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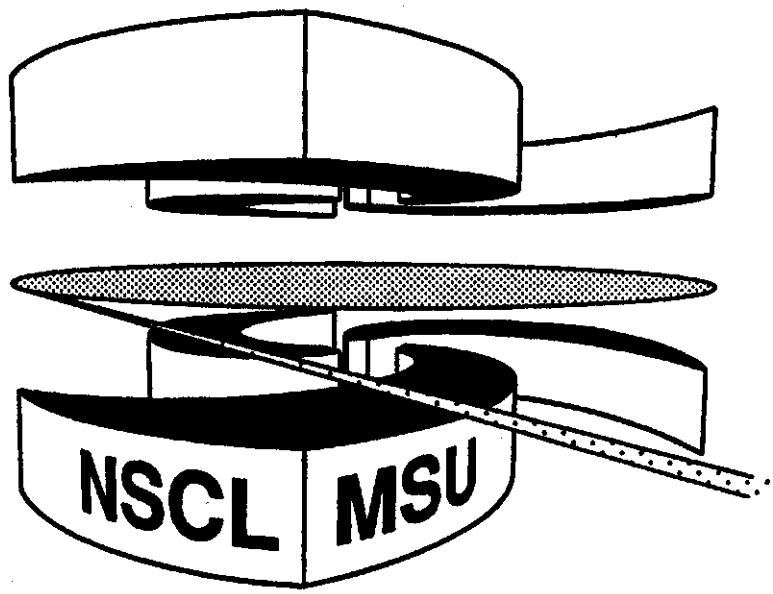

MSUCL-1057

JANUARY 1997

A Simple Mode on a Highly Excited Background: Collective Strength and Damping in the Continuum

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Abstract

Simple states, such as isobaric analog states or giant resonances, embedded into continuum are typical for mesoscopic many-body quantum systems. Due to the coupling to compound states in the same energy range, a simple mode acquires a damping width ("internal" dynamics). When studied experimentally with the aid of various reactions, such states reveal enhanced cross sections in specific channels at corresponding resonance energies ("external" dynamic8 which include direct decay of a simple mode and decays of intrinsic compound states through their own channels). We consider the interplay between internal and external dynamic8 using a general formalism of the effective nonhermitian hamiltonian and looking at the situation both from "inside" (strength functions and spreading widths) and from "outside" (S-matrix, cross sections and delay times). The restoration of isospin purity and disappearance of the collective strength of giant resonance8 at high excitation energy are discussed as important particular manifestations of this

complex interplay.

I. INTRODUCTION

Dynamical features of open mesoscopic quantum systems are characterized by the presence of "simple" (single-particle and collective) excitations, "complicated" (chaotic) intrinsic motion involving many degrees of freedom, and irreversible decay into continuum. The coexistence and interplay of these phenomena is the important aspect of all processes including excitation and deexcitation of the system. One of the questions of primary interest in nuclear physics, especially for future experiments with radioactive nuclear beams, is that of existence and purity of simple modes of nuclear excitation embedded into continuum. Similar problems arise in atomic and molecular physics, physics of atomic clusters and mesoscopic solid state devices.

During the last decade, a number of related phenomena were discovered in this area of nuclear physics, see for example [1,2]. Saturation of the spreading width of the giant dipole resonance (GDR) in hot nuclei [3–5], "disappearance" of the collective strength of the GDR at high excitation energy [6], and existence and relatively narrow width of the double GDR [7–9] are just few bright examples. In the physics of isobaric analog states (IAS), one can mention the evidence for existence of the so-called "broad poles" [10], very weak fluctuations of the spreading widths of the IAS throughout the periodic table [11–13], and restoration of isospin purity at high excitation energy [2].

In such problems, one always deals with a simple excitation mixed with the dense background of complicated states. The simple excitation is associated with a specific signal. It can be a quantum number which singles out the state in the ocean of surrounding states as it happens in the IAS case. As a rule, such a state is relatively pure with respect to this label when looked at in the entrance channel. The isospin purity is violated by the internal mixing [11] when, due to the high background level density, the statistical enhancement of perturbations becomes extremely important, similar to the well known enhancement of weak interactions observed in parity nonconservation [14]. The individuality of a "simple" mode can also be referred to its specific structure, for example in the case of a giant collective

vibration, whose coherence makes the state very different from the background. Such a special state is characterized by a large multipole moment which provides a strong collective gamma decay [1]. In all cases, the manifestations of the simple mode in specific reaction channels are intertangled with the chaotic mixing inside the system.

As a result of the mixing, the simple mode is fragmented over exact stationary states which form the fine structure of the spectrum. Being averaged over the unresolved fine structure, the excitation function is related to the strength distribution of the original "label" smoothly depending on excitation energy. More detailed statistical analysis of observed fluctuations, assuming generic correlations of energies and strengths for the invisible underlying states, is capable [15] of extracting their characteristics. In general, the strength functions and reaction cross sections represent two sides of the process, internal and external, and the relation between them is far from trivial. Thus, the strength distribution may or may not coincide with the width distribution seen in the reactions and decays [16].

The well known formalism [17] of the strength function proceeds as if the states under consideration were stable. However, all excited states, strictly speaking, have a finite lifetime and therefore belong to the continuum spectrum. The level widths of the resonances in the continuum [18,19] are governed by the interaction which is in general different from that forming the discrete spectrum inside the system. The effects of intrinsic mixing and coupling to and via continuum have to be considered simultaneously.

Below we formulate a consistent quantum-mechanical approach which fully accounts for the interplay of internal damping and decay and contains, as particular cases, the "disappearance" of the collective strength of the giant resonance [1] explained by kinetic arguments in [20], and the restoration of isospin purity at high excitation energy [2] in accordance with the old idea by Morinaga [21] and Wilkinson [22]. We discuss the general properties of the strength function of a simple mode embedded into continuum in its relation to what is observed in reactions. Our consideration, being intentionally schematic and less specific than in the well known review paper [11], is in many aspects complementary because of its generality and the simultaneous treatment of internal and external aspects of the problem.

II. EFFECTIVE HAMILTONIAN

We use the effective non-hermitian hamiltonian [19,24] in order to take into account internal and external interactions on equal footing. The intrinsic structure at high level density produces the set ("background") of the basis intrinsic states $|n\rangle$, $n = 1, \ldots N$, where N is supposed to be large. The simple state $|0\rangle$ is located in the the same range of energy. All $N + 1$ states have the same values of exact integrals of motion such as total angular momentum. We assume that the basis states are characterized also by quantities like isospin or parity which are approximate integrals of motion. The isospin mixing which is one of the subjects of our application is introduced explicitly by the off-diagonal elements of the hamiltonian. Parity nonconservation due to weak interactions can be another example of an approximate conservation law which can be included in a similar manner.

The effective hamiltonian in $(N + 1)$-dimensional space is the operator

$$\mathcal{H} = H - \frac{i}{2}W \tag{1}$$

containing two (real and symmetric for a time reversal invariant system) matrices H and W which describe internal and external coupling, respectively.

The antihermitian part W has a special structure [19,11,24] being originated by the on-shell decays into open channels $c = 1, 2, \ldots, k$,

$$W = \mathbf{A}\mathbf{A}^T \Rightarrow W_{nn'} = \sum_c A_n^c A_{n'}^c. \tag{2}$$

Here we introduced the $(N + 1) \times k$ matrices $\mathbf{A} = \{A_n^c\}$ of real transition amplitudes which are proportional to the matrix elements of the full original hermitian hamiltonian which connect intrinsic and channel subspaces of total Hilbert space.

The hermitian part H consists of the unperturbed energy ϵ_0 of the simple state $|0\rangle$, the internal $N \times N$ hamiltonian h describing the background states $|n\rangle$, and the coupling between the simple and complicated states. The real coupling matrix elements $H_{0n} = H_{n0} \equiv V_n, n \geq 1$, form an N-dimensional column vector $\mathbf{V}$. The spectrum h_n of the eigenvalues

of h is supposed to be very dense. Along with the similarity of generic complicated wave functions [25], this justifies the statistical approach.

