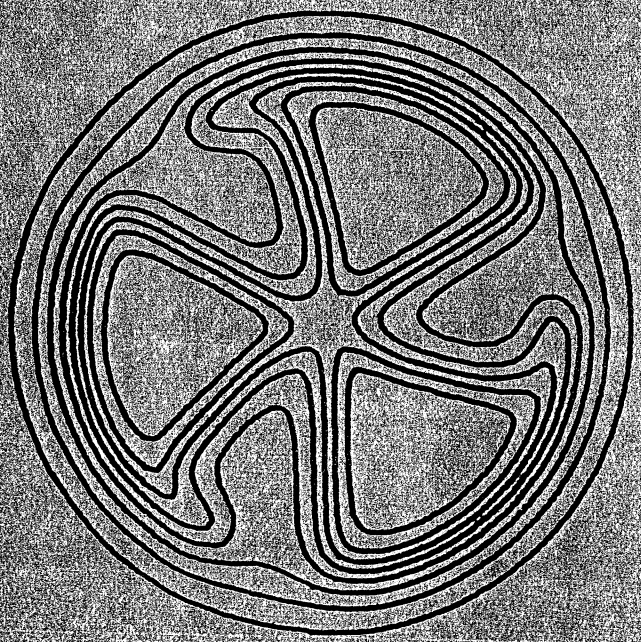
MICHIGAN STATE UNIVERSITY

CYCLOTRON LABORATORY

INELASTIC ELECTRON SCATTERING FORM-FACTORS CALCULATED FROM SHELL-MODEL WAVE FUNCTIONS

G. R. HAMMERSTEIN, DUANE LARSON,
and B. H. WILDENTHAL



MSUCL – 50

FEBRUARY 1972

Inelastic Electron Scattering Form-Factors Calculated from Shell-Model Wave Functions*

G.R. Hammerstein, Duane Larson, and B.H. Wildenthal

Cyclotron Laboratory, Physics Department
Michigan State University, East Lansing, Michigan 48823

ABSTRACT

Form-factors measured for (e,e') on ^{20}Ne, ^{24}Mg, ^{28}Si, and ^{32}S are well reproduced by calculations which use shell-model wave functions and scaling factors consistent with the usual effective charge assumptions.

---

*Work supported in part by the U.S. National Science Foundation.

Recent measurements[1] have yielded form-factors for inelastic electron scattering leading to the lowest 2^+ and 4^+ states of ^{20}Ne, ^{24}Mg, and ^{28}Si. Older data[2] are available for ^{32}S. In this note we present results of analyses of these data which use shell-model wave functions to generate the relevant transition densities. It has been shown[3-6] that presently available data on E2 transition strengths and electric quadrupole moments in the sd-shell can be understood in terms of these same wave functions if protons and neutrons are each assumed to carry an additional charge of 0.5e, an assumption not at variance with data available from the "single-particle" nuclei ^{17}O and ^{17}F. The question of interest here is whether the more detailed information contained in the experimental (e,e') form-factors can also be explained in the same terms.

The shell-model wave functions we use here have been obtained from calculations made in $0d_{5/2}$-$1s_{1/2}$-$0d_{3/2}$ basis spaces. The ^{20}Ne calculations employed the full set of states available in this space, but in the other calculations limits were placed on the numbers of allowable $d_{5/2}$ holes and $d_{3/2}$ particles so as to keep dimensions within practical bounds. The Ne and Mg shell-model calculations employed as a model Hamiltonian a (slightly) modified[5] version of Kuo's matrix elements.[8] The ^{28}Si and ^{32}S calculations were made with versions of the modified surface-delta interaction. Details of these calculations are available elsewhere.[3-5]

In the present work, these shell-model wave functions are used to generate the expectation values of the operators $(a_j^+ a_{j'})_J$ between the initial and final states of the target nucleus, where j,j' refer to the orbits of the active model space and J to the multipolarity of the transition. A specially adapted version[9] of the computer codes described by French, Halbert, McGrory, and Wong[10] is used for this procedure. These results are then appropriately combined with radial wave functions of the model orbits in order to construct transition densities $\rho_J(r)$. The form-factors for inelastic electron scattering then follow from the expression

$$|F(q)|^2 = \frac{4\pi}{Z^2} \frac{2J_f+1}{2J_i+1} \left| \int_0^\infty j(qr)\rho_J(r)r^2 dr \right|^2. \qquad (1)$$

