Shell-Model Study of ^{24}Ne[†]

R. G. H. Robertson and B. H. Wildenthal
Cyclotron Laboratory and Physics Department, Michigan State University, East Lansing, Michigan 48823
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Shell-model calculations for the $A=24$, $T=2$ system are presented which use an empirically modified version of Kuo's realistic two-body interaction and a large, but truncated, *sd*-shell basis space. Using the same two-body matrix elements, single-particle energies and effective charges that were found to be successful in the full *sd* space for $A=19-22$, good agreement with experimental measurements is obtained for ^{24}Ne levels and other members of the $A=24$, $T=2$ system. Some predictions are made for the unreported isotope ^{24}Si.

I. INTRODUCTION

This paper describes a large-basis shell-model calculation for the $A=24$, $T=2$ system, with particular consideration of $^{24}_{10}$Ne$_{14}$. Although ^{24}Ne is difficult to study experimentally, there is now a significant body of data obtained via the ^{22}Ne(t,p)-^{24}Ne reaction,[1-3] the ^{22}Ne$(t,p\gamma)^{24}$Ne reaction,[2,3] and the ^{3}H(^{22}Ne$,p\gamma)^{24}$Ne reaction.[4,5] Efforts to understand the ^{24}Ne spectrum in terms of projected Hartree-Fock[6] (HFP) and projected Hartree-Bogoliubov[7] (HBP) calculations have not been successful. It is thus of interest to see whether, as in the case of ^{24}Mg,[8] a shell-model calculation can reproduce simultaneously the collective and the individual-particle aspects of this nucleus. The main sections of the paper describe the calculation and diagonalization of the Hamiltonian matrices and the calculation of energy eigenvalues and electromagnetic properties. Two-nucleon transfer, and the β decays of ^{24}Ne and the unreported isotope ^{24}Si are also discussed briefly.

II. CALCULATION OF HAMILTONIAN

The two-body matrix elements used in this work are those described by Preedom and Wildenthal[9] and used in shell-model calculations for $A=19-22$,[9] and 23 and 24.[8] They are based on Kuo's realistic effective interaction,[10] but have been modified as described in Ref. 9 to produce better agreement of shell-model eigenvalues (calculated in the full *sd*-shell basis) with experimental level energies in the $A=18$ to 22 mass region. No data from nuclei with $A>22$ were used in the adjustment. Single-particle energies, taken directly from the ^{17}O spectrum, were equal to -4.15, -3.28, and $+0.93$ MeV for the $0d_{5/2}$, $1s_{1/2}$, and $0d_{3/2}$ orbitals, respectively. Although all three *sd*-shell orbitals were allowed to be active, it was necessary in the present calculation to restrict occupancy of the *d* orbitals in order to keep the dimensionality of the basis space manageable. Specifically, no fewer than four particles were permitted in the $0d_{5/2}$ shell, and no more than two in the $0d_{3/2}$ shell. With these restrictions, the dimensions of the largest matrix to be diagonalized were 442×442 for $J=3$.

The diagonalization itself was carried out at the Oak Ridge National Laboratory on an IBM 360 series computer using the Oak Ridge-Rochester shell-model code,[11] and the calculation of observables from the wave functions was done on the XDS Sigma-7 computer of the Michigan State University Cyclotron Laboratory.

The calculated energy spectrum for ^{24}Ne is shown in Fig. 1 compared to the experimental spectrum as given by Howard *et al.*[3] Also shown are the calculations of Khadkikar, Nair, and Pandya[6] and Goeke, Faessler, and Wolter.[7] There is good agreement between the shell-model spectrum and all known states of ^{24}Ne. In fact the correspondence is sufficiently close that one may assign the 4.89-MeV state a probable spin and parity of 3^+. Such an assignment is in accord with the experimental observation that the state is relatively weak in (t,p), as would be expected for an unnatural parity transition.

