# New Proton-Rich Nuclei in the $1 f_{7 / 2}$ Shell* 

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#### Abstract

We have measured the masses of ${ }^{43} \mathrm{Ti}$ and of the previously unknown nuclei ${ }^{47} \mathrm{Cr},{ }^{51} \mathrm{Fe}$, and ${ }^{55} \mathrm{Ni}$. These proton-rich members of $1 f_{7 / 2}$-shell mirror pairs are important for extensions of nuclear mass relationships to the $Z>N$ region.


In this Letter, we report mass measurements of $1 f_{7 / 2}$ shell nuclei which have $Z=N+1$, and are thus members of the heaviest mirror pairs known. The present measurements represent the first observation of ${ }^{47} \mathrm{Cr},{ }^{51} \mathrm{Fe}$, and ${ }^{55} \mathrm{Ni}$. The heaviest previously known nucleus with $Z>N$ was ${ }^{49} \mathrm{Mn}$, whose mass was found by Cerny et al. ${ }^{1}$ to be consistent with a predicted value of $-37.72 \pm 0.08$ MeV in an experiment with the ${ }^{40} \mathrm{Ca}\left({ }^{12} \mathrm{C}, t\right)$ reaction. The mass of ${ }^{43} \mathrm{Ti}$ has also been previously determined by the reaction ${ }^{40} \mathrm{Ca}(\alpha, n)^{43} \mathrm{Ti} .{ }^{2}$ The line of nuclear stability in the $1 f_{7 / 2}$ shell is at least two neutrons from the $N=Z$ line, and the observation of $\boldsymbol{T}_{z}=-\frac{1}{2}$ nuclei in this shell requires the transfer of three or more particles. Since all possible higher- $Z$ targets have a neutron excess of at least four, we have extended the $\left({ }^{3} \mathrm{He}\right.$, ${ }^{6} \mathrm{He}$ ) reaction to its maximal value as a probe for the study of proton-rich nuclei. ${ }^{3,4}$
Beams of $65-$ to $75-\mathrm{MeV}^{3} \mathrm{He}$ ions from the Michigan State University sector-focused cyclotron were used to perform these experiments. The ${ }^{6} \mathrm{He}$ particles were analyzed and detected in the focal plane of an Enge split-pole magnetic spectrograph. The energy scale was determined from a number of well-known $Q$ values including those for the ( ${ }^{3} \mathrm{He},{ }^{6} \mathrm{He}$ ) reaction on ${ }^{48} \mathrm{Ti}$, ${ }^{13} \mathrm{C}$, and ${ }^{12} \mathrm{C}$. Detection of the ${ }^{6} \mathrm{He}$ particles was performed variously with photographic emulsions, silicon position-sensitive detectors, and gas-filled resistive wire proportional counters. In each case different techniques were used to enhance the discrimination of the ${ }^{6} \mathrm{He}$ particles relative to the dominant background of protons, deuterons, and $\alpha$ particles.
Most of the data were taken with a wire propor-
tional counter. In this case a novel technique was employed to discriminate against the background particles. The time of flight of the particles was measured using a signal from a plastic scintillator behind the wire counter. The total energy signal from the plastic was also used to aid the identification. The flight path in the spectrometer (about 3 m ) is very well suited to this method since it produces about a $25-\mathrm{nsec}$ time difference between the ${ }^{6} \mathrm{He}$ particles and the major source of background, $\alpha$ particles. The stop signal for the clock was provided by the radio


FIG. 1. Energy spectrum of ${ }^{6} \mathrm{He}$ from the reaction ${ }^{54} \mathrm{Fe}\left({ }^{3} \mathrm{He},{ }^{6} \mathrm{He}\right){ }^{51} \mathrm{Fe}$. The ${ }^{3} \mathrm{He}$ bombarding energy is 70.8 MeV and the laboratory scattering angle is $9^{\circ}$.

TABLE I. Experimental results and comparison with calculations based on Coulomb-energy systematics.

|  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
| Residual <br> nucleus | $Q$ value <br> $(\mathrm{MeV})$ | Mass excess <br> $(\mathrm{MeV})$ |  |  |
|  |  | Experimental | Calculated ${ }^{\mathrm{b}}$ | $\Delta E_{c}$ <br> $(\mathrm{MeV})$ |
| ${ }^{43} \mathrm{Ti}$ | $-17.451 \pm 0.015$ | $-29.341 \pm 0.015^{\mathrm{c}}$ | $-29.29 \pm 0.05$ | 7.620 |
| ${ }^{47} \mathrm{Cr}$ | $-18.297 \pm 0.028^{\mathrm{d}}$ | $-34.625 \pm 0.028$ | $-34.51 \pm 0.07$ | 8.162 |
| ${ }^{51} \mathrm{Fe}$ | $-18.686 \pm 0.017$ | $-40.232 \pm 0.017$ | $-40.18 \pm 0.07$ | 8.790 |
| ${ }^{55} \mathrm{Ni}$ | $-17.532 \pm 0.015$ | $-45.369 \pm 0.015$ | $-45.35 \pm 0.08$ | 9.426 |

