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# MASS 25 ISOBARIC MULTIPLETS

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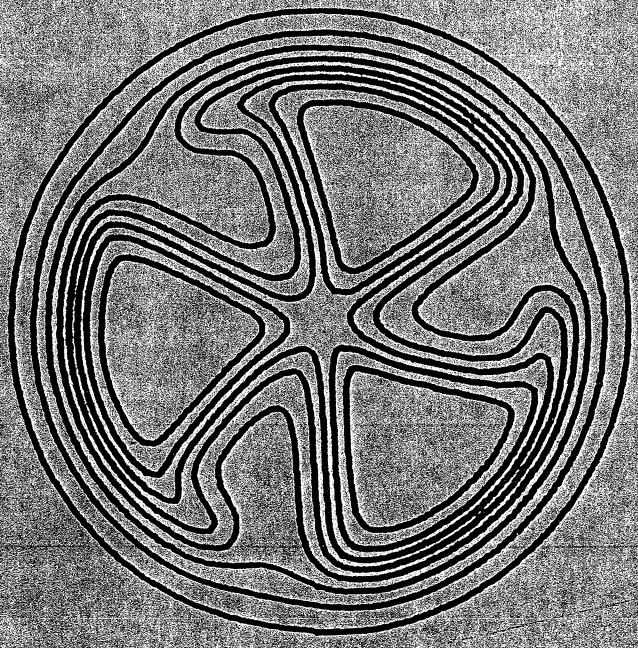

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# Mass 25 Isobaric Multiplets*

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## ABSTRACT

High precision mass determinations have now been made for all four members of the lowest two T=3/2 states in the A=25 system. The results are used to compute the coefficients of the isobaric mass multiplet equation.

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# I. INTRODUCTION

The isobaric mass multiplet equation relates the masses of the 2T+1 members of a multiplet with isobaric spin T. The equation is usually given as

$$M(T_z) = a + bT_z + cT_z^2$$

where $T_z$ is the z component of the isobaric spin, $T_z$=1/2 (N-Z). The coefficients a, b, and c are functions of the mass number, A, and spin, $J^\pi$, of the levels as well. The equation, in principle, relates the masses of any multiplet of analogous levels.

Deviations from the predictions of the isobaric multiplet equation are usually measured by citing the magnitude of d, the coefficient of a cubic term required to fit the masses. In terms of the measured masses $M(T_z)$, the d coefficient has a very simple form for a T=3/2 system, i.e.,

$$d = 1/6[(M(3/2) - M(-3/2) - 3(M(+1/2) - M(-1/2))]$$

where $M(T_z)$ is the mass of the level in the $T_z$ nucleus.

Thus the result d=0 implies that the spacing between inner members is 1/3 the spacing between the outer members. One can also see from the above equation that the uncertainty in d is three times more sensitive to the accuracy of the masses of the $T_z=\pm 1/2$, T=3/2 levels than it is to the $T_z=\pm 3/2$ levels.

In the present paper we are discussing the lowest two T=3/2 levels in the A=25 nuclei. These are the ground and first excited states of $^{25}$Si and $^{25}$Na and the lowest two T=3/2 states in $^{25}$Al

and $^{25}$Mg with spins of $5/2^+$ and $3/2^+$. The $^{25}$Si masses have been reported recently.[1] The present paper reports on the masses of the $^{25}$Al, $^{25}$Mg, and $^{25}$Na levels.

## II. EXPERIMENTAL

### Method

Table I lists the nuclear reactions used to study the levels in $^{25}$Al, $^{25}$Mg, and $^{25}$Na. Also listed are the calibration reactions which in every case were observed from impurities present in the target used for the primary measurement. All the data were taken at angles between 4° and 9° where kinematic effects are small.

The MSU split-pole spectrograph was used with either a single wire resistive readout gas proportional counter or a position sensitive silicon detector on the focal plane. The wire counter was backed up by a plastic scintillator, which was used for particle identification by time-of-flight relative to the time structure of the beam. The method for mass measurements consisted in determining the magnitude of the magnetic fields required to put the primary reaction and the calibration reaction at the same position along the focal plane. Since the Q-values for the primary and calibration reactions are very close, only small changes in magnetic field were required. The spatial resolution of the detectors was about 1 mm and was the dominant contribution to energy resolution of 15 to 20 keV.