The calculation for the $T=0$ states of ^{24}Mg with the same Hamiltonian has been described previously,[8] and there is good agreement with experiment for all but a few levels. However, when the positions of the lowest $T=1$ and $T=2$ levels in ^{24}Mg are calculated, the predicted excitation energies are too low by a few hundred keV for $T=1$ and 2 MeV for $T=2$. The source of the discrepancy is apparently the truncation of the basis space. For a few spins, $J=0$ and $J\geqslant8$, eigenvalues have been recalculated in the full *sd* basis, and the observed $T=0$ to $T=2$ splitting is then reproduced almost exactly (Fig. 2). Furthermore, the positions of excited 0^+ and 8^+ states do not change significantly relative to the lowest 0^+ state for each value of T. Thus there is some reason to believe that the effect of basis truncation is to shift the entire spectrum for a given value of T without greatly

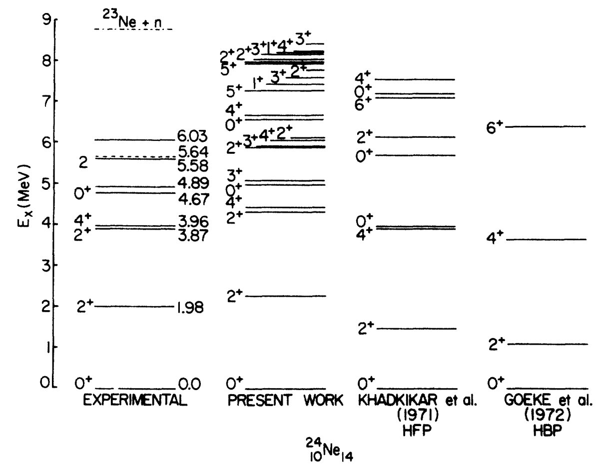

FIG. 1. Comparison of shell-model excitation energies with experimental levels of ^{24}Ne and with the calculations of Refs. 6 and 7.

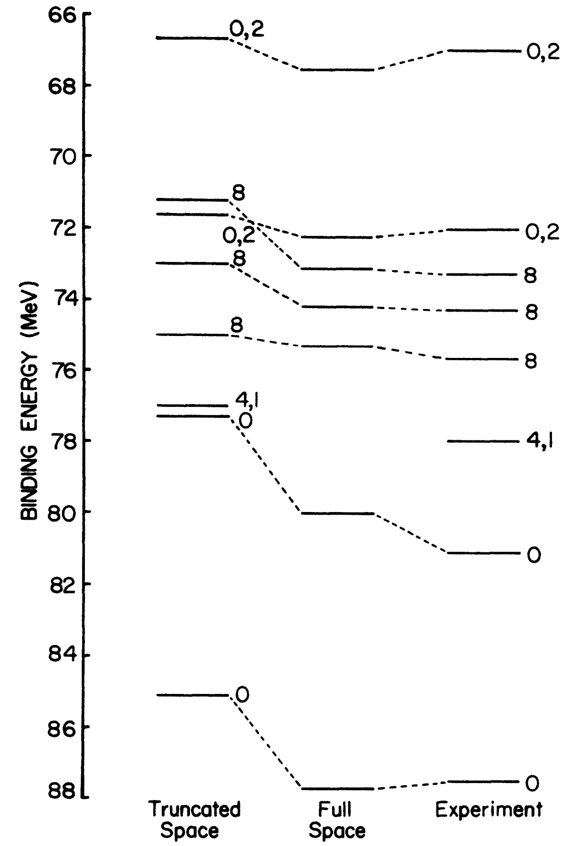

FIG. 2. Binding energies (relative to ^{16}O) for states of spin 0^+ and 8^+ in ^{24}Mg calculated in the truncated space and in the full space. Also shown is the location of the lowest $T=1$ state calculated in the truncated space. The experimental binding energies have been corrected for Coulomb effects.

altering the level spacings within that spectrum. In any event the effect of basis truncation is evidently more important for the $T=0$ states than for the $T=2$ states, and it seems probable that the ^{24}Ne spectrum calculated in the full basis would not be substantially different from that presented in Fig. 1. (In Fig. 2, the experimental positions of the 8^+ states have recently been established by Branford *et al.*[12] and the position of the second 0^+, $T=2$ state has been inferred from the ^{24}Ne spectrum.)