${ }^{a} Q$ values for the $\left({ }^{3} \mathrm{He},{ }^{6} \mathrm{He}\right)$ reaction.
${ }^{\mathrm{b}}$ Ref. 5.
${ }^{\mathrm{c}} \mathrm{A}$ value of $-29.321 \pm 0.010$ was previously measured, Ref. 2.
${ }^{\text {d }}$ The ground state of ${ }^{47} \mathrm{Cr}\left(\frac{3}{2}^{-}\right)$is very weakly excited in this reaction.
frequency of the cyclotron. This use of time of flight in a magnetic spectrometer has permitted measurements in the present experiment of ${ }^{6} \mathrm{He}$ cross sections as low as $20 \mathrm{nb} / \mathrm{sr}$ in the presence of an $\alpha$ background of approximately $1 \mathrm{mb} / \mathrm{sr}$.
A spectrum obtained from the reaction ${ }^{54} \mathrm{Fe}\left({ }^{3} \mathrm{He}\right.$, $\left.{ }^{6} \mathrm{He}\right){ }^{51} \mathrm{Fe}$ is shown in Fig. 1. The cross section for the strong $\frac{7}{2}^{-}$state at 0.27 MeV is only 0.4 $\mu \mathrm{b} / \mathrm{sr}$ at $9^{\circ}$. The ${ }^{51} \mathrm{Fe}$ ground state $\left(J^{\pi}=\frac{5}{2}{ }^{-}\right)$is even more weakly populated, but is unambigously identified in a number of spectra. The spin and parity assignments were derived from comparison with the known spectrum of the mirror nucleus ${ }^{51} \mathrm{Mn}$. The high-lying states in ${ }^{43} \mathrm{Ti},{ }^{47} \mathrm{Cr}$, ${ }^{51} \mathrm{Fe}$, and ${ }^{55} \mathrm{Ni}$ will be discussed in a later paper. The measured $Q$ values to the lowest observed state, the resulting mass excesses, and the Coulomb displacement energies are given in Table I. The measurement of ${ }^{43} \mathrm{Ti}$ by Aldridge, Plendl, and Aldridge ${ }^{2}$ is also shown for comparison. The experimental errors shown represent contributions from target-thickness uncertainty, spectrograph calibration, and statistical errors. Each of the values given represents at least four measurements of the specific reaction $Q$ value. Also given in Table I are the predicted excesses calculated by Harchol et al. ${ }^{5}$

As can be seen from Table I, the uncertainties in the measured $Q$ values are about 20 keV . Further reduction of experimental uncertainties is hindered by two factors. First, the very low yield obtained with cross sections which never exceed $1 \mu \mathrm{~b} / \mathrm{sr}$ makes it difficult to accumulate more than 50-100 events even in the strong peaks. Secondly, the high rigidity of the $50-60-\mathrm{MeV}{ }^{6} \mathrm{He}$ particles produced makes it necessary to run the spectrograph with a highly saturated field which is beyond the region in which the most accurate
measurements can be made.
The present measurements indicated several problems in understanding the effects of Coulomb energies in this region of the periodic table. One of the more interesting examples is the state dependence of the Coulomb interaction as observed in the mirror pair ${ }^{51} \mathrm{Mn}-{ }^{51} \mathrm{Fe}$. Since the ground states are $\frac{5}{2}^{-}$, seniority $\nu=3$, and the first excited states are $\frac{7}{2}^{-}, \nu=1$, one expects the $\Delta E_{c}$ for the $\frac{7}{2}^{-}-\frac{7}{2}^{-}$splitting to be larger than for the $\frac{5}{2}--\frac{5}{2}{ }^{-}$splitting. Indeed, the measurement shows a difference in these shifts of 30 keV . An estimate of the magnitude one would have expected can be made using $\mathrm{MBZ}^{6}$ wave functions for ${ }^{51} \mathrm{Mn}$ and values of the Coulomb pairing interaction, which include correlations calculated by Bertsch. ${ }^{7}$ Taking the increase in the interaction to be about 210 keV for two $f_{7 / 2}$ protons coupling to $0^{+}$and 70 keV for coupling to $2^{+}$, a $100-\mathrm{keV}$ difference is predicted. On the other hand, if we take the most naive view and consider only an uncorrelated pair of $f_{7 / 2}$ nucleons, the Coulomb effect is much smaller, i.e., about 80 keV for $0^{+}$and 20 keV for $2^{+}$, which thus yields a $35-\mathrm{keV}$ energy difference. This is indeed surprising and it would be very desirable to understand the phenomena. A similar puzzle related to Coulomb effects in these mirror nuclei exists in the position of the $d_{3 / 2}$ hole state observed in the present investigation at 0.32 MeV in ${ }^{43} \mathrm{Ti}$. Calculations using the realistic model of Nolen and Schiffer ${ }^{9}$ with a Woods-Saxon potential of radius $1.29 A^{1 / 3}$ and a charge distribution fit to electron scattering ${ }^{9}$ predicts this level at 0.55 MeV .
The present results constitute a large portion of the mass measurements necessary to apply the highly successful Garvey-Kelson ${ }^{8}$ mass relationship to the proton-rich side of the periodic
table in the $1 f_{7 / 2}$ shell. The additional masses required may be measured using the ( $p,{ }^{6} \mathrm{He}$ ) reaction, and current plans are to use the techniques we have developed for the $\left({ }^{3} \mathrm{He},{ }^{6} \mathrm{He}\right)$ measurements in investigating the ( $p,{ }^{6} \mathrm{He}$ ) reaction.
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# Three-Nucleon Bound State from Faddeev Equations with a Realistic Potential 

