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THE EFFECTIVE TWO-NUCLEON INTERACTION FROM INELASTIC PROTON SCATTERING*

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Since inelastic scattering cross-sections depend on a nuclear matrix element containing both the wave functions of the nuclear states involved and the effective interaction V_{eff} between the projectile and the target nucleons,[1] it is necessary to have an a priori value for V_{eff} before one can use inelastic scattering as a tool for nuclear spectroscopy. An approach to determining V_{eff} which I will explore in detail in this paper, is entirely empirical in nature. One examines transitions where the wave functions are well known and adjusts the strength of the two-body interaction to fit the observed cross-sections. If the results for a representative sample of cases are consistent one has obtained a workable interaction for spectroscopic studies.

This approach was tried by Satchler[2] in his early studies of inelastic scattering, but was unsuccessful for reasons which we now understand fairly well. The major difficulty was that the states involved were collective in nature so that the cross sections were greatly enhanced. On the other hand,only simple wave functions were used in the theoretical calculations, so single-particle-size cross sections were predicted, and when the two-body potential was adjusted to fit the measured cross sections, the resulting potential was unphysically large. Since the collective enhancement depends on the nucleus, the state involved, and the angular-momentum transfer (L) of the transition, the effective interaction obtained was also nucleus, state and L dependent. In addition the calculations neglected exchange contributions which depend on L and the bombarding energy,[1] and again the empirical V_{eff} mirrored this dependence. For these reasons it was not possible to find a consistent effective-interaction for the strong, spin independent part of the force. Studies[2-9] of L=0 transitions involving spin-flip or isospin-flip were more successful

since both collective enhancements and exchange effects are relatively small in these cases.

Faced by this difficulty several investigators[10-12] introduced "realistic" effective interactions derived in a more-or-less plausible fashion from the free two-body force. As Schaeffer and McManus have shown at this symposium, this approach has been relatively successful and has led to an understanding of the importance of collective enhancement and of the contribution of exchange processes. However, the derivation of these forces is not free of uncertainties. For example, in approaches[10,12] using the Moszkowski-Scott[13] separation method it is necessary to neglect the odd-state forces since the method cannot be applied to repulsive forces. The effect of this omission is not clear and is particularly serious for the parts of the force responsible for spin-flip and isospin-flip transitions since they are given as the small differences of large numbers. It is worth noting that the validity of the realistic forces has been checked in detail only for the spin-isospin-independent part of the force. It is also difficult to evaluate the accuracy of approximations made in obtaining the interaction and elsewhere in the theory. In the empirical approach, on the other hand, the consistency of the results obtained provides an immediate measure of the accuracy of the calculation, with the additional advantage that inaccuracies which affect all transitions in the same fashion are automatically corrected through a renormalization of the effective force.

For several reasons the time seems ripe for a return to the empirical procedure. First, we now understand the importance of the exchange process and can do calculations including exchange; secondly, we have a good understanding of when wave functions are adequate and in many cases electron scattering data is available as a check; and thirdly, enough data are available so we have some chance of isolating individual parts of the two-body interaction. The next section of this paper contains a brief outline of the theoretical formalism. This is followed by a review of experimental information on the central (or scalar) part of V_{eff}. Finally, in the last part of the paper, recent work on the tensor and spin-orbit parts of the force is discussed and available information is summarized.

THEORETICAL CONSIDERATIONS

In the distorted wave approximation (DWA) neglecting exchange, the cross-section for a reaction A(a,b)B is proportional to the square of a transition amplitude

$$T_{ba} = \int \chi_b^{(-)} \langle \psi_f | V_{eff} | \psi_i \rangle \chi_a^{(+)} d\vec{r}, \qquad (1)$$

where the χ's are distorted waves generated in an appropriate optical potential. The form factor $\langle \psi_f | V_{eff} | \psi_i \rangle$ depends on both the wave functions ψ of the states involved and the effective interaction.[14] It is assumed that V_{eff} can be written as the sum of the two-body interactions between the projectile "p" and the target nucleons "i",

$$V_{eff} = \Sigma V_{ip} \qquad (2)$$

where the sum is over the valence nucleons.

For the central part of the force we have

$$V_{ip}(r) = V_{00}(r) + V_{10}(r)\vec{\sigma}_i\cdot\vec{\sigma}_p + V_{01}(r)\vec{\tau}_i\cdot\vec{\tau}_p + V_{11}(r)(\vec{\sigma}_i\cdot\vec{\sigma}_p)(\vec{\tau}_i\cdot\vec{\tau}_p). \qquad (3)$$

The subscripts on the V_{ST} are the spin and isospin transferred in the reaction. For mneumonic purposes we will often write $V_o=V_{00}$, $V_\sigma=V_{10}$, $V_\tau=V_{01}$, $V_{\sigma\tau}=V_{11}$, since for example, V_{11} is responsible for a reaction involving both spin and isospin flip.