The effective hamiltonian $\mathcal{H}$ can be studied with standard matrix methods [24,26]. Its diagonalization gives complex eigenvalues

$$\mathcal{E}_j = E_j - \frac{i}{2}\Gamma_j, \quad j = 0, \ldots, N, \tag{3}$$

and the quasistationary eigenstates $|j\rangle$ with a pure exponential decay law $\sim \exp(-i\mathcal{E}_j t)$. The construction of the effective hamiltonian guarantees the unitarity of the scattering matrix, see below Sect. 5.2.

III. STANDARD MODEL OF THE STRENGTH FUNCTION

The description of the mixing of stable internal states, which forms complicated stationary superpositions and spreads the srength of original simple states, is well known [17]. With the antihermitian part W omitted, the intrinsic propagation within the closed system is described by the Green function $G(E)$ of the hermitian part of the hamiltonian,

$$G(E) = \frac{1}{E - H}. \tag{4}$$

The eigenvalues of the intrinsic hamiltonian H are given [17] by the $(N+1)$ poles $E = \epsilon_\alpha$ of the Green function (4). They are the roots of the secular equation

$$G_{00}^{-1}(E) \equiv F(E) = E - \epsilon_0 - \sum_{n \geq 1} \frac{V_n^2}{E - h_n} = 0. \tag{5}$$

Each eigenfunction $|\alpha\rangle$ of H carries a fraction

$$f^\alpha = |C_0^\alpha|^2 = \left(\frac{dF}{dE}\right)_{E=\epsilon_\alpha}^{-1} = \left[1 + \sum_n V_n^2/(\epsilon_\alpha - h_n)^2\right]^{-1} \tag{6}$$

of the collective strength determined by the weight of the corresponding component C_0^α in the expansion over the basis states,

$$|\alpha\rangle = C_0^\alpha |0\rangle + \sum_{n=1}^{N} C_n^\alpha |n\rangle. \tag{7}$$

The smooth strength function of the simple excitation is defined in terms of the average local level spacing D of background states,

$$P_0(\epsilon) = [f^\alpha / D(\epsilon)]_{\epsilon_\alpha = \epsilon}. \tag{8}$$

It is normalized according to $\sum_\alpha f^\alpha = \int d\epsilon P_0(\epsilon) = 1$.

The formal solution (6) requires the knowledge of statistical properties of the background spectrum h_n and coupling matrix elements V_n. The simplest ansatz used in the standard model [17] assumes a roughly equidistant dense spectrum of h_n and interaction intensities V_n^2 uncorrelated with energies h_n and slightly fluctuating around their mean value $\langle V^2 \rangle$. For convenience of the reader, we collect the results of the uniform model in the Appendix, along with a brief discussion.

At $\langle V^2 \rangle > D^2$, the strength function of this uniform model has the Breit-Wigner shape

$$P_0(\epsilon) = \frac{1}{2\pi} \frac{\Gamma^\downarrow}{(\epsilon - \epsilon_0)^2 + (\Gamma^\downarrow)^2/4}. \tag{9}$$

with the spreading width given by the golden rule,

$$\Gamma^\downarrow \equiv \Gamma_s = 2\pi \frac{\langle V^2 \rangle}{D}. \tag{10}$$

The standard model just described is valid [27–29] if Γ_s does not exceed the energy range ΔE of coupling strength V_n^2 (ΔE is defined by the spread of the doorway states which provide the gates for the further mixing of the state $|0\rangle$). This is expected to be a good approximation for the IAS with the typical spreading width ≤ 100 keV. In the case of giant resonances $\Gamma_s \simeq \Delta E$ and the uniform model should be corrected [30]. However the difference influences mainly the shape of the wings of the strength function which is of minor importance for our purpose; here we use the uniform model for definiteness.

IV. SIMPLE STATE EMBEDDED INTO CONTINUUM

A. Formulation of the problem

Now we take into account the openness of the system. The simple state $|0\rangle$ is open to the direct decay (channels c which display specific signatures of the simple mode, for example collective γ-radiation from the giant reasonance or pure isospin of the IAS). Due to the intrinsic coupling to compound states, the simple state also acquires access to many "evaporation channels" labeled by the superscript e; partial widths depend on the distribution of strength of the simple mode carried by specific compound states.

When applied to the IAS with isospin $T_>$, we have to consider the surrounding background states $|n\rangle$ which belong mainly to the isospin $T = T_> - 1$. The isospin mixing occurs mostly through intrinsic interaction [11] so that the decay channels for the decoupled simple mode and evaporation channels for compound states carry different isospins. In many cases, the effects we are interested in can be studied using one direct channel which will be labeled as $c = 0$. Then we have in the hamiltonian (1) the amplitudes $A^0_0 \equiv \sqrt{\gamma_0}$, where γ_0 is the "natural" width of the simple state, and $A^e_n, n \geq 1$. All A^e_n are assumed to be of the same order of magnitude.

At low energies (for example, for neutron resonances), only few decay channels are open and the narrow compound states do not overlap. Their widths $\gamma_n = \sum_e (A^e_n)^2$ are small compared to their mean energy spacing D. As energy and level density increase, we pass the region of strong coupling via continuum where the width collectivization occurs and broad "Dicke resonances" [23,24,26] form the contribution of direct processes. The situation changes again when many uncorrelated decay channels are open, and the off-diagonal elements (2) of the antihermitian part W of the effective hamiltonian are averaged out. Then the states $|0\rangle$ and $|n\rangle$ simply acquire finite widths. These unstable states are coupled through the hermitian interaction V and this is what bridges the gap between the intrinsic strength function and its manifestation in the resonance reactions.

To describe the open compound states, we introduce the $N \times N$ Green function

$$g(z) = \frac{1}{z - h + (i/2)w}. \qquad (11)$$

N complex poles $z = \tilde{\epsilon}_\nu$ of $g(z)$ determine energies and evaporation widths of compound resonances still decoupled from the simple mode. In eq.(11), w stands for the $(N \times N)$ submatrix of W, eq.(2), which acts in the compound subspace and describes the evaporation together with the interaction between the compound states through common decay channels. The latter is characterized by the off-diagonal matrix elements of w. Being the sums of uncorrellated contributions of many evaporation channels, $k \gg 1$, these elements, due to mutual cancellations, are small in comparison with the diagonal elements, $|w_{nn'}/w_{nn}| \sim 1/\sqrt{k}$ (see [24]). The corresponding corrections are of order of γ_{ev}^2/kD^2 where γ_{ev} is the typical evaporation width. We will neglect them below assuming $\gamma_{ev} \ll \sqrt{k}D$. Under this condition, partial decay widths of the compound states to specific evaporation channels are small, $\gamma_{ev}/k \ll D$.

The complex energies of compound resonances in this approximation are equal to $\tilde{\epsilon}_\nu = h_\nu - (i/2)\gamma_{ev}, \nu = 1, 2, ..., N$, supposing on the statistical grounds that the fluctuations of widths of compound states are weak since the number k of evaporation channels is large. The simple state has its own complex energy $\tilde{\epsilon}_0 = \epsilon_0 - (i/2)\gamma_0$ where γ_0 is the direct decay width.

Let us now switch on an interaction between the simple and compound states through the hermitian coupling operator $\mathbf{V}$. The mixing proceeds in competition with the decays of intrinsic states, both via direct and evaporation channels. Whence, we need to generalize the standard procedure of determination of the strength function, Sect. 3, for the decaying system. In our schematic although quite generic model, it could be done exactly.

B. Decay widths in the presence of intrinsic damping

The diagonalization of the total non-hermitian hamiltonian (1) leads to $N + 1$ complex eigenvalues (3) which are the roots $z = \mathcal{E}_j$ of the secular equation (compare to (5))

$$\mathcal{F}(z) \equiv z - \tilde{\epsilon}_0 - \mathbf{V}^T g(z)\mathbf{V} = 0, \tag{12}$$

or, in the explicit form,

$$\mathcal{E}_j - \tilde{\epsilon}_0 - \sum_\nu \frac{V_\nu^2}{\mathcal{E}_j - \tilde{\epsilon}_\nu} = 0. \tag{13}$$

The interaction amplitudes V_ν, which couple the unstable simple state $|0\rangle$ with complicated (and decaying as well) intrinsic states $|\nu\rangle$, are still real in the approximation taken above (we neglected the off-diagonal part of the continuum coupling w).

