$＇s） S－S seniorities

[^9]:    
    
    ํㅛㅁㅇㅇ O

    ロののN゙ロー
    $A=32,2 J^{\pi}=2^{+}, 2 T=0, E=178.481$ ，eigenvector 1 of this $A, J, T$. Model core $=16$ ，dimension $=128$.
    
    
    Listed components account for $79.9 \%$ of the wave function．
    Occupation $(D 5)=11.35,(S 1)=2.68,(D 3)=1.97$ ．

[^10]:    MNOONHO
    會がらOONNO
    ロップ○ ○

    ジロOHHO
    

    シットロ～が
    

    ロのツ○のが
    
    

    NNOONHO
    
    I

    붑웅 N
    
    Listed components account for $88.6 \%$ of the wave function．
    $\begin{aligned} & \text { Occupation } \\ & (D 5)=11.25,(S 1)=2.13,\end{aligned}(D 3)=2.61$ ．

[^11]:    Amplitude
    S-shell labels
    
    解
    2 (S-shell $T$ 's)
    2 (coupled $T$ 's)
    $\mathrm{S}-\mathrm{S}$ seniorities

[^12]:    
    
    8
    7
    $\div$
    $i$
    
    
    
    
    
    
    
    Listed components account for $88.3 \%$ of the wave function．
    Occupation $(D 5)=11.08,(S 1)=2.55,(D 3)=4.37$ ．