The selection rules describing the scattering then determine which of the V_{ST} are important in any particular reaction. For the direct (non-exchange) process these are[14]

$$\vec{J}=\vec{J}_f-\vec{J}_i \qquad \vec{T}=\vec{T}_f-\vec{T}_i$$
$$\vec{S}=\vec{s}_i-\vec{s}_f \qquad \pi_i\pi_f=(-1)^L. \qquad (4)$$
$$\vec{L}=\vec{J}-\vec{S}$$

Here $\vec{J}$, $\vec{S}$ and $\vec{L}$ are the total, spin and orbital angular momenta transferred in the reaction and T is the transferred isospin. The subscripted quantities refer to initial(i) and final(f) states, π denoting the parity of these states. For the case of spin-one-half projectiles we have

$$S=0,1 \qquad (5)$$
$$T=0,1.$$

As an example we apply these rules to a transition from a $J_i=1^+$, $T_i=0$ state to a $J_f=0^+$, $T_f=1$ state (denoted as $(1^+,0)\to(0^+,1)$). We find J=1; L=0,2; S=1; T=1, which means that only $V_{ST}=V_{11}$ can contribute to the cross-section. Table I shows transitions which isolate other parts of the force. The important case of the excitation of a collective natural parity state, $(0^+,0)\to(2^+,0)$ for example, does not precisely isolate V_o since both S=0 and 1 are allowed. However, the S=1 amplitude is not enhanced by collective effects and is usually negligible so only V_o is important.

TABLE I.--Transitions Isolating V_o, V_σ, V_τ, $V_{\sigma\tau}$.

V_{ST}	Transition[a]	Reaction
V_o	$(0^+,0)\rightarrow(0^+,0)$	(p,p')
V_σ	$(0^+,0)\rightarrow(2^-,0)$	(p,p')
V_τ	$(0^+,1)\rightarrow(0^+,1)$	(p,n) to analog state
$V_{\sigma\tau}$	$(1^+,0)\rightarrow(0^+,1)$	(p,p')

[a]The notation is $(J_i^{\pi_i}, T_i)\rightarrow(J_f^{\pi_f}, T_f)$.

Of course the selection rules apply strictly only to the direct part of the scattering amplitude, and in the general case all V_{ST} can contribute to the exchange amplitude. However, if the effective interaction acts only in even states (or only in odd states) the force contributing to the exchange amplitude is the same (within a sign) as that in the direct amplitude.[12,15,16] We will find that the force obtained from our analysis is quite similar to a Serber force (V_o: V_σ: V_τ: $V_{\sigma\tau}$=-3:1:1:1) which is an even state force, so we are still able to isolate single V_{ST}. Angular momentum transfers not satisfying $\pi_i\pi_f=(-1)^L$ are also allowed in exchange, but in most cases studied to date these amplitudes are small[15] and we shall not consider them here.

Although the discussion has been carried out for inelastic scattering reactions, we have used an isospin formalism, so the calculations for (p,n) reactions are formally the same, only isospin quantum numbers being changed.

EXPERIMENTAL INFORMATION ON THE V_{ST}

A large number of analyses of inelastic scattering and charge exchange data are available in the literature. However, the great bulk of these were done with inadequate wave functions and neglecting exchange, or have other defects which prevent the inclusion of their results in this review.

The assumptions and criteria used to select data were:

1) The interaction has a Yukawan radial dependence with a range μ of 1.0F, or else the reaction has L=0 so we can obtain an estimate of the strength of an equivalent 1.0F Yukawa by matching the volume integrals of the forces. Calculations in a few special cases have shown that this procedure is not accurate for L>0.[16,17] The choice of 1.0F as the standard range was made primarily because it is the most commonly used range and shapes of angular distributions are not strongly affected by changes of μ, at least for μ between 0.7 and 1.4F.[17] However, Table II shows that this value of the range also yields a mean-square radius for the potential which is roughly equal to that for the long-range part of the Hamada-Johnston potential.[18]

2) The transition isolates a single one of the V_{ST}. This effectively restricts us to self-conjugate nuclei (whose ground states have T=0) since otherwise both V_{S0} and V_{S1} can contribute to the cross section.

TABLE II.--Yukawas Matched to the Hamada-Johnston Potential[a].

ST	00	01	10	11
V_{ST}(MeV)[b]	24.8	12.8	4.6	8.3
μ_{ST}(F)[b]	1.06	0.98	1.18	1.06

[a]These Yukawas $\frac{e^{-r/\mu}}{r/\mu}$ have the same volume integral and value of of $\langle r^2\rangle$ as the long range part of the Hamada-Johnston potential.

[b]Adapted from Table I of Ref. 12.

3) In the case of V_o, where collective effects enhance the relevant cross sections, we require that the wave functions properly describe the electromagnetic transitions between the states involved. In some cases electron scattering data were directly used in the analysis[19] and in some other cases effective charge techniques[15] were used to renormalize the inelastic scattering amplitudes.

4) Exchange effects were small (L=0), or the calculation included exchange (L≠0). In the L=0 case, values of V_{ST} obtained were decreased by 20% (15% above 30 MeV), corresponding to the average contribution[12,15] of exchange to such cross sections. This requirement unfortunately eliminated the large number of analyses using macroscopic core-polarization techniques. In a few cases data were reanalyzed to include exchange effects using the code DWBA 70.[20]

5) A Serber exchange mixture was assumed in the analysis. This choice is roughly consistent with what one expects from the Hamada-Johnston force (see Table II) and with the results of the present review.