III. ELECTROMAGNETIC PROPERTIES

The total transition rate for emission of electromagnetic radiation of multipolarity L may be written[13]:

$$T(L)=\frac{8\pi(L+1)}{L[(2L+1)!!]^2}\frac{k^{2L+1}}{\hbar}B(L),$$

where

$$B(L)=(2J_i+1)^{-1}\sum_{M_i M_f m}|\langle J_f M_f T_f T_{zf}|\mathcal{O}^L_m|J_i M_i T_i T_{zi}\rangle|^2 .$$

The electromagnetic operator $\mathcal{O}^L_m$ of rank L, projection m, has the following forms (with the usual notation):

Electric:

$$\mathcal{O}^L_m(E)=\sum_{i=1}^{A}\left[e_n(i)\frac{\tau_0^0(i)-\tau_0^1(i)}{2}+e_p(i)\frac{\tau_0^0(i)+\tau_0^1(i)}{2}\right]r_i^L Y^L_m(\hat{r}_i) .$$

Magnetic:

$$\mathcal{O}^L_m(M)=\sum_{i=1}^{A}[\operatorname{grad} r_i^L Y^L_m(\hat{r}_i)]\cdot\left[\left(g_{nl}(i)\frac{\tau_0^0(i)-\tau_0^1(i)}{2}+g_{pl}(i)\frac{\tau_0^0(i)+\tau_0^1(i)}{2}\right)\frac{2\vec{l}_i}{L+1}\right.$$
$$\left.+\left(g_{ns}(i)\frac{\tau_0^0(i)-\tau_0^1(i)}{2}+g_{ps}(i)\frac{\tau_0^0(i)+\tau_0^1(i)}{2}\right)\vec{s}_i\right]\mu_N .$$

In these expressions, $\tau_0^1(i)$ is the isospin operator[13] of rank 1 and projection 0, defined to have eigenvalues of +1 or −1 when operating on a proton or a neutron state, respectively, while $\tau_0^0(i)$ is the isospin identity operator. The quantity μ_N is the nuclear magneton. One can express each electromagnetic operator in JT space as a sum of isoscalar and isovector operators:

$$\mathcal{O}_m^L = \sum_{k=0}^{1} (-1)^k \Omega_{m0}^{Lk} ,$$

where Ω_{m0}^{Lk} is an electromagnetic operator of rank L, projection m in J space, and rank k, projection 0 in T space:

$$\Omega_{m0}^{Lk}(E) = \tfrac{1}{2} \sum_{i=1}^{A} [e_n(i) + (-1)^k e_p(i)] r_i^L Y_m^L(\hat{r}_i)\, \tau_0^k(i) ,$$

$$\Omega_{m0}^{Lk}(M) = \frac{1}{2} \sum_{i=1}^{A} [\mathrm{grad} r_i^L Y_m^L(\hat{r}_i)] \cdot \left\{ [g_{nl}(i) + (-1)^k g_{pl}(i)] \frac{2\vec{l}_i}{L+1} + [g_{ns}(i) + (-1)^k g_{ps}(i)] \vec{s}_i \right\} \mu_N \tau_0^k(i) .$$

The reduced transition probability then becomes:

$$B(L) = (2J_i+1)^{-1}(2T_f+1)^{-1}(\langle J_f T_f ||| \Omega^{L0} ||| J_i T_i \rangle \delta_{T_i T_f} - \langle T_i T_{zi} 1\, 0 | T_f T_{zf} \rangle \langle J_f T_f ||| \Omega^{L1} ||| J_i T_i \rangle)^2 ,$$

where the triple bars indicate that the matrix element is reduced with respect to both J and T.