Similar to (7), the quasistationary eigenstates $|j\rangle$ can be represented as superpositions of decoupled unstable states $|0\rangle, \ldots, |\nu\rangle$

$$|j\rangle = \tilde{C}_0^j |0\rangle + \sum_\nu \tilde{C}_\nu^j |\nu\rangle. \tag{14}$$

The fraction $\tilde{f}^j = |\tilde{C}_0^j|^2$ of the strength of the simple state $|0\rangle$ carried by the quasistationary state $|j\rangle$ is equal, as in eq.(6), to

$$\tilde{f}^j = \frac{1}{1 + L^j}, \quad L^j = \mathbf{V}^T g^\dagger(\mathcal{E}_j) g(\mathcal{E}_j) \mathbf{V}. \tag{15}$$

With $\mathcal{E}_j = E_j - (i/2)\Gamma_j$, the loops L^j can be written as

$$L^j = \sum_\nu \frac{V_\nu^2}{|\mathcal{E}_j - \tilde{\epsilon}_\nu|^2} = \frac{2}{\Gamma_j - \gamma_{ev}} \operatorname{Im} \sum_\nu V_\nu^2 \frac{1}{\mathcal{E}_j - \tilde{\epsilon}_\nu}. \tag{16}$$

Using the secular equation (13) we arrive at a very simple expression

$$L^j = \frac{\gamma_0 - \Gamma_j}{\Gamma_j - \gamma_{ev}}, \tag{17}$$

leading to the individual strengths (15)

$$\tilde{f}^j = \frac{\Gamma_j - \gamma_{ev}}{\gamma_0 - \gamma_{ev}}. \tag{18}$$

In other words, the resulting width of the quasistationary state $|j\rangle$ can be found from simple probabilistic arguments,

$$\Gamma_j = \gamma_0 \tilde{f}^j + \gamma_{ev}(1 - \tilde{f}^j). \tag{19}$$

The direct decay width is distributed over all quasistationary states according to their fractions of the strength of the original simple state. It is easy to check the normalization of the weights (18):

$$\sum_j \tilde{f}^j = \frac{1}{\gamma_0 - \gamma_{ev}} \Big(\sum_j \Gamma_j - (N+1)\gamma_{ev} \Big) = 1 \tag{20}$$

where the last step follows from the invariance of the imaginary part of the trace of the hamiltonian (1), $\sum_j \Gamma_j = \gamma_0 + N\gamma_{ev}$. We have to notice that the probabilistic interpretation emerges here as a result of a strict quantum-mechanical calculation, with no ensemble averaging or transition to a kinetic description.

C. "Broad pole"

Explicit expressions for the properties of the strength function, including the spreading width along with the decay widths into continuum can be obtained if the average characteristics of the intrinsic spectrum and of the coupling matrix elements are specified.

In the uniform model [17] used earlier for the stable states, eq.(13) gives a pair of coupled equations for the real and imaginary parts of the complex energy (3), see Appendix,

$$E_j = \epsilon_0 + \frac{1}{2}\Gamma_s \frac{x_j(1-y_j^2)}{1+x_j^2 y_j^2}, \quad \Gamma_j = \gamma_0 - \Gamma_s \frac{(1+x_j^2)y_j}{1+x_j^2 y_j^2}. \tag{21}$$

Here Γ_s stands for the standard spreading width (10), and notations

$$x_j = \cot\left(\pi \frac{E_j}{D}\right), \quad y_j = \tanh\left(\frac{\pi}{2}\frac{\Gamma_j - \gamma_{ev}}{D}\right) \tag{22}$$

are introduced.

At moderate values of the interaction V, the simple state keeps an appreciable fraction of the collective strength and preserves its individuality, see eq. (24) below. Such a state was called the broad pole in [10]. The problem of IAS can serve as a typical example. The unperturbed analog state $|0\rangle$ arises at the energy ϵ_0 carrying almost pure isospin $T_>$. Its direct decay width γ_0 is much larger than the evaporation width γ_{ev} of background states with isospin $T_<$ [19]. The isospin-violating interaction V mixes these states.

Assuming that the resulting width $\Gamma_0 \equiv \Gamma_{j=0}$ satisfies the condition $(\Gamma_0 - \gamma_{ev}) \gg D$ we have from (22) $y_0 \approx 1$, so that eqs. (21) give for the complex root corresponding to the broad pole

$$E_0 \approx \epsilon_0, \quad \Gamma_0 = \gamma_0 - \Gamma_s. \tag{23}$$

The second expression reads $\Gamma = \Gamma^{\uparrow} - \Gamma^{\downarrow}$ in the notations chosen in [10]. The state $T_>$ can be observed only if it decays before mixing, $\gamma_0 > \Gamma_s$. The collective strength (18) carried by the broad pole is then

$$\tilde{f}^0 = \frac{\gamma_0 - \gamma_{ev} - \Gamma_s}{\gamma_0 - \gamma_{ev}} = 1 - \frac{\Gamma_s}{\gamma_0 - \gamma_{ev}} \tag{24}$$

which remains of order of unity as long as Γ_0 noticeably exceeds γ_{ev}. This formula extends to the case of unstable compound states the measure introduced in [10,31] of the "purity of analog spin" of the broad pole. On the other hand, the typical values $\tilde{f}^j$ for $j \neq 0$ are small.

D. General strength function

The energy dependence of the strengths (18) is hidden in the secular equations (21). Exclusion of x_j leads after simple algebra to the general equation for the strength function which depends only on the absolute value $|\gamma_0 - \gamma_{ev}|$,

$$\tilde{f}^j = \frac{D}{2\pi|\gamma_0 - \gamma_{ev}|} \ln \frac{(E_j - \epsilon_0)^2 + \frac{1}{4}\left[\Gamma_s + |\gamma_0 - \gamma_{ev}|(1 - \tilde{f}^j)\right]^2}{(E_j - \epsilon_0)^2 + \frac{1}{4}\left[\Gamma_s - |\gamma_0 - \gamma_{ev}|(1 - \tilde{f}^j)\right]^2}, \tag{25}$$

or, for small $\tilde{f}_j$,

$$\tilde{f}^j = \frac{D}{2\pi|\gamma_0 - \gamma_{ev}|} \ln \frac{(E_j - \epsilon_0)^2 + \frac{1}{4}(\Gamma_s + |\gamma_0 - \gamma_{ev}|)^2}{(E_j - \epsilon_0)^2 + \frac{1}{4}(\Gamma_s - |\gamma_0 - \gamma_{ev}|)^2}. \tag{26}$$

Substituting summation over j by integration over energy, one can easily check that this distribution is normalized as

$$\int \frac{dE_j}{D} \tilde{f}(E_j) = \begin{cases} \dfrac{\Gamma_s}{|\gamma_0 - \gamma_{ev}|}, & \Gamma_s < |\gamma_0 - \gamma_{ev}|; \\ \\ 1, & \Gamma_s > |\gamma_0 - \gamma_{ev}|. \end{cases} \tag{27}$$

In the upper case, the contribution $\tilde{f}^0 = 1 - \Gamma_s/|\gamma_0 - \gamma_{ev}|$ (compare with eq. (24)) of the simple state (23) is lost in the integral. Indeed, the small factor in front of the logarithm in eq. (25) is in this case compensated due to the small value of the denominator of the expression under the logarithm so that eq. (26) is not valid for this special state. On the other hand, when the increasing mixing rate characterized by the spreading width prevails upon the influence of direct decays, the simple mode fully dissolves in the sea of compound states.