Empirical optical model fits to elastic scattering data were also used to provide estimates of V_o and V_τ. The real part U of the optical model potential for a self-conjugate nucleus is given in first order by folding the nuclear matter density $\rho(r')$ with the two-body interaction between the projectile and the target nucleons[21]

$$U(r)=\int\rho(r')V_o(|\vec{r}-\vec{r}'|)d\vec{r}'. \tag{6}$$

One can then show that

$$\int V_o(r)d\vec{r}=\frac{1}{A}\int U(r)d\vec{r} \tag{7}$$

where A is the atomic number. Thus from the volume integral of U one can obtain the volume integral of V_o and hence V_o itself. A similar relationship connects the optical model symmetry potential and V_τ. An energy dependent correction of between -11 and -26%[21,22] was applied to these values of V_o and V_τ to account for the contribution of exchange processes.

The results are shown in Figs. 1-4, where the laboratory bombarding energy is plotted along the abscissa and the strength of the 1.0F Yukawa potential along the ordinate. The relative signs of the V_{ST} have not all been fixed by experiment, and the examples studied in this review shed no light on them, since only a single V_{ST} is important in each case. The signs chosen are those universally predicted by the realistic models, namely, V_o is attractive while V_σ, V_τ, and $V_{\sigma\tau}$ are repulsive. Also shown on each figure as lines labelled HJ, KK or KKD are comparisons with the so-called realistic forces. These lines denote the Yukawa with the same volume integral as the long range parts of the Hamada-Johnston force (HJ),[18] of the Kallio-Kolltveit force (KK),[23] and of the density-dependent[24] Kallio-Kolltveit force averaged over the lead nucleus (KKD). The numbers near the points are the mass numbers of the targets. Some comments on individual V_{ST} follow.

V_o: The spin-isospin independent part of the effective interaction appears to be independent of bombarding energy and of L, and is in essential agreement with the theoretical estimates. It is particularly reassuring that the results obtained from inelastic scattering and from the optical model are in such close agreement. The two values for the ^{90}Zr point[15] correspond to different experimental values of the B(E2) used to fix the effective charge.

V_σ: There is rather little solid information available here. The two lowest energy points from the $^{16}O(p,p')^{16}O(2^-)$ reaction[17] are upper limits because of compound nucleus contributions. It now seems likely, as we shall discuss below, that the spin-orbit force contributes to this cross section, so the other points may also be upper limits. Both V_σ and $V_{\sigma\tau}$ contribute to the spin-flip part of the cross section for the ^{89}Y point; it was assumed in the analysis[25] that $V_\sigma=V_{\sigma\tau}$.

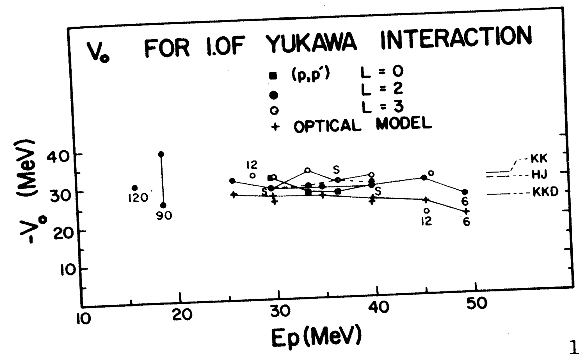

Figure 1.--Values of V_o. Points not numbered are from ^{16}O. S means two points are superimposed.

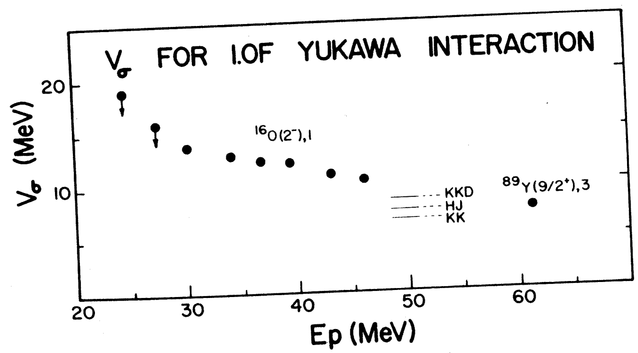

Figure 2.--Values of V_σ. The annotation $^{16}O(2^-)$,1 means the data is from $^{16}O(p,p')^{16}O(2^-)$ and that L=1.

V_τ: Only a few analyses are available above 20 MeV and the results of these are not particularly consistent. The points labelled <1p> are the average 1p-shell results of Clough et al.,[8,9] and have been used to argue that V_τ is strongly energy dependent between 30 and 50 MeV. Recent experiments, analyzed using a macroscopic-model form factor, indicate that any energy dependence is much less dramatic than indicated by the <1p> points. One can see, by comparing the lines labelled KK and KKD, that V_τ is particularly sensitive to any density dependence in the two-body force. It is not clear how important such effects should be in charge exchange reactions, but the effect would tend to damp contributions from the high density interior region and lead to a more strongly surface-peaked form factor.

$V_{\sigma\tau}$: There is a substantial amount of data available from both (p,p') and (p,n) reactions. $V_{\sigma\tau}$ seems fairly well determined and independent of energy, but all these transitions are dominated by L=0, so we have no information on a possible L-dependence.