We note that, since $j_J(qr) \to \frac{1}{(2J+1)!!} q^J r^J$ as $q \to 0$, the expression for the form factor becomes, in the limit of $q \to 0$, equal to the definition of the B(EJ) of an electric transition of multipolarity J:

$$\frac{Z^2[(2J+1)!!]^2}{4\pi} \lim_{q\to 0} \frac{|F(q)|^2}{q^{2J}} = \frac{2J_f+1}{2J_i+1} \left| \int_0^\infty \rho_J(r) r^{J+2} dr \right|^2 \equiv B(EJ) \qquad (2)$$

However, the complete (e,e') form factors provide a more detailed test of the transition density than is available from just the B(EJ), since ρ is sampled for varying values of r as q varies. Provided the calculated and observed shapes of $|F(q)|^2$ are in reasonable agreement, the square of the isoscalar effective charge $e(eff)=e_p(eff)+e_n(eff)$ (all the present transitions

are $\Delta T=0$) may be defined as the number which normalizes the form factor calculated with $e_p=1.0$, $e_n=0.0$ to the experimental distribution (i.e., $e_{eff}^2=\frac{|F(q)|^2_{expt}}{|F(q)|^2_{th}}$.)

Our principal aim in the present study is to test the shell-model wave functions by comparing calculated form-factors to the experimental curves. In principle, this could not be done unambiguously unless the correct radial forms of the single-particle wave functions, which are not available from the shell-model work, are also known. Happily, from our point of view at least, it appears that the effective charge extracted from such a comparison is insensitive not only to the particular prescription by which these radial forms are calculated, but also even to whether the resulting form factor is a very good fit to the data or not. In Fig. 1, the solid lines are form factors calculated with harmonic oscillator wave functions, $\hbar\omega=45A^{-1/3}-25A^{-2/3}$. Dashed lines result from $\hbar\omega=41A^{-1/3}$ and dot-dashed lines from $\hbar\omega=32A^{-1/3}$. Results very similar to these three sets are obtained with radial functions calculated by binding $d_{5/2}$, $s_{1/2}$, and 3/2 protons by 12, 11, and 6 MeV in Woods-Saxon wells of a=0.75F and r_o=1.33F, 1.25F, and 1.40F, respectively. Also, a reduction of the binding energy per proton by 6 MeV can be compensated for by simultaneously reducing r_o by $\sim$10%. As illustration of the insensitivity of extracted effective charges to these details, six of the different prescriptions noted yield effective charges of 1.96, 1.93, 1.91, 1.97, 1.97, and 1.95 for ^{20}Ne (2^+) if experimental and calculated maximum cross sections are set euqal.

In the remaining discussion we refer to the results obtained with harmonic oscillator functions generated according to $\hbar\omega = 45A^{-1/3} - 25A^{-2/3}$. In Fig. 1 are shown the experimental form factors for the 2^+ first excited states of ^{20}Ne, ^{24}Mg, ^{28}Si, and ^{32}S, and for the 4^+ states of ^{20}Ne and ^{28}Si, and our calculated shapes. The effective charges for these transitions are listed in Table I together with representative information available from other studies of the B(E2)'s for the 2^+ states, also expressed in terms of the effective charge necessary to normalize experimental values and the present shell model predictions. No other B(E4) data are available. We see that there is, in general, good agreement between the results of our analysis of the (e,e') data and the B(E2) data obtained with such techniques as Doppler-shift attenuation, resonance fluorescence, Coulomb excitation, and recoil distance.[11-20]

We conclude that the shell model wave functions are capable of encompassing the more complex phenomena of inelastic electron scattering in a way self-consistent with prior analyses of B(E2)'s. It is important to note that the effective charges extracted from the analyses of the electron scattering data are essentially free from any ambiguity resulting from the choice of single-particle radial wave functions. This situation is to be contrasted to the usual case in which effective charges are extracted by taking the ratios of calculated and measured B(E2) values. Details of the tails of the radial

wave functions are of comparable importance to the effective charge in arriving at a calculated B(E2) and hence any effective charge deduced from this method (i.e., almost any previously determined value) is very much a function of the assumed single-particle wave functions. The electron scattering measurements thus seem to constitute a fundamental improvement in obtaining this kind of information.

Also important is the window that these results open onto the E4 excitation phenomena. The shell-model results predict strong and moderately strong E4 excitation in ^{20}Ne and ^{28}Si and a very weak transition in ^{24}Mg. These predictions are qualitatively consistent with the data, but further experimental work is needed to reach a definite conclusion about the ^{24}Mg transition. Extensive (e,e') data of moderately good energy resolution should prove extremely informative as regards nuclear structure in this region.