The results of calculations of $E2$ and $M1$ electromagnetic transition rates in ^{24}Ne are presented in Table I. Free-nucleon g values have been employed in the $M1$ operator. Effective charges of $e_p = 1.5e$ and $e_n = 0.5e$ and harmonic-oscillator wave functions ($\hbar\omega = 41A^{-1/3}$) are assumed for the $E2$ operator. These choices have given a good account of electromagnetic properties for other sd-shell nuclei.[8,9] The calculated transition rates for ^{24}Ne are in excellent agreement with the limited experimental data available.

One of the objectives of the investigation was to learn whether the effective charges which gave good results for isoscalar transitions in $N=Z$ nuclei would also be satisfactory for transitions with isovector components. While that appears to be the case for ^{24}Ne, it is possible that future, more detailed experimental data may reveal a need for changes in e_p and e_n. Therefore, Tables II and III list separately the isoscalar and isovector reduced matrix elements so that such an adjustment may be made. From the form of the operators $\Omega_{m0}^{Lk}(E)$ it may be seen that the isoscalar term is proportional to $(e_n + e_p)$ and the isovector term to $(e_n - e_p)$. Hence values of $B(M1)$ and $B(E2)$ may be obtained for any choice of e_p and e_n, and for any T_z, by application of the expressions given above. (The reduced matrix elements shown in Table III include factors of $e_p + e_n = 2.0$ and $e_n - e_p = -1.0$. The ith excited state of a given J is identified as J_i.) Generally, $E2$ transitions are dominated by the isoscalar component, and only $e_p + e_n$ can be well de-

TABLE I. Electromagnetic decay properties and mean lifetimes of ^{24}Ne levels.

Initial levels				Final levels			
	E_x	τ (psec)			E_x	Branch (%)	
J^π	(MeV)	Calc.	Exp.[a]	J^π	(MeV)	Calc.	Exp.[b]
2^+	1.9808	0.71	$1.0^{+0.2}_{-0.4}$	0^+	0.00	100	100
2^+	3.867	0.02	<0.1	0^+	0.00	4	10
				2^+	1.9808	96	90
4^+	3.96	0.90		2^+	1.9808	100	100
0^+	4.6757	3.53	>1.5	2^+	1.9808	98	100
				2^+	3.867	2	0
3^+	4.89	0.04		2^+	1.9808	93	100[c]
				2^+	3.867	6	0
				4^+	3.96	1	0
2^+	5.58	0.03		0^+	0.00	3	5
				2^+	1.9808	95	100
				2^+	3.867	2	5

[a] Reference 5.
[b] Reference 3.
[c] Weak.

TABLE II. $M1$ transitions in the $A=24$, $T=2$ system.

Initial level	Final level	$\langle J_f T_f \,\|\|\|\, \Omega^{1k}(M) \,\|\|\|\, J_i T_i \rangle$ $k=0$	$k=1$	$B(M1)(T_z=-2)$ (μ_N^2)
2_2	2_1	−0.021	3.88	0.407
3_1	2_1	0.149	1.85	0.053
3_1	2_2	0.198	2.36	0.085
3_1	4_1	−0.009	−0.84	0.013
2_3	2_1	0.142	−3.46	0.352
2_3	2_2	−0.105	−1.64	0.061
2_3	3_1	−0.051	−2.66	0.180
3_2	2_1	0.052	−0.87	0.017
3_2	2_2	−0.076	−3.78	0.259
3_2	4_1	−0.041	−1.88	0.064
3_2	3_1	0.122	1.28	0.024
4_2	4_1	−0.084	−3.32	0.153
4_2	3_1	−0.031	1.97	0.060
2_4	2_1	0.002	−0.19	0.001
2_4	2_2	−0.100	−1.80	0.075
2_4	3_1	0.048	0.81	0.015

termined. However, it is interesting that the ground-state transition from the second excited 2^+ state in ^{24}Ne is predicted to be almost entirely isovector. A more accurate experimental determination of its partial lifetime would thus be of value in establishing $e_n - e_p$, and hence e_n and e_p individually. There is at present no experimental information on static magnetic dipole and electric quadrupole moments of states in ^{24}Ne. For the first excited state (2^+) these are predicted to be $1.88\mu_N$ and 0.02 b, respectively. Such a small quadrupole moment is in marked contrast to the predictions of Hartree-Bogoliubov theory.[7]