To assess the accuracy to be expected in analyses such as this, one must have some idea of the sensitivity of the calculations to the input assumptions, in addition to those already considered. In the light nuclei considered here, the optical model parameters are suspect. This problem has not been thoroughly studied, but the shapes of the angular distributions do not seem to be sensitive to reasonable changes in the optical model.[17] However, the magnitude of a predicted cross section has been observed to change by as much as 30% when different sets of optical model parameters were used,[17] thus introducing a 15% change in V_{ST}. No damping (due to non-local potential effects, for example) was applied to the distorted waves in the quoted calculations.

Although harmonic oscillator (HO) wave functions were used in most of the calculations quoted here, Woods-Saxon (WS) wave-functions were used occasionally. The shapes and magnitudes of the cross sections are not usually very sensitive to either the range of the HO potential or to the choice of a HO or WS radial dependence.

The sources of information for the figures are tabulated in Table III.

One can summarize these results by stating that V_o and $V_{\sigma\tau}$ are fairly well determined, are nearly independent of energy, and have the values V_o=27±5 MeV and $V_{\sigma\tau}$=12±2.5 Mev, while V_σ and V_τ are poorly determined. Very few transitions isolate V_σ and other spin-dependent forces may contribute to these, so it will be difficult to get precise numbers for this small part of the force. On the other hand new (p,n) data is becoming available in the 20-50 MeV range and it seems likely that reliable estimates of V_τ will be available soon.

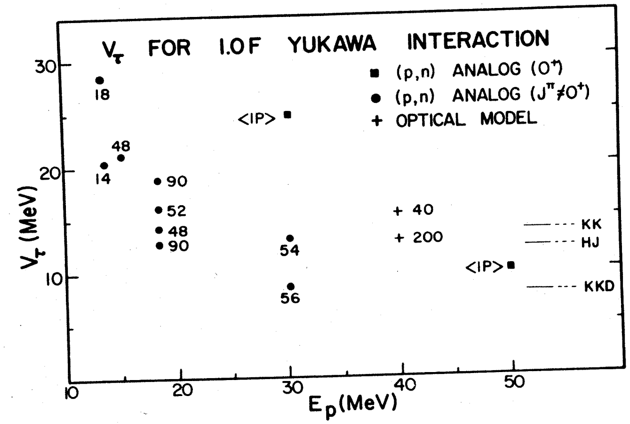

Figure 3.--Values of V_τ.

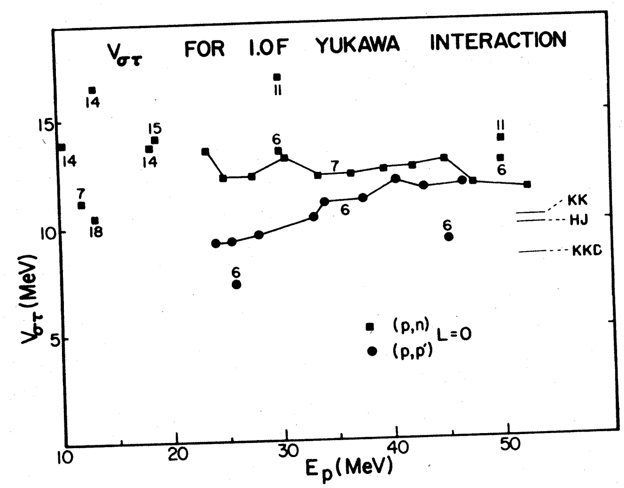

Figure 4.--Values of $V_{\sigma\tau}$.

TABLE III.--Sources of information on V_{ST}.

Inter-action	Nucleus(L)[b]	Ep(MeV)	Reference	Modifications[a]
V_o	^6Li(OM)	25.9-49.5	22	E
	^6Li(2)	25.9-49.5	22	E
	^{12}C(3)	28.05,45.5	10	
	^{16}O(0,2,3)	29.8-40.1	19	E
	^{16}O(3)	30.1,46.1	17	E
	^{58}Ni to ^{208}Pb(OM)	30,40	21,26,27	E
	^{90}Zr(2)	18.8	15	
	^{120}Sn(2)	16	15	
V_σ	^{16}O(1)	24.5-46.1	17	E
	^{89}Y(3)	61	25	
V_τ	<1p>	30,50	8,9	R,E
	^{14}C(0)	13.7	7	R,E
	^{18}O(0)	11.9,13.5	7	R,E
	^{48}Ti(0)	15.25,18.5	7,2	R,E
	^{52}Cr(0)	18.5	2	E
	^{54}Fe(0)	30.2	28	R,E
	^{56}Fe(0)	30.2	28	R,E
	^{90}Zr(0)	18.5	2,7	R,E
	A=40,200(OM)		21,29	E
$V_{\sigma\tau}$	^6Li(0)	30,50	8,9	R,E
	^6Li(0)	25.9,45.1	22	E
	^6Li(0)	24.3-46.5	6	E
	^7Li(0)	12.0-52.3	4,6	E
	^{11}B(0)	30,50	8,9	R,E
	^{14}C(0)	10.4,13.3,18.3	7,30	R,E
	^{15}N(0)	18.8	7	R,E
	^{18}O(0)	13.2	7	R,E

a) E or R means the results of the reference were modified to account for the effects of exchange or a different range, respectively.

b) In the case of (p,p') or (p,n) reactions, L is the momentum transfer. For optical model studies L=OM.