### Sources of Error

Since the primary and calibration reactions were measured with the same beam energy, detector angles, target, and position

on the focal plane, most sources of error cancel. The uncertainty of the masses of the target and the projectiles as well as the uncertainties of the Q-value of the calibration reaction generally amounting to about 1-2 keV. .The other error contributions are somewhat larger. They are, in order of decreasing importance, centroid determination, target thickness corrections, total field integral in the spectrometer, beam energy and detection angle. The total error ends up to be approximately 4 keV for all of the cases studied except $^{25}$Na. The measurements were repeated on different days with different angles, beam energies, detector, etc. and the results were found to reproduce to within the random errors. The procedures were also checked by comparing accurately known levels to the calibrations. Several high resolution runs ($\Delta E \sim 4$ keV) were taken on plates, and these results were also found to be consistent with the data taken with the position detectors.

## III. RESULTS

### The Lowest T=3/2 States in $^{25}$Al

The $J=5/2^+$, T=3/2 state in $^{25}$Al was excited by the $^{27}$Al(p,t)$^{25}$Al reaction at 40 MeV. This state has been observed previously by this reaction and shown to have a characteristic strong L=0 angular distribution shape.[2] The $^{12}$C(p,t)$^{10}$C (g.s.) reaction is also L=0, hence both calibration and primary reactions are very strong at the forward angles used. The target was 50 $\mu g/cm^2$ Al on a 20 $\mu g/cm^2$ carbon backing. The energy loss of protons and tritons in the target was small, and no significant

shifts in the position of the reaction peaks were observed when the target was rotated by 180°.

Table II gives the mass excess and excitation energy measured in the present experiment, together with results of previous measurements. The comparisons are considerably clouded by the fact that the $^{25}$Al ground state mass has shifted by 17 keV between the 1964[3] and 1971[4] mass tables, and the results quoted by other authors for the excitation energy had to be corrected.

The second T=3/2 state in $^{25}$Al($3/2^+$) was also observed but was considerably weaker. Its excitation energy was found to be $70\pm1$ keV above that of the lowest T=3/2 state. This result is in excellent agreement with previous results.[5,6]

## The Lowest T=3/2 States in $^{25}$Mg

The $J=5/2^+$, T=3/2 state in $^{25}$Mg was excited by the $^{27}$Al(p,$^3$He)$^{25}$Mg and $^{26}$Mg(p,d)$^{25}$Mg reactions. The state appeared very strongly in the (p,$^3$He) reaction since it is a very forward peaked L=0 like the (p,t) reaction leading to the analogous state in $^{25}$Al. Targets were 50 μg/cm$^2$ on a 20 μg/cm$^2$ carbon backing. Target thickness corrections were not negligible for the $^3$He's produced. The measurements were repeated several times and found to be consistently reproducible.

## $^{25}$Na Ground State

The ground state mass of $^{25}$Na was determined by means of the $^{26}$Mg(d,$^3$He) reaction at 24 MeV. The error for this mass experiment

was 10 keV, which is considerably larger than the other measurements. This was due to target thickness uncertainties and the relatively larger separation between the calibration and primary reaction which makes the measurement quite sensitive to beam energy. The rigidity of the $^{3}$He's produced was so low that it fell in the range in which the calibration of the spectrograph has not been as carefully mapped out as the higher fields. However, the agreement with the previous measurement is good as can be seen in Table II.

## IV. ANALYSIS

### Determination of the Coefficients

The isobaric mass equation was fit to the mass excesses measured in the present experiment with the inclusion of a cubic term of the form, $dT_z^3$. The results are given in Table III. Also given in Table III are the coefficients calculated with the average mass from Table II. There is indication that d has a small positive value.

The spacing of the $5/2^+$-$3/2^+$ doublet is more accurately known in the four nuclei then are the total masses. That this spacing (90, 80, 70, 40 keV) cannot be fit with a quadratic is thus responsible for the difference between the d coefficients of the $5/2^+$ and $3/2^+$ multiplets. This results comes predominantly from the relatively low value of the energy of the first excited state of $^{25}$Si compared to its analogs.

## The d-coefficient

A non-vanishing d coefficient can arise from several mechanisms. The mixing of the $T_z=\pm 1/2$ members with states of T=1/2 has been proposed by Janecke.[7] This effect is shown to be real and configuration dependent in the calculations of McGrory and Wildenthal presented in Ref. 1. This was a shell-model calculation in which neutrons and protons are treated separately, and isobaric spins is no longer a good quantum number. They predict levels in the $T_z=\pm 1/2$ nuclei that have wave functions very similar to the $T_z=\pm 3/2$ levels and lie at about the right energy for a T=3/2 state. These states are mixed with nearby T=1/2 levels and have been shifted enough to give d coefficients of the order of 5 keV.

Another type of wave function change has been calculated by Bertsch and Kahana.[8] In this case, the stronger Coulomb field in the most proton rich nucleus leads to larger proton orbits and thus a weaker two-body Coulomb interaction. This effect gives a positive d coefficient of about 0.25 to 0.50 keV.