The γ decay of the lowest $T=2$ state in ^{24}Mg at 15.436 MeV to $T=1$ states at 10.03 and 10.80 MeV has been observed by Riess *et al.*[14] The calculated values of $B(M1)$ for the population of these two states are 1.15 and $0.75\mu_N{}^2$, respectively. The corresponding width for the transition to the 10.03-MeV state is 2.2 eV, which may be compared with the experimental estimate[14] of 1.7 eV.

IV. TWO-NUCLEON TRANSFER

Glendenning[15] has shown that the differential cross section for a direct two-nucleon transfer reaction may be expressed in the form:

$$\frac{d\sigma}{d\theta} = \sum_{MLSJT} \left|\sum_N G_{NLSJT} B^M_{NL}(\vec{k}_1, \vec{k}_2)\right|^2 .$$

TABLE III. $E2$ transitions in the $A=24$, $T=2$ system.

Initial level	Final level	$\langle J_f T_f \,\|\|\| \Omega^{2k}(E) \,\|\|\| J_i T_i \rangle$ $k=0$	$k=1$	$B(E2)(T_z=-2)$ ($e^2\,\mathrm{fm}^4$)
2_1	g.s.	31.3	1.3	36.7
2_2	g.s.	1.3	10.3	2.0
2_2	2_1	29.1	3.3	27.8
4_1	2_1	35.6	−1.7	30.4
0_2	2_1	−4.8	−2.7	1.4
0_2	2_2	−9.1	−2.6	9.8
3_1	2_1	−2.7	12.5	4.7
3_1	2_2	−7.9	−0.4	1.6
3_1	4_1	3.6	0.5	0.3
2_3	g.s.	−5.2	1.1	1.5
2_3	2_1	22.7	−2.6	24.6
2_3	2_2	−22.1	−3.5	14.8
2_3	4_1	−9.2	4.2	6.3
3_2	2_1	−0.8	11.9	3.2
3_2	2_2	−46.5	−2.4	56.8
3_2	4_1	−20.5	−7.1	6.1
4_2	2_1	−24.2	−8.1	6.9
4_2	2_2	−21.0	3.6	12.7
4_2	4_1	−16.8	−7.1	2.7
2_4	g.s.	−2.3	1.1	0.4
2_4	2_1	7.8	−2.0	3.6
2_4	2_2	−6.9	−4.0	0.5
2_4	4_1	−6.7	−0.1	1.8

The structure factors G contain microscopic nuclear information, and in particular the amplitudes for finding two nucleons in various shell-model states coupled to $LSJT$. The quantities B^M_{NL} depend on details of the reaction mechanism, however, and cannot as yet be calculated with great reliability. Nevertheless by making some plausible assumptions[15] one can test the shell-model predictions for the structure factors by comparing relative cross sections to states of given JT, because many of the uncertain aspects of B^M_{NL} are the same for all such states. The structure factors G may be written

$$G_{NLSJT} = g\sum_\gamma Z_{(j_1 j_2)JT} \begin{pmatrix} l_1 & \frac{1}{2} & j_1 \\ l_2 & \frac{1}{2} & j_2 \\ L & S & J \end{pmatrix} \times \Omega_n \langle n0, NL; L | n_1 l_1, n_2 l_2; L\rangle ,$$