So far we have neglected the contribution of the tensor and spin-orbit parts of the effective interaction. In most cases this is a reasonable assumption. However, we shall find that in certain circumstances either the long-range tensor force or the short-range spin-orbit force can dominate an inelastic scattering reaction. We examine first the evidence for the tensor force.

THE TENSOR FORCE

In the simplest case, the one-pion-exchange-potential (OPEP), the tensor force has the form

$$\text{OPEP:}\quad V_T(r)=V_{T\tau}\vec{\tau}_1\cdot\vec{\tau}_2\, S_{12}f(\alpha) \tag{8}$$

$$f(\alpha)=\left(1+\frac{3}{\alpha r}+\frac{3}{(\alpha r)^2}\right)\frac{e^{-\alpha r}}{\alpha r},$$

where S_{12} is the usual tensor operator and $\alpha^{-1}=(\frac{h}{m_\pi c})=1.415F$ is the Compton wavelength of the pion. Because of the strongly divergent form of the OPEP as $r\to 0$, alternative shapes have usually been chosen. The most common of these are the regularized OPEP or ROPEP form[30,31]

$$\text{ROPEP:}\quad V_T(r)=(V_T+V_{T\tau}\vec{\tau}_1\cdot\vec{\tau}_2)S_{12}[f(\alpha)-\frac{\beta^3}{\alpha^3}f(\beta)], \tag{9}$$

and the r^2-Yukawa form[20]

$$r^2\text{-Y:}\quad V_T(r)=(V_T+V_{T\tau}\vec{\tau}_1\cdot\vec{\tau}_2)S_{12}r^2\frac{e^{-\alpha r}}{\alpha r}. \tag{10}$$

In Eq. 9, β is taken to be substantially larger than α so the potential resembles the OPEP at large r, while in both these forms a term V_T which can act in no-isospin-transfer cases is allowed.

The first two terms in the expansion of the Fourier transform of the tensor and spin-orbit forces are proportional to the integrals[32]

$$J_4 = \int r^4V(r)dr$$
$$J_6 = \int r^6V(r)dr. \tag{11}$$

Schaeffer and I[38] estimated the strength and range for the r^2-Y form by matching J_4 and J_6 for the r^2-Y form of Eq. 10 to those for the OPEP or the Hamada-Johnston potential. In the latter case we find $V_T{\sim}0.05\ V_{T\tau}$, so one would expect the tensor force to contribute strongly only to transitions in which the isospin changes.

Some special selection rules apply to the tensor force, namely[30,33,34]

$$S=1$$
$$L=\lambda,\lambda\pm 2. \tag{12}$$

The first equation states that only spin-flip transitions are possible. The second reflects the fact that the nature of S_{12} allows the two angular momentum transfers involved in the problem, that to the valence nucleon (λ) and that to the projectile (L), to differ by ±2. For the central-force case λ=L.

Most of the available information on $V_{T\tau}$ comes from analyses of the $^{14}C(p,n)^{14}N$(g.s.) reaction, which is the analog of the strongly inhibited β^- decay of ^{14}C. Since this is a $(0^+,1)\rightarrow(1^+,0)$ transition we have S=T=1; L=0,2 and for a central force one would expect the L=0 amplitude to dominate the reaction. However, it has been shown by Madsen[3] that the L=0 amplitude is approximately proportional to the β-decay matrix element which is very small in this case. One then expects the central-force contribution to the cross-section to be small and to have an L=2 shape. Adding a tensor force relaxes these selection rules. The amplitude with angular momentum λ=0 transferred to the nucleus is still small but the amplitude with (λ=2, L=0), to which only the tensor force can contribute, is now expected to be important.

Measurements and calculations of cross sections and polarizations for $^{14}C(p,n)$ have been performed by Wong, et al.[30] at a variety of energies below 20 MeV. Inclusion of a tensor force of the ROPEP form greatly improved the fits to cross sections, but perhaps because of compound nucleus effects or poorly known optical model parameters, quantitative fits were not generally obtained. Fits to the polarization data were very poor either with or without a tensor force. Exchange processes were not included in this analysis, but they should not be large for this L=0 transition.

The analogous transition in inelastic proton scattering leaves ^{14}N in its first excited state ($J=0^+$,T=1) at 2.311 MeV. Crawley, et al.[31] measured the cross section for this transition at Ep=24.8 MeV and analyzed it in terms of a force of ROPEP shape, again without including exchange processes. A good fit to the data was obtained for $\theta<80^\circ$ by including a tensor force of roughly OPEP magnitude, but not with a central force alone. The data at back angles were not fitted, even qualitatively, but the cross sections were so small (~30 μb/sr) that it was not possible to exclude compound nucleus processes in such a light nucleus.