Acknowledgements

We express our appreciation to G. Bertsch, H. McManus, J.B. McGrory, F. Petrovich, and A. Arima for stimulating and helpful conversations.

REFERENCES

1. Y. Horikawa et al., Phys. Letts. 36B(1971)9.
2. R. Lombard, P. Kossanyi-Demay, and G. R. Bishop, Nucl. Phys. 59(1965)398.
3. E. C. Halbert, J. B. McGrory, S. P. Pandya, and B. H. Wildenthal, Advances in Nuclear Physics, Vol. IV (M. Baranger and E. Vogt, eds.) Plenum Press, New York (1971).
4. B. H. Wildenthal, E. C. Halbert, J. B. McGrory, and T. T. S. Kuo, Phys. Rev. C4(1971)1266.
5. B. H. Wildenthal, J. B. McGrory, E. C. Halbert, and H. D. Graber, Phys. Rev. C4(1971)1708.
6. B. H. Wildenthal, J. B. McGrory and P. W. M. Glaudemans, Phys. Rev. Letters 26(1971)96.
7. B. H. Wildenthal, to be published.
8. T. T. S. Kuo, Nucl. Phys. A103(1967)71.
9. Duane Larson, unpublished.
10. J. B. French, E. C. Halbert, J. B. McGrory and S. S. M. Wong in Advances in Nuclear Physics, Vol. III, (M. Baranger and E. Vogt, eds.) Plenum Press, New York (1969).
11. D. Schwalm and B. Povh, in Contributions to ICPNS, Montreal 1969, p. 15 of the Addendum (University of Montreal Press, 1969).
12. O. Häusser, et al., Nucl. Phys. A168(1971)17.
13. D. Vitoux, R. C. Haight and J. X. Saladin, Phys. Rev. C3, (1971)718.