Except for an exponentially narrow domain of parameters around the point $\Gamma_s = |\gamma_0 - \gamma_{ev}|$, the width (FWHM) Γ of the distribution (26) is determined by

$$\Gamma^2 = |\Gamma_s^2 - (\gamma_0 - \gamma_{ev})^2|. \tag{28}$$

The tails of the strength function, $E \gg (\Gamma_s + |\gamma_0 - \gamma_{ev}|)$, are universal and given by the standard model, $\tilde{f}^j \approx (D/2\pi)\Gamma_s/E^2$. In the limits $\Gamma_s \gg |\gamma_0 - \gamma_{ev}|$ or $\Gamma_s \ll |\gamma_0 - \gamma_{ev}|$, eq.(26) reduces to the Breit–Wigner distribution

$$\tilde{f}^j = \frac{D}{2\pi} \begin{cases} \dfrac{\Gamma_s}{(E_j - \epsilon_0)^2 + \Gamma_s^2/4} \leq \dfrac{2}{\pi}\dfrac{D}{\Gamma_s}, \\ \dfrac{\Gamma_s}{|\gamma_0 - \gamma_{ev}|}\dfrac{|\gamma_0 - \gamma_{ev}|}{(E_j - \epsilon_0)^2 + |\gamma_0 - \gamma_{ev}|^2/4} \leq \dfrac{2}{\pi}\dfrac{D\Gamma_s}{(\gamma_0 - \gamma_{ev})^2}, \end{cases} \tag{29}$$

respectively. Near the point $\Gamma_s = |\gamma_0 - \gamma_{ev}|$, (26) is invalid and eq.(25) gives

$$\tilde{f}^0 = \frac{D}{\pi\Gamma_s}\left(\ln\frac{2\pi\Gamma_s}{D} - \ln\ln\frac{2\pi\Gamma_s}{D} + ...\right). \tag{30}$$

The strength $\tilde{f}^0$ is still larger than all $\tilde{f}^j$ for $j \neq 0$ but this cannot influence the normalization (27). Fig. 1 illustrates the relation between the exact expression for the strength function, eq.(25), the approximation (26), which is invalid in the center of the spectrum, and more crude approximations (29).

The strength function gives an average description of the fragmentation of individual simple configurations in the intrinsic space. In the next section we study the problem as it is seen in continuum properties.

V. SCATTERING CHARACTERISTICS

A. Scattering matrix

Up to now we concentrated on the "inside" view of a simple unstable mode mixed with complicated fine structure states. The "outside" world was present as a reservoir for irreversible decay through numerous open channels. Now we take a glimpse of the same system from the viewpoint of reaction amplitudes and cross sections where only asymptotic states are observed.

The scattering matrix $\hat{S} = \{S^{cc'}\}$ at energy E can be written as [19]

$$\hat{S}(E) = \hat{s}^{1/2}\{1 - i\hat{T}(E)\}\hat{s}^{1/2}, \tag{31}$$

$$\hat{T}(E) = \mathbf{A}^T \mathcal{G}(E)\mathbf{A}. \tag{32}$$

Here $\hat{s}$ includes the potential scattering as well as channel coupling and direct reactions in the continuum. Those effects being unrelated to intrinsic dynamics are irrelevant for our purpose, and $\hat{s}(E)$ can be considered as a diagonal matrix with phase shift elements $\exp(2i\delta^c)$ smoothly depending on E. The Green function in eq.(32),

$$\mathcal{G}(z) = \frac{1}{z - \mathcal{H}}, \tag{33}$$

describes the propagation governed by the total hamiltonian (1). It differs from the intrinsic Green function by the antihermitian part of the effective hamiltonian. Both $\mathcal{G}(z)$ and the scattering matrix (31) have poles at the complex energies (3).

It is a straightforward exercise to establish, with the aid of the factorized structure (2) of the antihermitian part W, the relation

$$\mathcal{G}(E) = G(E) - \frac{i}{2}G(E)\mathbf{A}\frac{1}{1 + (i/2)\hat{R}(E)}\mathbf{A}^T G(E) \tag{34}$$

between the two Green functions (33) and (4). The $\hat{R}$-matrix in eq.(34) is familiar from nuclear reaction theory [32],

$$\hat{R}(E) = \mathbf{A}^T G(E)\mathbf{A}. \tag{35}$$

It describes the propagation inside the closed system between two acts of coupling to the continuum; the poles of $\hat{R}(E)$ correspond to the energies ϵ_α of intrinsic states with the mixing V fully accounted for. The reaction matrix $\hat{T}(E)$ of eq.(32) is similar to (35) but includes all intermediate couplings to the continuum. Finally, for the scattering matrix (31, 32) the substitution (34) gives

$$\hat{T}(E) = \frac{\hat{R}(E)}{1 + (i/2)\hat{R}(E)}, \quad \hat{S}(E) = \hat{s}^{1/2}\frac{1 - (i/2)\hat{R}(E)}{1 + (i/2)\hat{R}(E)}\hat{s}^{1/2}. \tag{36}$$

B. Scattering wave function, delay time and unitarity

The scattering wave function $|\Psi_E^c\rangle$ with the incident wave in the channel c at energy E can be presented by the superposition of intrinsic, $|n\rangle$, and continuum channel, $|c; E\rangle$, components,

$$|\Psi_E^c\rangle = \sum_n b_n^c(E)|n\rangle + \sum_{c'} \int_{E^{c'}}^{\infty} dE' \chi^{cc'}(E, E')|c'; E'\rangle \tag{37}$$

where $E^{c'}$ is the threshold energy in the channel c'. Recall that the decay amplitudes A_n^c are the matrix elements of the total original hamiltonian between the states $|n\rangle$ and $|c; E\rangle$. By a direct substitution of (37) into the Schrødinger equation, we find [19] the $N \times k$ matrix $\mathbf{b}(E)$ of the intrinsic components b_n^c as

$$\mathbf{b}(E) = \mathcal{G}(E)\mathbf{A}\hat{s}^{1/2}. \tag{38}$$

The diagonal elements of the $k \times k$ matrix $\mathbf{b}^\dagger(E)\mathbf{b}(E)$ determine the norm of the internal part of the wave function initiated in the channel c at energy E. Therefore this matrix should characterize the fraction of delay time in this reaction due to intrinsic resonances. Indeed, the Smith's time delay matrix is defined [33] as

$$\hat{\tau}(E) = -i\hat{S}^\dagger(E)\frac{d\hat{S}(E)}{dE}. \tag{39}$$

Taking into account only the resonance energy dependence via the $\hat{R}$-matrix in eq.(36), we find

$$\hat{\tau}_{res}(E) = -\hat{s}^{-1/2}\frac{1}{1-(i/2)\hat{R}(E)}\frac{d\hat{R}(E)}{dE}\frac{1}{1+(i/2)\hat{R}(E)}\hat{s}^{1/2}. \tag{40}$$

In the same resonance approximation one can neglect the energy dependence of amplitudes $\mathbf{A}$ to get from (35)

$$(d\hat{R}/dE)_{res} = -\mathbf{A}^T G^2(E)\mathbf{A}. \tag{41}$$

Using the relation (34) between the Green functions G and $\mathcal{G}$, we obtain

$$\hat{\tau}_{res} = \hat{s}^{-1/2}\mathbf{A}^T\mathcal{G}^\dagger(E)\mathcal{G}(E)\mathbf{A}\hat{s}^{1/2}. \tag{42}$$

Thus, the time delay matrix (40) coincides with the intrinsic norm matrix found from (38),

$$\mathbf{b}^\dagger\mathbf{b} = \hat{\tau}_{res}. \tag{43}$$

The total Green function (33) describes the propagation in the open system and, therefore, the delay time as well.

We can now define the normalized probability $p_n^c(E)$ to find the system in the intrinsic state $|n\rangle$ in the "elastic" reaction $c \to c$,

$$p_n^c(E) = \frac{1}{\tau_{res}^{cc}(E)}|b_n^c(E)|^2, \quad \sum_n p_n^c(E) = 1. \tag{44}$$

The probability $p_0^c(E)$ characterizes the weight of the simple state $|0\rangle$ in the channel c. In the problem of the IAS this quantity measures the isospin purity in a given channel.

The full scattering matrix (31,32) is unitary provided the potential scattering matrix $\hat{s}$ is unitary. It follows from the fact that the decay amplitudes $\mathbf{A}$ in the entrance and exit channels of eq.(32) are the same which appear in all intermediate processes described by the total propagator $\mathcal{G}(E)$ with the aid of the effective hamiltonian (1,2).