Because this reaction promises perhaps the cleanest measure of $V_{T\tau}$, Fox, et al.[35] at Michigan State have measured cross sections at 29.8, 36.6, and 40.0 MeV. The angular distribution at 29.8 MeV and a preliminary analysis are shown in Fig. 5. Theoretical cross sections including exchange processes were calculated using the code DWBA 70.[20] The central force ($V_{\sigma\tau}$) had a strength consistent with the results of Fig. 4; it is clear from the curve labeled "central (exchange)" that the central force alone cannot describe the data. The tensor force had an r^2-Y form. The range of $V_{T\tau}$ was obtained by matching the integrals of Eq. 11 to the OPEP potential as described

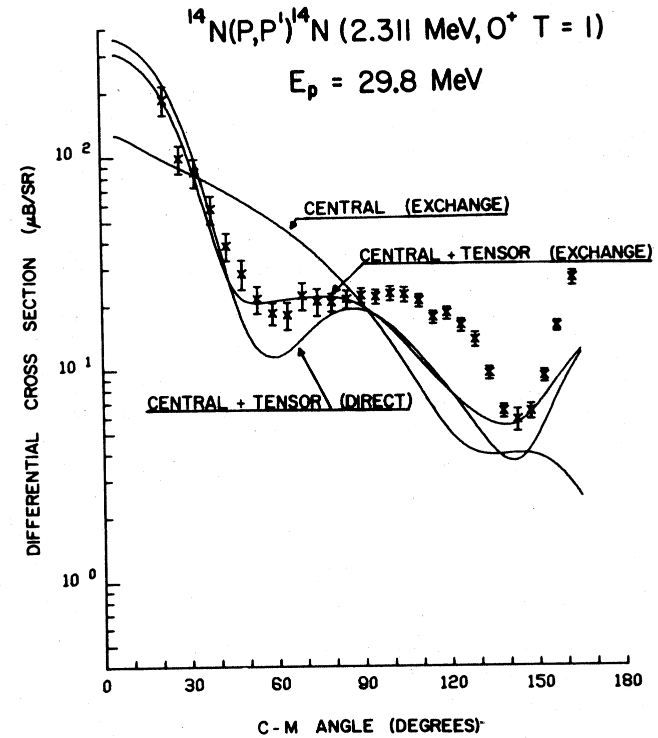

Figure 5.--Cross sections for the $^{14}N(p,p')^{14}N(2.311$ MeV, 0^+,T=1) reaction at 29.8 MeV.

earlier and its strength was adjusted to fit the data. The calculations including central and tensor forces are labeled "Central + OPEP". The curve labeled "direct" contains only the direct amplitude, while that labeled "exchange" contains both direct and exchange amplitudes. The fit is good out to $\theta=90^\circ$ and even the back angle dip is qualitatively predicted. An additional calculation including a spin-orbit force of reasonable size produced an overall 10% decrease in the cross section, but did not change the shape. The curves shown were calculated for $V_T=0$, but the results are not sensitive to this assumption. It should be noted that the asymmetries predicted by the present calculation are not in good agreement with the measurements of Escudie, et al.[36] at 24 MeV.

Love and Parish[34] found that the $^{12}C(p,p')^{12}C$(15.1 MeV, 1^+, T=1) reaction at 45.5 MeV was sensitive to tensor force contributions, and that a good fit to the angular distributions was obtained with a tensor force of reasonable magnitude. This calculation included the exchange amplitude. The first calculations to do so were performed by Love et al.[33] on the $^{14}N(p,n)^{14}O$(g.s.) and $^{13}C(p,n)$ ^{13}N(g.s.) reactions at 12.2 MeV, but for reasonable values of the force, the predicted cross sections were much too small.

The results discussed above are summarized in Table IV. The parameters are defined in Eqs. 8-11, and the data are compared using the J_4 integral of Eq. 11. The results all lie fairly close to the value J_4=318 MeV-F^5 for the OPEP, and to the estimates[38] obtained from the Hamada-Johnston potential. We may conclude that although the tensor force is not yet well determined from the data, its value is consistent with one's qualitative expectations.

THE SPIN-ORBIT FORCE

We next examine the spin-orbit part (V_{LS}) of the effective interaction. One might qualitatively expect that this part of the force would be important in the study of spin-dependent phenomena such as asymmetries, in transitions to high-spin states, and at high energies. We shall examine these cases in turn and find that there is as yet no firm empirical information on the LS interaction, although there are promising possibilities for further investigation.

Raynal[39] has made a fairly extensive study of the effect of a microscopic LS force on the nature of the predicted asymmetries in inelastic proton scattering. He finds that inclusion of V_{LS} improves the fits, but that one does not come close to a quantitative fit. Consequently it is difficult to obtain a measure of the strength of the force from asymmetry measurements.

The situation is more encouraging in the case of high spin states. Love[40] has found that an LS force is important in the excitation of 6^+ and 8^+ states in $^{90}Zr(p,p')$ at 61.2 MeV. In the case of the 8^+ state a form of V_{LS} obtained by Gogny, et al.[41] in a fit to the free-two-body system, is sufficient by itself, to reproduce the experimental cross section. The difficulty here is not a lack of sensitivity to V_{LS} as it apparently dominates the transition. Rather one needs a better understanding of the importance of collective enhancements in this high multipolarity transition[42] so that the magnitude of the cross sections can be used to empirically determine the spin-orbit force.