where γ ($\equiv n_1 l_1 j_1 n_2 l_2 j_2$) implies summation over all states of the two nucleons, and $g=1$ if $n_1 l_1 j_1 = n_2 l_2 j_2$ and $\sqrt{2}$ otherwise. The bracket ( ), defined by Glendenning,[15] transforms from j-j to L-S coupling; Ω_n, also defined by Glendenning, is an overlap integral between the relative motions of the transferred particles in the projectile and the residual nucleus; and $\langle\, |\, \rangle$ is a Moshinsky bracket. The pair transfer amplitude $Z_{(j_1 j_2)JT}$ is the mixed-configuration two-particle coefficient of fractional parentage (cfp) defined by Cohen and Kurath.[16] It depends on the cfp's between pure j-j configurations and on the amplitudes of those configurations in the target and residual nucleus vectors. For pure configurations it is equivalent to the $B(J, j_1 j_2)$ used in Ref. 15 and elsewhere.

In the (t,p) reaction on an even-even nucleus, the quantum numbers $LSJT$ of the transferred pair of neutrons are uniquely determined, and only the summation over the radial quantum numbers need be carried out. Table IV presents pair transfer amplitudes $Z_{(j_1 j_2)JT}$ calculated from the shell-model wave functions, as well as an "enhancement factor" A_i defined as

$$A_i = \frac{G_{N(\max)LSJ_iT_i}{}^2}{G_{N(\max)LSJ_1T_1}{}^2} .$$

Use of this factor exploits the tendency for both G and B to be largest for the largest allowed value of N, and permits a crude comparison with experiment without reference to detailed reaction mechanisms. The comparison in Table IV is with the ^{22}Ne$(t,p)^{24}$Ne data of Silbert and Jarmie.[1] The agreement is only qualitative at best, but Q-value effects have been ignored and Silbert and Jarmie's data were taken at $E_t = 2.6$ MeV where it is unlikely that the reaction is entirely direct. Indeed the pu-

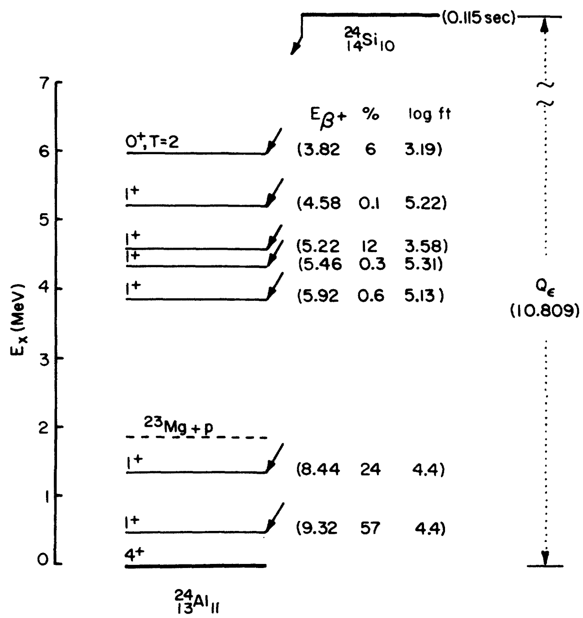

FIG. 3. Predicted decay scheme for the unreported isotope ^{24}Si.

tative 3^+ state at 4.89 MeV, although weak, is still stronger than the 3.96-MeV 4^+ state and the 5.58-MeV $2^{(+)}$ state (at 30°_{lab}), and that would argue against a completely direct one-step mechanism. An investigation of the ^{22}Ne$(t,p)^{24}$Ne reaction at higher energies is in progress.[17]

It is of interest that the calculation predicts the second and third 4^+ states to be substantially stronger than the lowest, and the unassigned level at 6.03 MeV is seen in (t,p) with a cross section 7 times that of the 3.96-MeV state. The calculated level scheme (Fig. 1) lends some support to a 4^+ assignment for this state since the nearby levels, 0^+, 2^+, and 3^+, are predicted to be weakly populated.