The unitarity condition $\hat{S}\hat{S}^\dagger = \hat{S}^\dagger\hat{S} = 1$ gives for the reaction matrix (32)

$$\hat{T}^\dagger\hat{T} = i(\hat{T} - \hat{T}^\dagger) \tag{45}$$

which can be transformed, with the help of (2) and (38), into

$$\hat{s}^{1/2}\mathbf{b}^{\dagger}(E)W\mathbf{b}(E)\hat{s}^{-1/2} = i\{\hat{T}(E) - \hat{T}^{\dagger}(E)\}. \tag{46}$$

VI. A SIMPLE CASE: STABLE BACKGROUND STATES

The simplest situation corresponds to the stable background states with no direct access to open channels, $\gamma_{ev} \to 0$, when the intrinsic evolution for the reaction in the channel c starts and ends at the simple state. The background states are involved by the internal coupling only at the intermediate stages of the reaction. Calculating the diagonal element of the resonance time delay matrix (42) we obtain for the probability (44),

$$p_0^c(E) \equiv f(E) = \Big[1 + \sum_n \frac{V_n^2}{(E-h_n)^2}\Big]^{-1} = [dF/dE]^{-1}. \tag{47}$$

This is nothing but the continuous generalization of the strengths $f^\alpha = |C_0^\alpha|^2$ defined above by eq.(6) in discrete points ϵ_α of the intrinsic energy spectrum, $f^\alpha = f(E = \epsilon_\alpha)$. Since the intrinsic states are coupled to continuum through the state $|0\rangle$ and the probabilities p_n^c are normalized, eq.(44), the decay (or population) partial widths γ_0^c do not appear in (47). If several direct decay channels c are open, the energy behavior (47) is identical for all of them being determined by intrinsic dynamics only.

The probability (47) vanishes at energies $E = h_n$ of the unperturbed background states which are located intermittently with the actual energies ϵ_α. In the vicinity of h_n the complicated states dominate the intrinsic part of the scattering wave function.

Another, though equivalent to (47), representation of the time delay in terms of the complex energies (3) of quasistationary states can be derived from (42),

$$\tau_{res}^{cc}(E) = -2\frac{\gamma_0^c}{\gamma_0}\mathrm{Im}[\mathrm{Tr}\mathcal{G}(E)] = \frac{\gamma_0^c}{\gamma_0}\sum_j \frac{\Gamma_j}{(E-E_j)^2 + \Gamma_j^2/4}. \tag{48}$$

The delay times for different channels c are proportional to the corresponding partial widths of the state $|0\rangle$ and have the identical energy dependence determined by the complex energy spectrum of intrinsic unstable states.

The representation (48) is useful when the vicinity of the broad pole (23) is considered. It follows from (48) that the contribution of this pole is a smooth function of energy superimposed onto the picket fence of the δ-like peaks with the average value proportional to the Weisskopf recurrence time π/D for a long-lived wave packet. At the energy $E = E_0$, the time delay in a channel c due to excitation of the broad pole is equal to $4\gamma_0^c/\gamma_0\Gamma_0$. On the other hand, one gets

$$|b_0^c(E_0)|^2 = \gamma_0^c|\mathcal{G}_{00}(E_0)|^2 \approx 4\gamma_0^c/(\gamma_0)^2 \tag{49}$$

since the energy E_0 is very close to the unperturbed energy of the state $|0\rangle$. Therefore the probability maximum is determined by the fraction of the total width γ_0 of the original mode which still resides at the broad pole,

$$p_0(E_0) = \frac{\Gamma_0}{\gamma_0} = \frac{\gamma_0 - \Gamma_s}{\gamma_0}. \tag{50}$$

in agreement with (24) taken at $\gamma_{ev} = 0$.

One should have in mind that the distribution (47) wildly fluctuates on the fine structure energy scale. With the energy resolution worse than the level spacing D, one sees only a smooth behavior coinciding with that of the strength function $P_0(E)$, eq.(8). It is quite natural because here the intrinsic mixing is the only source for the spreading of the strength, or for isospin impurity in the case of IAS. An average magnitude of the probability to find the original isospin can be easily estimated in the standard model with the uniform background. Eq.(47) gives here ($\Gamma^{\downarrow} = \Gamma_s$)

$$p_0(E) = \frac{\sin^2(E\pi/D)}{\sin^2(E\pi/D) + (\pi\Gamma^{\downarrow}/2D)}, \tag{51}$$

or, after averaging over fine structure, and taking $\Gamma^{\downarrow} \gg D$,

$$\overline{p_0(E)} = \frac{1}{\pi}\frac{D}{\Gamma^{\downarrow}}. \tag{52}$$

This natural estimate (inverse number of fine structure states within the spreading width) coincides with that used by von Brentano [10].

VII. MIXING WITH OPEN COMPOUND STATES

A. Purity of a simple state

The situation changes in the realistic case with many open evaporation channels. Strong fluctuations of the probability $p_0(E)$ are smeared out since the compound poles are displaced to the complex energy plane even with no coupling to the simple mode. This probability remains considerable in a finite vicinity of the point ϵ_0 ensuring a noticeable isospin purity of the internal part of the scattering wave function in this region.

If the simple mode and the compund states have no common decay channels, the nonzero decay amplitudes are $A_0^c = \sqrt{\gamma_0}$ (consider for simplicity a single direct decay channel) and A_n^c. The reaction amplitudes are equal to

$$T^{cc}(E) = \gamma_0 \mathcal{G}_{00}(E), \quad T^{ce}(E) = \sqrt{\gamma_0} \sum_\nu \mathcal{G}_{0\nu}(E) A_\nu^e \tag{53}$$

where now $\mathcal{G}_{00}^{-1}(E) = \mathcal{F}(E)$ (see (12)) whereas

$$\mathcal{G}_{0\nu}(E) = \frac{V_\nu}{E - \tilde{\epsilon}_\nu} \mathcal{G}_{00}(E). \tag{54}$$

The delay time in the elastic process, according to (42) and (38), is given by

$$\begin{aligned} \tau_{res}^{cc}(E) &= \gamma_0 |\mathcal{G}_{00}(E)|^2 \left(1 + \sum_\nu \frac{V_\nu^2}{|E - \tilde{\epsilon}_\nu|^2}\right) \\ &\equiv \gamma_0 |\mathcal{G}_{00}(E)|^2 \left[1 + L(E)\right] \end{aligned} \tag{55}$$

where the loop $L(E)$ is the analog of L^j, eq.(16), taken at the running real energy E rather than at the complex energy $\mathcal{E}_j$. Therefore we find instead of (47)

$$p_0(E) \equiv \tilde{f}(E) = \frac{1}{1 + L(E)} = \left[1 + \sum_\nu \frac{V_\nu^2}{|E - \tilde{\epsilon}_\nu|^2}\right]^{-1}. \tag{56}$$

The function $\tilde{f}(E)$ extends the strength function (15) of the quasistationary states to a running real energy E (compare with the similar correspondence between the functions (6) and (47) in the case of stable compound states). Note that, by definition (44), the

resonance envelope $|\mathcal{G}_{00}|^2$ is divided out of normalized probabilities $p_0(E)$ which behave uniformly within the spreading width.

The loop function (56) can be calculated similar to (17). Under the same assumptions, it is equal to

$$L(E) = \frac{\Gamma_s}{\gamma_{ev}} y \frac{(1+x^2)}{1+x^2y^2}. \tag{57}$$

where, instead of (22), we now have

$$x = \cot\Big(\pi\frac{E}{D}\Big), \quad y = \tanh\Big(\frac{\pi}{2}\frac{\gamma_{ev}}{D}\Big). \tag{58}$$

For a small evaporation width, $\gamma_{ev} \ll D$, the expression (57) reduces to

$$L(E) = \frac{\pi\Gamma_s}{2D}(1+x^2). \tag{59}$$

The results in this limit do not depend on the evaporation width at all and therefore coincide with those following from (47). In particular, the weight of the simple state in the intrinsic part of the scattering wave function is in average of order of $D/\Gamma_s \ll 1$.

As level density and number of open channels increase, the ratio γ_{ev}/D rapidly grows together with the argument of y, eq.(58). One has a fast transition to the limit of the overlapping background states when $y \approx 1$ and $L(E) \to \Gamma_s/\gamma_{ev}$. The probability (56) in this case is noticeably greater than in (52),

$$\tilde{f} = \frac{\gamma_{ev}}{\gamma_{ev}+\Gamma_s} \gg \frac{D}{\Gamma_s}. \tag{60}$$

The fluctuations disappear, and the simple state preserves its individuality in the intrinsic wave function across the whole region of the giant or analog resonance. This behavior is demonstrated in Fig. 2.