We now examine the energy dependence of cross sections dominated by various parts of V_{eff}. Using the code DWBA 70,[20] I have calculated total cross-sections σ_T, for the $^{16}O(p,p')^{16}O$(8.87 MeV, 2^-, T=0) reaction when mediated by Yukawan central forces (V_σ) with ranges of 1.0F and 1.4F, by a r^2-Y tensor force (V_T), and by a Yukawan spin-orbit force. Figure 6 shows the ratio of the cross section at Ep to that at 20 MeV for the direct amplitude only and for the direct-plus-exchange amplitudes. The contribution of V_{LS} increases with energy while that due to other components decreases, the relative contribution of V_{LS} compared to V_σ(1.0F) changing by a factor of five between 20 and 60 MeV. This result allows one to hope that one can disentangle the effects of V_{LS} by studying inelastic scattering as a function of energy.

TABLE IV.--Values of the tensor force.[a]

Determination	$V_{T\tau}$ (MeV)	$\alpha(F^{-1})$	$\beta(F^{-1})$	J_4(MeV-F^5)
$^{14}C(p,n)^{14}N$(g.s.)[b]				
Ep=10.4 MeV	5.4	0.707	4.0	444
12.7	5.1	0.707	4.0	420
13.3	5.1	0.707	4.0	420
18.3	3.9	0.707	4.0	321
$^{12}C(p,p')^{12}C$(15.1MeV,1^+,1)[c]				
Ep=45.5 MeV	2.35	0.707	-	200*
$^{14}N(p,p')^{14}N$(2.311MeV,0^+,1)[d]				
Ep=24.8 MeV	3.9	0.707	2.0	290
29.8	14.6	1.23	-	421*
1p-shell, two body[e] matrix elements	5.1	0.707	4.0	420
OPEP				318
HJ[f] (r_c=1.0F)				288
(r_c=0.6F)				294

a) Only numbers marked (*) include exchange.
b) Ref. 30. ROPEP form.
c) Ref. 34. OPEP form.
d) 24.8 MeV: ref. 31, ROPEP form. 29.8 MeV: ref. 35, r^2-Y form.
e) Determined by Schmittroth, ref. 37, from Cohen-Kurath 1p-shell two-body matrix-elements involving the 1^+, T=0 and 0^+, T=1 states only.
f) Ref. 38. For the part of the Hamada-Johnston potential with $r \geq r_c$.

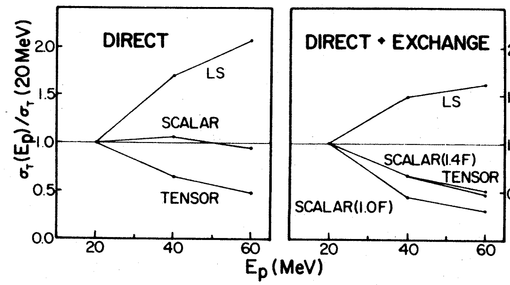

Figure 6.--Energy dependence of cross sections mediated by central, tensor and spin-orbit forces.

The excitation of an unnatural parity state, such as the 2^- state at 8.87 MeV in ^{16}O, would seem a likely place to look for spin-orbit effects, since the competing central interaction, V_σ, is relatively small. Data on this reaction has recently become available[17] at energies between 23.4 MeV and 46.1 MeV, and the analysis of this data in terms of central forces yielded a puzzling anomaly. At low energies relatively good fits to the data were obtained as is shown in Figure 7. As the energy increased, however, the fits became worse, the DWA calculations continuing to predict an L=1 shape while the experimental angular distribution began to resemble that for an L=3 transfer. The character of the discrepancy is shown in Figure 8, where one should compare the data points and the curve labeled "Central". We have used the code DWBA 70[20] to perform calculations with exchange, and an effective interaction including V_σ, a tensor force obtained[38] from the Hamada-Johnston potential[18] as discussed earlier, and a spin-orbit force somewhat larger than that used by Love[40] in his studies of high spin states. The spin-orbit force was expressed as the sum of two Yukawas with ranges of 0.392F and 0.595F, and its strength was adjusted to roughly fit the data. The curve labeled "Central + tensor + LS" shows the results of this calculation displaced upward 20% for display purposes. It is clear that a good fit is obtained for $\theta<90°$. The asymmetry data of Benenson, et al.[43] at Ep=40.5 MeV are also fairly well described at forward angles by the same forces. It should be pointed out that one can obtain a cross section peak at roughly the correct angle without an LS force if one includes a tensor force with $|V_T| \approx |V_{TT}|$. There seems little justification for such an approach, since the Hamada-Johnston potential gives $V_T \approx 0$. The tensor force used in the present calculation had very small V_T, and hence contributes only to