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A method for solving Faddeev equations in configuration space is used to study the state of three nucleons bound in a Reid potential. Including $D$ states for the spectator particle gives a binding energy of 6.6 MeV , probabilities $P_{\mathcal{S}^{\prime}}=1.9 \%, P_{D}=7.9 \%$, and a charge radius $r_{c}\left({ }^{3} \mathrm{He}\right)=1.97 \mathrm{fm}$; the dip in the charge form factor is found at $q^{2}=14.5 \mathrm{fm}^{-2}$.

Exact treatment of the three-nucleon bound state can provide a significant test for realistic two-nucleon interactions.

We here give an exact solution of the Faddeev equations for three nucleons bound via the Reid soft-core potential ${ }^{1}$ acting on the states ${ }^{3} S_{1},{ }^{3} D_{1}$, ${ }^{3} D_{2},{ }^{1} S_{0}$, and ${ }^{1} D_{2}$. We consider the six components $\left[\psi_{l \lambda L}{ }^{\alpha}(x, y) e_{S}{ }^{\alpha}\right]_{1 / 2}$ of the Faddeev amplitude listed in Table I. Distances and orbital momenta are denoted by $x$ and $l$ for the interacting pair, and $y$ and $\lambda$ for the spectator particle; $l$ and $\lambda$ are coupled to total momentum $L$. The spin isospin state $e_{S}{ }^{\alpha}$ is characterized by total spin $S$ and by $\alpha$ which stands for $A,-$, or + according to whether $e_{S}{ }^{\alpha}$ has complete antisymmetry, mixed symmetry and antisymmetry, or symmetry, respectively, under the exchange of two interacting par-

TABLE I. Independent components of the Faddeev amplitude included in the complete computation.

| Component | $l$ | $\lambda$ | $L$ | $S$ | $\alpha$ |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | 0 | 0 | 0 | $1 / 2$ | $A$ |
| 2 | 2 | 2 | 0 | $1 / 2$ | $A$ |
| 3 | 0 | 0 | 0 | $1 / 2$ | - |
| 4 | 2 | 2 | 0 | $1 / 2$ | - |
| 5 | 2 | 0 | 2 | $3 / 2$ | - |
| 6 | 0 | 2 | 2 | $3 / 2$ | - |

ticles. Then, writing the Faddeev equations in configuration space ${ }^{2}$ turns them into a set of six partial differential equations ${ }^{3}$ for the six unknown functions of Table I. Typically, the first equation, which involves the spatially symmetric component, has the following form ${ }^{2}$ :

$$
\begin{equation*}
\left[\frac{\hbar^{2}}{m}\left(\frac{\partial^{2}}{\partial x^{2}}+\frac{\partial^{2}}{\partial y^{2}}\right)+E-V(x)\right] \psi_{000}^{A}(x, y)=V(x) \int_{-1}^{1} \frac{x y}{x^{\prime} y^{\prime}} \psi_{000}^{A}\left(x^{\prime}, y^{\prime}\right) d u+\Phi(x, y) \tag{1}
\end{equation*}
$$

where

$$
x^{\prime}=\frac{1}{2}\left(x^{2}-2 \sqrt{3} x y u+3 y^{2}\right)^{1 / 2}, \quad y^{\prime}=\frac{1}{2}\left(y^{2}+2 \sqrt{3} x y u+3 x^{2}\right)^{1 / 2} .
$$

Here the potential $V(x)$ is the average interaction in ${ }^{3} S_{1}$ and ${ }^{1} S_{0}$ states; the function $\Phi(x, y)$ is a short-