The purity of the intrinsic part becomes perfect when $\gamma_{ev} \gg \Gamma_s$; the depletion of admixed states of the opposite isospin occurs faster than their population. This gives a microscopic justification of the isospin purity at high excitation energy predicted in [21,22] and recently observed experimentally [2]. At the same conditions, the fraction of the simple mode carried by a generic compound state,

$$1 - \tilde{f} = \frac{\Gamma_s}{\gamma_{ev} + \Gamma_s}, \tag{61}$$

is small. This means that the compound processes have no time to explore the presence of the exceptional simple state.

The equivalent result was formulated in terms of the kinetic balance between the processes of decay and mixing in [20] where the mechanism for the disappearance of the collective strength of the GDR at high energies was suggested. The authors showed that the probability of excitation of a collective mode in an initially heated nucleus is equal, using our notations, to $\Gamma_s/(\gamma_{ev} + \Gamma_s)$ and therefore diminishes as Γ_s/γ_{ev}, when the temperature exceeds a critical value determined by the condition $\gamma_{ev} \sim \Gamma_s$. Complementary to the somewhat qualitative kinetic arguments of [20], here the analogous conclusion follows from a full quantum-mechanical consideration.

We need to mention parenthetically that such statements assume the saturation of the spreading width $\Gamma^{\downarrow} \approx \Gamma_s$ as a function of temperature. The absence of a considerable dependence on excitation energy is well known for the IAS [11–13,34]. The saturation of the intrinsic spreading width presumably takes place for the GDR as well [1,3–5]. General theoretical arguments in favor of such a saturation [11,28,29,34,30] are based on the chaotization of the intrinsic dynamics and they will not be repeated here.

B. Excitation and decay of a simple mode

Here we compare the cross sections of various processes initiated in the channel c. They start with the excitation of the simple mode. The "elastic" scattering, $c \to c$, competes with the evaporation $c \to e$ through numerous compound channels e. These branches are described by the amplitudes T^{cc} and T^{ec}, respectively, see eq.(53).

On the real energy axis, $z = E$, the uniform model leads to the inverse Green function $\mathcal{G}_{00}^{-1}(E) = \mathcal{F}(E)$, compare eq.(5),

$$\mathcal{F}(E) = E - \epsilon_0 - \Gamma_s \frac{x(1-y^2)}{2(1+x^2y^2)} + \frac{i}{2}\Big[\gamma_0 + \Gamma_s y \frac{(1+x^2)}{1+x^2y^2}\Big]. \tag{62}$$

At $y = 0$ (no evaporation), the elastic cross section

$$|T^{cc}|^2 = \gamma_0^2/|\mathcal{F}(E)|^2 = \gamma_0^2 \left[\left(E - \epsilon_0 - \frac{\Gamma_s}{2} \cot \frac{\pi E}{D} \right)^2 + \gamma_0^2/4 \right]^{-1} \tag{63}$$

reveals fine structure fluctuations. In the case of small $y \neq 0$, these fluctuations are enhanced in a vicinity of the point $E = \epsilon_0$ due to the energy dependence of the imaginary part of $\mathcal{F}(E)$, eq.(62). However, the fluctuations are washed away when evaporation becomes strong, $\gamma_{ev} \gg D$, so that $y \to 1$ and (62) simplifies to

$$\mathcal{F}(E) = E - \epsilon_0 + \frac{i}{2}(\gamma_0 + \Gamma_s). \tag{64}$$

Note that here the decay width γ_0 and the spreading width Γ_s are combined into the total width of the resonance on the real energy axis. In Fig. 3 we illustrate the energy dependence of the elastic cross section $\sigma^{cc} = |T^{cc}|^2$ for different values of relevant parameters.

Using the optical theorem, one obtains from eqs. (53,62) and (57)

$$-2\mathrm{Im}\, T^{cc}(E) = |T^{cc}(E)|^2 \left[1 + \frac{\gamma_{ev}}{\gamma_0} L(E) \right] \tag{65}$$

for the total cross section initiated in the channel c. The fraction of $|T^{cc}|^2$ in the total cross section determines the branching ratio of the simple decay mode,

$$\mathcal{B}^{cc}(E) = \frac{\gamma_0}{\gamma_0 + \gamma_{ev} L(E)} = \frac{\gamma_0 \tilde{f}(E)}{\gamma_0 \tilde{f}(E) + \gamma_{ev} \left[1 - \tilde{f}(E) \right]}, \tag{66}$$

in agreement with the probabilistic interpretation of the function $\tilde{f}(E)$. This function rather than its discrete counterpart (15) is relevant when an actual reaction process is considered.

The amplitude $T^{ec}(E)$, eq.(53), for evaporation in a given channel e after the simple state is excited in the entrance channel c, strongly fluctuates together with the exit amplitudes A_ν^e. This amplitude vanishes in average. Assuming many uncorrelated statistically equivalent decay channels, we can use a natural statistical suggestion [11,24]

$$\langle A_\mu^e A_\nu^{e'} \rangle = \delta^{ee'} \delta_{\mu\nu} \gamma_{ev}/k. \tag{67}$$

Taking into account eqs. (53),(54), we obtain

$$\langle|T^{ec}(E)|^2\rangle = \gamma_0|\mathcal{G}_{00}(E)|^2\sum_{\mu\nu}\frac{V_\mu V_\nu\langle A^e_\mu A^e_\nu\rangle}{(E-\tilde{\epsilon}^*_\mu)(E-\tilde{\epsilon}_\nu)} = |T^{cc}|^2\frac{\gamma_{ev}}{k\gamma_0}L(E), \tag{68}$$

so that the corresponding branching ratio is equal to

$$\mathcal{B}^{ec} = \frac{1}{k}\frac{\gamma_{ev}L(E)}{\gamma_0+\gamma_{ev}L(E)} = \frac{1}{k}\frac{\gamma_{ev}\left[1-\tilde{f}(E)\right]}{\gamma_0\tilde{f}(E)+\gamma_{ev}\left[1-\tilde{f}(E)\right]}. \tag{69}$$

Eqs. (66) and (69) give $\mathcal{B}^{cc}+k\mathcal{B}^{ec}=1$ in accordance with the unitarity condition. The statistical ansatz (67) is self-consistent because an equivalent approximation was in fact introduced earlier when the off-diagonal elements of the antihermitian operator w in the compound space were substituted by the average evaporation width, see the discussion after eq.(11).

In the case of considerable evaporation and overlapping compound resonances, $\gamma_{ev}/D \gg 1$, the branching ratios saturate at, see (60),

$$\mathcal{B}^{cc} = \frac{\gamma_0}{\gamma_0+\Gamma_s}, \qquad \mathcal{B}^{ec} = \frac{1}{k}\frac{\Gamma_s}{\gamma_0+\Gamma_s}. \tag{70}$$

For the saturated spreading width Γ_s, these limiting values cease to be sensitive to the level density of compound states and depend on excitation energy or temperature only through the direct width γ_0. Under such conditions, only the simple state with the total width $\gamma_0+\Gamma_s$, corresponding to the two possible ways of its decay, escape and internal dissipation, is seen in the scattering in the entrance channel c. Here again the background of compound states serves as a reservoir for irreversible decay, equivalent by its properties to decay into continuum.

C. Reactions initiated in compound channels

The processes started in the compound channels e, for example, driven by a nuclear interaction of heavy ions, can populate the simple mode through internal mixing. The corresponding amplitude T^{ce} is the same as the amplitude T^{ec} considered above, eqs. (53) and (68). The competing compound-compound processes are described by the set of the amplitudes

$$T^{e'e}(E) = \sum_\nu \frac{A^{e'}_\nu A^e_\nu}{E - \tilde{\epsilon}_\nu} + \sum_\nu \frac{A^{e'}_\nu V_\nu}{E - \tilde{\epsilon}_\nu} \mathcal{G}_{00} \sum_\mu \frac{V_\mu A^e_\mu}{E - \tilde{\epsilon}_\mu}. \tag{71}$$

The second term in (71) accounts for the virtual excitation of the simple mode with the subsequent deexcitation again via compound channels.