The state dependence of the d coefficient is evidence that it is the mixing with the lower T states that is producing the d coefficient. The effect depends on the location of T=1/2 states of the same spin and has not yet been calculated accurately.

## The b- and c-coefficients

The b and c coefficients measure the Coulomb energy as well as its rate of change between isobaric analogs. Under the assumption of uniformly charged sphere of radius $R_c$

$$b = -0.6 \frac{(A-1)e^2}{r_c A^{1/3}} + (M_n - M_p).$$

This gives for both the $5/2^+$ and $3/2^+$ states, $r_c$=1.37 F.

The c coefficient in the same model is given by

$$c = \frac{0.6\ e^2}{r_c A^{1/3}}$$

which gives $r_c$=1.35 F. However both of these radii should be reduced by 3-5% to account for the effects of the Pauli exclusion principle. This is normally called the exchange contribution.

The c-coefficient for the $5/2^+$ multiplet has been calculated by Auerbach, Lev, and Kashy.[9] The b-coefficient was used to determine the radius of the Woods-Saxon potential in which the particles are bound. The identical radius and wave function was then used to calculate the c-coefficient. This calculation, which used the simplest possible configuration, gave c=.240 MeV as compared with the experimental value c=.221 MeV.

The calculations of McGrory and Wildenthal give the b- and c-coefficients for both the $5/2^+$ and $3/2^+$ multiplet. Except for the b-coefficient of the $5/2^+$ multiplet, the agreement is very good. The b-coefficient is about 1% too high which is enough to make the $5/2^+$-$3/2^+$ spacing increase with $T_z$ instead of the experimentally observed decrease.

The present experimental situation for isospin quartets is that only two, the A=9 ground state and A=25 first excited state, require a significant d-coefficient. However, the measured

coefficients are quite small and it is not difficult to find mechanisms which provide shifts of the measured magnitude. Neither theory nor experiment is sufficiently accurate at this point to give quantitative information on the higher order effects which produce the required d-coefficients.

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TABLE I. Reactions Used and Their Q-values in MeV.

| Nucleus | Reaction | Beam Energy | Q-value | Calibration | Q-values |
|---|---|---|---|---|---|
| $^{25}$Al | $^{27}$Al(p,t) | 40 MeV | -23.878 | $^{12}$C(p,t)(g.s.) | -23.364 |
| $^{25}$Mg | $^{27}$Al(p,$^{3}$He) | 40 MeV | -19.426 | $^{16}$O(p,$^{3}$He)(3.945 MeV) | -19.188 |
| | $^{26}$Mg(p,d) | 36 MeV | -16.649 | $^{12}$C(p,d)(g.s.) | -16.497 |
| $^{25}$Na | $^{26}$Mg(d,$^{3}$He) | 24 MeV | - 8.649 | $^{16}$O(d,$^{3}$He)(g.s.) | - 6.634 |
| | | | | $^{12}$C(d,$^{3}$He)(g.s.) | -10.463 |

TABLE II. Comparison to Previous Experiments

| Nucleus | M.E. (MeV) | $E_x$(MeV) | Ref. |
|---|---|---|---|
| $^{25}$Al | -1.008(5) | 7.904 | Present Work |
| | -1.013(6) | 7.899 | a) |
| | -1.010(4) | 7.902 | Average |
| $^{25}$Mg | -5.412(4) | 7.780 | Present Work |
| | -5.410(7) | 7.782 | b) |
| | -5.400(4) | 7.792 | c) |
| | -5.4072(1.3) | 7.7843 | d) |
| | -5.4077(1.3) | 7.7838 | Average Excluding c) |
| $^{25}$Na | -9.356(10) | 0 | Present Work |
| | -9.361(9) | 0 | d) |
| | -9.359(7) | 0 | Average |

[a]Ref. 5.

[b]C. Detraz and R. Richter, Nucl. Phys. A158, 393(1970).

[c]B.L. Berman, R.J. Baglan, and C.D. Bowman, Phys. Rev. Letters 24, 319(1970).

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TABLE III. Coefficients of Isobaric Mass Multiplet Equation in keV

| A | J | a* | b | c | d | |
|---|---|---|---|---|---|---|
| 25 | $5/2^+$ | -3266±4 | -4405±7 | 221±4 | 5.3±4.0 | Present Measurement |
| | | -3264±3 | -4398±5 | 221±3 | 1.9±2.9 | Average |
| 25* | $3/2^+$ | -3189±4 | -4396±8 | 217±4 | 8.6±4.2 | Present Measurement |
| | | -3187±3 | -4388±6 | 216±4 | 4.8±3.6 | Average |

*a is determined for the mass excess