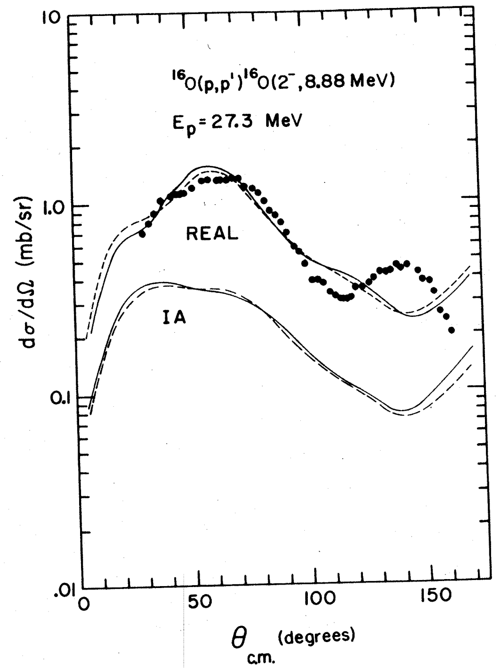

Figure 7.--Cross-sections for the $^{16}O(p,p')^{16}O(8.88\ MeV,\ 2^-,T=0)$ reaction at Ep=27.3 MeV. The upper two curves are for a Yukawa form of V_σ with a range of 1.0F and two different optical model potentials. The two lower curves should be ignored.

the exchange amplitude, giving the forward peaked cross-section shown in Figure 8.

A summary of the forces which have been used in various calculations is shown in Table V and compared with the results implied by the empirical optical model potential.

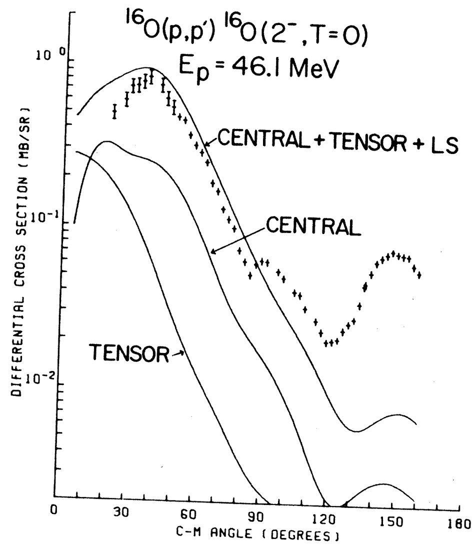

Figure 8.--Cross sections for the $^{16}O(p,p')^{16}O$(8.88 MeV, 2^-, T=0) reaction at 46.1 MeV.

THE IMAGINARY PART OF THE EFFECTIVE INTERACTION

Finally, I will discuss briefly a recent and interesting development. Satchler[44] has suggested that the microscopic effective interaction may contain an important imaginary component and has given a recipe for determining it from the imaginary part of the optical model potential. In the specific case of a transition to the 3^- state at 3.73 MeV in ^{40}Ca, inclusion of this imaginary effective interaction substantially improved the fit to the cross-section. Recently Howell and Hammerstein[45] have examined the effect on cross sections, asymmetries and spin-flip fractions of adding an imaginary form factor to the usual real microscopic form factor. They conclude that an imaginary component is definitely required, but that a simple collective model is more reliable than the Satchler[44] prescription.

TABLE V.--Values of the spin-orbit force.[a]

Determination	J_4(T=0) ($MeV-F^5$)	J_4(T=1) ($MeV-F^5$)
Optical Model V_{so}=6.7MeV[b)]	-80	-
Raynal-Asymmetries[c)]	-61.5	-35.8
Love-$^{90}Zr(p,p')$[d)]	-37.6	-15.2
$^{16}O(p,p')^{16}O(8.87,2^-,0)$[e)]	-50.8	-32.2
HJ^f (r_c=1.0F)	- 7.3	- 6.5
(r_c=0.6F)	-27.7	-13.7

a) The spin-orbit force has the form $[V_{LS}(T{=}0)+V_{LS}(T{=}1)\vec{\tau}_1\cdot\vec{\tau}_2]\vec{\ell}\cdot\vec{S}$
b) Ref. 26.
c) These are twice the numbers quoted in ref. 39, but correspond to values actually used (ref. 32).
d) Ref. 40.
e) See the text of this paper.
f) Ref. 38. From the part of the Hamada-Johnston potential (Ref. 18) with $r>r_c$.

SUMMARY

The present situation is summarized in Table VI. It seems likely it will be difficult to determine V_σ accurately, the present results being in the nature of upper limits. High quality (p,n) data are now available from Colorado[46] at 23 MeV and from MSU[47] at 22, 30, and 40 MeV. Careful analysis of these data should lead to reliable values of V_τ. Though the tensor force appears to be important in only a few cases, its effects are distinctive and more detailed fits to existing data should pin down this part of the force reasonably well. Almost nothing is presently known about V_{LS}, but circumstances have been found in which this part of the force is important and the situation looks hopeful.

Since ^{40}Ca is the heaviest stable N=Z nucleus and its wave functions are well known, the recent high resolution $^{40}Ca(p,p')$ experiments of Gruhn, et al.[48] at Ep=25, 30, 35, and 40 MeV may prove particularly useful in future studies of V_{eff}.

TABLE VI.--Our present knowledge of V_{ST}, V_T, V_{LS}.

Force	Value (for 1.0F range Yukawa)
V_o	27±5 MeV
V_σ	poorly determined
V_τ	poorly determined, hopeful
$V_{\sigma\tau}$	12±2.5 MeV
$V_{T\tau}$	consistent with OPEP
V_{LS}	poorly determined, hopeful

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