To evaluate the total cross section of compound-compound reactions,

$$\sigma^e \equiv \sum_{e'} \langle |T^{e'e}|^2 \rangle, \tag{72}$$

we perform here the statistical averaging as in (67). Neglecting the numerical corrections of the order $1/k$, and using the notations of Appendix for the sums over the spectrum of the compound states, we obtain

$$\sigma^e = \frac{\gamma^2_{ev}}{k^2} \{ |\mathcal{S}|^2 + k\mathcal{S}_{11} + 2\mathrm{Re}\left[\mathcal{G}^*_{00} \langle V^2 \rangle (\mathcal{S}\mathcal{S}_{02} + k\mathcal{S}_{12}) \right]$$

$$+ |\mathcal{G}_{00}|^2 \langle V^2 \rangle^2 (|\mathcal{S}_{20}|^2 + k\mathcal{S}^2_{11}) \}. \tag{73}$$

The terms proportional to the number k of open channels appear as a result of pairwise coherent averaging of random decay amplitudes. Only these terms survive in the limit $k \gg 1$. Taking the sums of Appendix in the overlapping limit $y \to 1$ and recalling that in the same limit, according to (68),

$$\sigma^{ce} \equiv |T^{ce}|^2 = \frac{1}{k} |\mathcal{G}_{00}|^2 \gamma_0 \Gamma_s, \tag{74}$$

we come, after many cancellations, to a simple expression for the total cross section of all reactions initiated in a generic compound channel e,

$$\sigma^{ce} + \sigma^e \equiv |T^{ce}|^2 + \sum_{e'} \langle |T^{e'e}|^2 \rangle \tag{75}$$

$$= \frac{2\pi}{k} \left[(E - \epsilon_0)^2 + \frac{1}{4} (\gamma_0 + \Gamma_s)^2 \right] |\mathcal{G}_{00}|^2 \frac{\gamma_{ev}}{D}.$$

This gives the branching ratio for the deexcitation into the channel c carrying the signature of the simple mode,

$$\mathcal{B}^{ce} = \frac{1}{2\pi} \frac{\gamma_0 \Gamma_s}{(E - \epsilon_0)^2 + (\gamma_0 + \Gamma_s)/4} \frac{D}{\gamma_{ev}}. \tag{76}$$

The resonance at the simple state is suppressed by the inverse number γ_{ev}/D of the background states on the typical evaporation width. As it was discussed above, the observation of the signal of the simple mode in the reaction started in a compound channel becomes less probable with increasing γ_{ev}, in agreement with the kinetic arguments of [20].

The same result can be expressed with the aid of the function $\tilde{f}$, eq.(56),

$$\frac{\tilde{f}\sigma^{ce}}{(1-\tilde{f})\sigma^{e}} = \frac{1}{2\pi}\gamma_0\Gamma_s|\mathcal{G}_{00}|^2\frac{D}{\Gamma_s}. \tag{77}$$

Here $\tilde{f}\sigma^{ce}$ determines the fraction of the cross section of the process $e \to c$ due to the intrinsic simple state; the denominator is the similar fraction of the compound-compound cross section due to the complicated intrinsic states, with no excitation of the simple mode. The right hand side of (77) is the resonance curve of the simple mode excited through the background (entrance factor Γ_s) and deexcited through its own exit channel (factor γ_0). The integral of the left hand side ratio over the energy region covered by the spreading width gives the inverse number of fine structure states in this region, D/Γ_s.

D. Common decay channels

One of the objections raised against the kinetic explanation [20] of the disappearance of the collective strength of the giant resonance is related to the possibility of preequilibrium excitation of the giant mode [35]. In this case the intrinsic evolution would start with the state which already carries some amount of collective strength. In our language such a possibility can be taken into account via the presence of the reaction channels a connected both to the simple mode and to the background states. For such channels, all amplitudes, A_0^a and A_ν^a do not vanish; until now we assumed that, before the internal mixing, the simple state and the fine structure states have no common decay channels. For the case of the IAS, this situation is associated with the extrnal isospin mixing which is apparently of minor importance [11]. However, for the giant resonance this effect can change the situation.

The common decay channels can be incorporated into theory without problems. Here we consider the simplest case of a single common channel which can be easily analyzed by

the standard means. The corresponding real amplitudes will be denoted as a_0 and a_ν for the simple state and bacground states, respectively. A many-channel case brings in many amplitudes of such type. Being uncorrelated, they should not lead to any effects of coherent enhancement.

In the single channel case, the exact algebraic solution gives the matrix elements of the total Green function (33)

$$\mathcal{G}_{00}(E) = \left[E - \tilde{\epsilon}_0 - \sum_\nu \frac{V_\nu^2}{E - \tilde{\epsilon}_\nu} + \frac{i}{2}\frac{\alpha_0^2}{1 + (i/2)R^a}\right]^{-1}, \tag{78}$$

$$\mathcal{G}_{0\nu}(E) = \mathcal{G}_{\nu 0}(E) = \mathcal{G}_{00}\frac{u_\nu}{E - \tilde{\epsilon}_\nu}, \tag{79}$$

$$\mathcal{G}_{\mu\nu}(E) = \frac{\delta_{\mu\nu}}{E - \tilde{\epsilon}_\nu} + \frac{1}{E - \tilde{\epsilon}_\mu}\left(\frac{i}{2}\frac{a_\mu a_\nu}{1 + (i/2)R^a} + u_\mu \mathcal{G}_{00} u_\nu\right). \tag{80}$$

Here the renormalized amplitudes are introduced for the decay of the simple state through the channel a,

$$\alpha_0(E) = a_0 + \sum_\nu \frac{V_\nu a_\nu}{E - \tilde{\epsilon}_\nu}, \tag{81}$$

and for the mixing between the simple state and the background including the intermediate continuum states,

$$u_\nu(E) = V_\nu - \frac{i}{2}a_\nu \frac{\alpha_0}{1 + (i/2)R^a}. \tag{82}$$

The analog of the R-matrix, eq.(35), for the a channel is

$$R^a(E) = \sum_\nu \frac{a_\nu^2}{E - \tilde{\epsilon}_\nu}. \tag{83}$$

Using these exact expressions we evaluate the reaction amplitudes. We are interested in reactions starting in the channel a and ending either in the channel c specific for our signature of the simple mode or in any of the other channels, a or e. The elastic $a \to a$ amplitude is given by (compare (36)

$$T^{aa} = \frac{R^a}{1 + (i/2)R^a} + \left(\frac{\alpha_0}{1 + (i/2)R^a}\right)^2 \mathcal{G}_{00}. \tag{84}$$

The deexcitation through the special channel c is governed by the amplitude

$$T^{ca} = \sqrt{\gamma_0}(\mathcal{G}_{00}a_0 + \sum_\nu \mathcal{G}_{0\nu}a_\nu) = \sqrt{\gamma_0}\mathcal{G}_{00}\frac{\alpha_0}{1+(i/2)R^a}. \tag{85}$$

To make a conclusion of the importance of the excitation through the common channel, we assume that, similar to the amplitudes A^e_ν, eq.(67), the new amplitudes a_ν are uncorrelated quantities with a large magnitude which contributes significantly to the total width γ_{ev} of the compound states, $\langle a^2\rangle \equiv \gamma_a \sim \gamma_{ev} \gg D$. Using the estimates of the Appendix for the sums over the fine structure states in the overlapping limit, we obtain $R^a \approx -i\pi(\gamma_a/D)$. Thus, R^a is a large imaginary quantity determined by the number of compound states in the interval γ_a. According to the same estimates, the first term in (84) dominates, and the contribution of terms containing the sum with the cross products $V_\nu a_\nu$ is relatively small. Finally,

$$\frac{\langle|T^{ca}|^2\rangle}{\langle|T^{aa}|^2\rangle} \approx \left(\frac{D}{\pi\gamma_a}\right)^2 \gamma_0\gamma_0^a|\mathcal{G}_{00}|^2. \tag{86}$$

The average partial width for the decay of the simple state into the channel a is equal to

$$\gamma_0^a \equiv \langle|\alpha_0|^2\rangle = a_0^2 + \frac{\gamma_a}{\gamma_{ev}}\Gamma_s. \tag{87}$$

This, quite general, result shows that, in the case of the common channel capable of populating both, simple and compound states, the statistical branching for the deexcitation via the simple mode drops with the increasing level density $\rho \sim 1/D$ of compound states. Whence, strong common channels with $\gamma_a \sim \gamma_{ev}$ cannot recover the disappearing simple mode. Weak channels, $\gamma_a \simeq \gamma_{ev}/k$, are useless because of their small total cross sections.