14. S.I. Baker, private communication.

15. C.P. Swann, Phys. Rev. C4(1971)1489.

16. O. Hausser, et al., Phys. Rev. Lett. 23(1969)3201.

17. F.C.P. Huang and D.K. McDaniels, Phys. Rev. C2(1970)1342.

18. K. Nakai, J.L. Quebert, F.S. Stephens, and R.M. Diamond, Phys. Rev. Lett. 24(1970).

19. F. Ingebretsen, et al., Nucl. Phys. A164(1971)433.

20. E.C. Booth, B. Chassan and K.A. Wright, Nucl. Phys. 57(1964) 403.

14. S.I. Baker, private communication.

15. C.P. Swann, Phys. Rev. C4(1971)1489.

16. O. Hausser, et al., Phys. Rev. Lett. 23(1969)3201.

17. F.C.P. Huang and D.K. McDaniels, Phys. Rev. C2(1970)1342.

18. K. Nakai, J.L. Quebert, F.S. Stephens, and R.M. Diamond, Phys. Rev. Lett. 24(1970).

19. F. Ingebretsen, et al., Nucl. Phys. A164(1971)433.

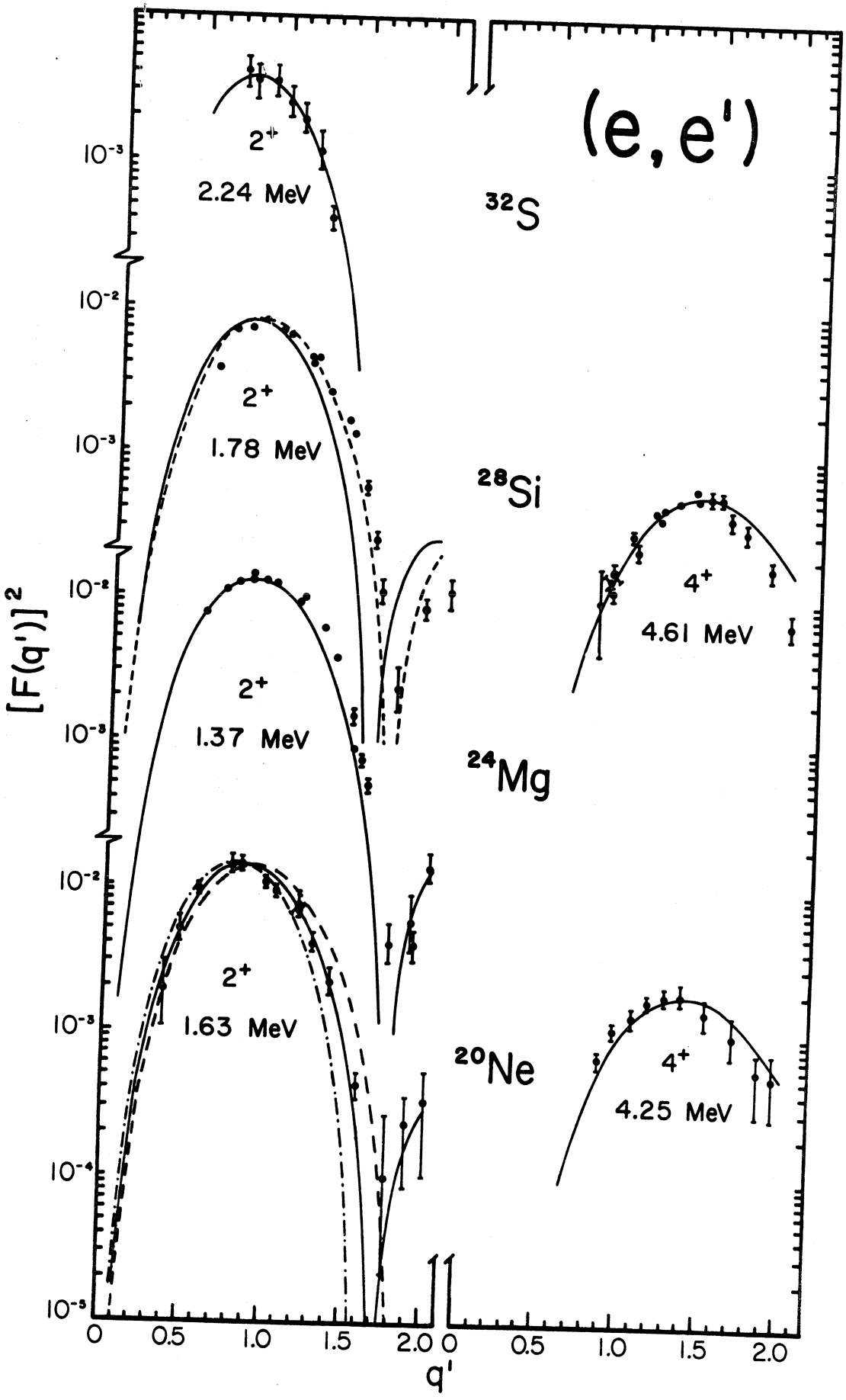
20. E.C. Booth, B. Chassan and K.A. Wright, Nucl. Phys. 57(1964) 403.

Transition	$(e_p+e_n)_{eff}$					Predicted B(EJ) $\hbar\omega=45A^{-1/3}-25A^{-2/3}$ $(e_p+e_n)=2.00$ (e^2fm^{2J})
	(e,e')	Coulomb Ex.	Doppler Shift	Recoil Distance	Resonance Fluorescence	
$^{20}Ne, 0_1^+ \to 2_1^+$	1.93	1.92[a]	1.96[b]	1.92[a]	-	308
$0_1^+ \to 4_1^+$	1.88	-	-	-	-	29200
$^{24}Mg, 0_1^+ \to 2_1^+$	1.99	2.06[c]	2.15[d]	-	2.10[e]	400
$0_1^+ \to 4_1^+$		-	-	-	-	2000
^{28}Si $0_1^+ \to 2_1^+$	2.22	1.92[f]	2.08[g]	-	-	342
$0_1^+ \to 4_1^+$	2.49	-	-	-	-	13450
^{32}S $0_1^+ \to 2_1^+$	2.28	2.40[h]	1.91[i]	-	2.16[j]	230
$0_1^+ \to 4_1^+$		-	-	-	-	41900

[a] Ref. 11
[b] Ref. 12
[c] Ref. 13
[d] Ref. 14
[e] Ref. 15
[f] Ref. 16
[g] Ref. 17
[h] Ref. 18
[i] Ref. 19
[j] Ref. 20

FIGURE CAPTION

Figure 1 Form factors measured for inelastic electron scattering to 2^+ and 4^+ levels in ^{20}Ne, ^{24}Mg, and ^{28}Si(Ref. 1) and ^{32}S(Ref. 2) and predictions based on shell model wave functions.


(e, e')
^{32}S
^{28}Si
^{24}Mg
^{20}Ne
2+
2.24 MeV
2+
1.78 MeV
2+
1.37 MeV
2+
1.63 MeV
4+
4.61 MeV
4+
4.25 MeV
[F(q')]^2
q'
10^{-3}
10^{-2}
10^{-4}
10^{-5}
0
0.5
1.0
1.5
2.0