V. β DECAYS OF ^{24}Ne AND ^{24}Si

The β^- decay of 3.38-min ^{24}Ne populates two 1^+ states in ^{24}Na. The calculation of $\log ft$ values for these transitions has been described previously,[18] and the results are in excellent agreement with experiment. Calculated $\log ft$ values for population of the 0.47- and 1.35-MeV states are 4.29 and 4.30, respectively, while measured values are 4.4 in both cases.

The proton-rich, $T_z = 2$ isotope, ^{24}Si has not been observed, but an experimental measurement of its mass would be of considerable interest for testing the isobaric multiplet mass equation (IMME) in an isospin quintet. On the basis of experimental data for ^{24}Ne, ^{24}Na, ^{24}Mg, and ^{24}Al, and the present shell-model calculations, a number of predictions can be made for ^{24}Si. Its mass may be estimated directly from the IMME, assuming that the lowest $T=2$ state in ^{24}Na lies at 5.97 MeV excitation.[19] The resulting mass excess, 10.76 MeV, implies that ^{24}Si is stable against proton decay by 3.30 MeV, against two-proton decay by 3.43 MeV, and against α decay by 9.18 MeV. The β^+ decay to the lowest 1^+ states is analogous to the decay of ^{24}Ne. Additional states at higher excitation are accessible, and the $\log ft$ values of transitions leading to the next four 1^+ states have been calculated from the shell-model wave functions. These states,

TABLE IV. Calculated pair transfer amplitudes for ^{22}Ne$(t,p)^{24}$Ne.

E_x (MeV)	J_i^π	$(d_{5/2})^2$	$(s_{1/2})^2$	$(d_{3/2})^2$	$d_{5/2}s_{1/2}$	$d_{5/2}d_{3/2}$	$s_{1/2}d_{3/2}$	A_i	$(\sigma_i/\sigma_1)_{\text{exp}}$
0.0	0_1^+	−0.7921	−0.2844	−0.1775				1	1
4.76	0_2^+	0.0725	0.6885	0.0654				0.69	1.92
	0_3^+	0.2844	−0.0315	0.0771				0.05	
1.99	2_1^+	−0.1039		0.0509	0.2019	0.0520	0.0438	1	1
3.87	2_2^+	0.2859		0.0135	0.3783	−0.0476	0.0286	5.56	1.52
5.58	2_3^+	0.0685		0.0128	−0.1082	0.1585	−0.0439	0.06	0.13
	2_4^+	−0.1721		−0.0261	0.2351	0.0381	−0.1148	0.08	
3.96	4_1^+	0.0154				−0.0560		1	1
	4_2^+	−0.2351				−0.0582		14	
	4_3^+	−0.1565				−0.1892		31	
	4_4^+	−0.0081				−0.0414		1	

and the lowest $T=2$ state, are above the proton breakup threshold. The results of these considerations are summarized in Fig. 3. The half-life of ^{24}Si (from the seven decay branches shown) is estimated to be 115 msec.

VI. CONCLUSIONS

Essentially all of the experimentally known properties of ^{24}Ne have been reproduced by the shell-model calculations presented here. The agreement between theory and experiment is particularly close for energy eigenvalues and electromagnetic transition rates. A similar comparison for two-nucleon transfer cross sections is vitiated by uncertainties in the reaction theory, but qualitative trends appear to be satisfactorily accounted for. The microscopic structure of ^{24}Ne is evidently rather complex, for it is not well explained in terms of Hartree-Fock[6] or Hartree-Bogoliubov[7] calculations. Although the spectra of ^{24}Ne and ^{18}O are quite similar, as noted by Howard *et al.*,[3] which might be explained by closure of the $0d_{5/2}$ neutron shell at $N=14$, this attractive analogy is not supported by detailed examination of the calculated wave functions. They show little evidence for shell closure, and it seems the resemblance is partly fortuitous.

It is particularly gratifying that these results have been obtained with a model Hamiltonian that makes use of no data from the mass region $A>22$. One is encouraged to believe that the effective Hamiltonian approach will have very wide applicability.

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