VIII. CONCLUSION

In the paper we considered the most general properties of an open quantum system where a simple mode of excitation interacts with the background of very complicated states. Both, simple and compound, states are coupled to the continuum and have finite lifetimes. Internal dynamics (mixing) and external dynamics (decays) are intertangled in a nontrivial

way. The intrinsic dynamics in the presence of the continuum are governed by the effective nonhermitian hamiltonian. The widths of intrinsic states modify the strength function of the simple mode. This view from "inside" has to be supplemented by that from "outside" for determining the observables measured in a real scattering experiment, such as cross sections and delay times. In a formal language, here we project the dynamics of quasistationary intrinsic states back to the real energy axis. The effective hamiltonian by its construction guarantees the correct properties of the scattering matrix including unitarity. Therefore it becomes possible to use the knowledge of internal dynamics in order to compare cross sections of competing processes.

The general although schematic character of the analysis allows one to draw the conclusions concerning the manifestations of the simple mode in various situations. A typical example is given by the IAS which can be seen as a broad pole [10] or to be dissolved in the sea of the fine structure levels of another isospin. The analysis, analogous to that in [11], confirms the old idea [21,22] of increasing isospin purity of the IAS at high excitation energy. The experimental data [2] agree with this conclusion. The isospin purity is restored because of the very fast depopulation of the admixed background states when their decay width γ_{ev} increases compared to the spreading width Γ_s of the simple state (IAS in this case). Here Γ_s is assumed to be a slow changing or saturating function of excitation energy [5] as predicted by the analysis based on the chaotic character of the intrinsic dynamics [29].

Such a consideration is not specific for the IAS and can be applied to other simple modes embedded into continuum. The giant dipole resonance is known to preserve its individuality up to high excitation energy or temperature [1]. In particular, this is clearly seen in the observation of the nearly harmonic double-phonon excitations [9]. The new phenomenon of disappearance of the collective strength of the GDR [1] is still a debatable subject. Such a behavior was qualitatively explained in [20] as a result of a shift of the kinetic equilibrium in favor of compound decays when the ratio γ_{ev}/Γ_s increases. Our general quantum-mechanical analysis confirms this result. Moreover, we made the arguments which demonstrate that the conclusion is still valid when the simple mode can be excited from the reaction channels

which are common for the simple mode and the background states.

To complete the analysis, it would be interesting to consider the situation when several simple states can share the collective strength and the decay width into the channel which signals the deexcitation of the simple mode. This is the case in the realistic calculation of the GDR. The collective peak accumulating a large part of the isovector dipole strength is shifted to high energies compared to the unperturbed shell model position. However, some strength is still concentrated at the unshifted energy. This "configuration splitting" leads to specific interference phenomena [16] which again can be described with the use of the effective nonhermitian hamiltonian. The distribution of the dipole strength and the width evolves with the increasing excitation energy which should be taken into account when the interplay of the internal interaction and external decays is considered. Typically, this results in the quenching of the collective strength and its redistribution in favor of the low energy component. These effects are seen experimentally [36] and discussed in [37].

The authors are indebted to P. von Brentano who initiated this work and made an important impact by numerous discussions at the initial stage. We thank D.V.Savin for constructive discussions and assistance. One of us (V.S.) is grateful to Y.Fyodorov, F.Izrailev, I.Rotter and H.-J.Sommers for interesting discussions. He also thanks the Köln University and the National Superconducting Cyclotron Laboratory for generous hospitality. This work was supported by the National Science Foundation, through grants 94-03666 and 95-12831, and by the INTAS grant 94-2058.

Appendix

A uniform model of compound spectra

Assuming the equidistant spectrum of unstable background states with the level spacing D and the decay width γ, which corresponds to γ_{ev} of the main text, and substituting actual coupling matrix elements V_ν^2 by their average $\langle V^2 \rangle$, we have to deal with the sums as the

trace of the Green function (11)

$$\mathcal{S} = \sum_{\nu=-\infty}^{\infty} \zeta_\nu(E). \tag{88}$$

For the calculations of the scattering processes, the energy E is real and

$$\zeta_\nu(E) = \frac{1}{E - \tilde{\epsilon_\nu}} = \frac{1}{E - \nu D + (i/2)\gamma}. \tag{89}$$

The summation in (88) leads to

$$\mathcal{S} = \frac{\pi}{D} \cot\left[\frac{\pi}{D}\left(E + \frac{i}{2}\gamma\right)\right] = \frac{\pi}{D}\,\frac{x - iy}{1 + ixy} \tag{90}$$

where the parameters are introduced

$$x = \cot\left(\frac{\pi E}{D}\right), \quad y = \tanh\left(\frac{\pi\gamma}{2D}\right). \tag{91}$$

As the decay width γ increases, the quantity y changes very rapidly from a small value $y \approx \pi\gamma/2D$ for isolated long-lived states, when $\gamma/D \ll 1$, to a value exponentially close to 1 for overlapping levels, when $\gamma/D \gg 1$. In practice it is sufficient to consider just these two limiting cases. At small γ, the imaginary part of $\mathcal{S}$ is small, $\propto y \approx (\pi\gamma/2D)$, and the real part of $\mathcal{S}$ is equal to $\pi x/D$ as for stable levels [17]. In the opposite case of large γ/D, the real part vanishes $\sim (1 - y^2)$ whereas $\mathrm{Im}\mathcal{S} \approx -\pi/D$. Both cases have a general meaning being not limited by restrictions of the uniform model. Thus, the result for the overlapping case follows immediately after substituting the $\sum(E - \epsilon_\nu)^{-1}$ by the integral over the levels with a level density $1/D$ and using a small shift of energies into the complex plane. This expression is routinely used in statistical theory of nuclear reactions [11]. A similar consideration is valid for the sums as in (13) and (22) taken at a fixed complex energy $\mathcal{E}_j = E - (i/2)\Gamma_j$ instead of running real energy E.

More complex sums can be easily analyzed in the same way. Here we give some examples used in the text (the notation $\mathcal{S}_{mn}$ corresponds to m factors ζ_ν and n factors ζ_ν^* so that the basic sum $\mathcal{S} \equiv \mathcal{S}_{10}$):

$$\mathcal{S}_{20} = \sum_\nu \zeta_\nu^2 = \left(\frac{\pi}{D}\right)^2 \frac{(1 - y^2)(1 + x^2)}{(1 + ixy)^2}, \tag{92}$$

$$\mathcal{S}_{11} = \sum_{\nu} |\zeta_\nu|^2 = \frac{2\pi}{D\gamma} \frac{y(1+x^2)}{1+x^2y^2}, \tag{93}$$

$$\mathcal{S}_{12} = \sum_{\nu} |\zeta_\nu|^2 \zeta_\nu^* = \frac{i\pi}{D\gamma} \frac{1+x^2}{1+x^2y^2} \left[\frac{2y}{\gamma} - \frac{\pi}{D}(1-y^2)(1-x^2y^2+2ixy) \right]. \tag{94}$$

In the overlapping limit $y \to 1$, these sums go to 0, $2\pi/D\gamma$ and $2i\pi/D\gamma^2$, respectively. The first sum (92) vanishes in accordance with the fact that both poles in the equivalent integral are located on the same side of the real axis. The nonvanishing sums are proportional to the level density $\rho = 1/D$, i.e. they have a coherent component growing at high excitation energy.

The sum $\mathcal{S}_{22}$ can be calculated as

$$\mathcal{S}_{22} = \sum_{\nu} |\zeta_\nu|^4 = \left[\frac{\partial^2}{\partial E \partial E'} \sum_{\nu} \zeta_\nu(E) \zeta_\nu^*(E') \right]_{E'=E}. \tag{95}$$

After simple algebra, we obtain

$$\mathcal{S}_{22} = -\frac{4}{\gamma^3} \mathrm{Im}\mathcal{S} + \frac{2}{\gamma^2} \mathrm{Re} \frac{d\mathcal{S}}{dE}. \tag{96}$$

In the overlapping limit, the first term in (96) gives $4\pi/D\gamma^3$ whereas the second one is proportional to $(1-y^2)/D^2\gamma^2$ and therefore it is small compared to the first term since the exponential smallness of $(1-y^2)$ overcompensates an extra factor γ/D.

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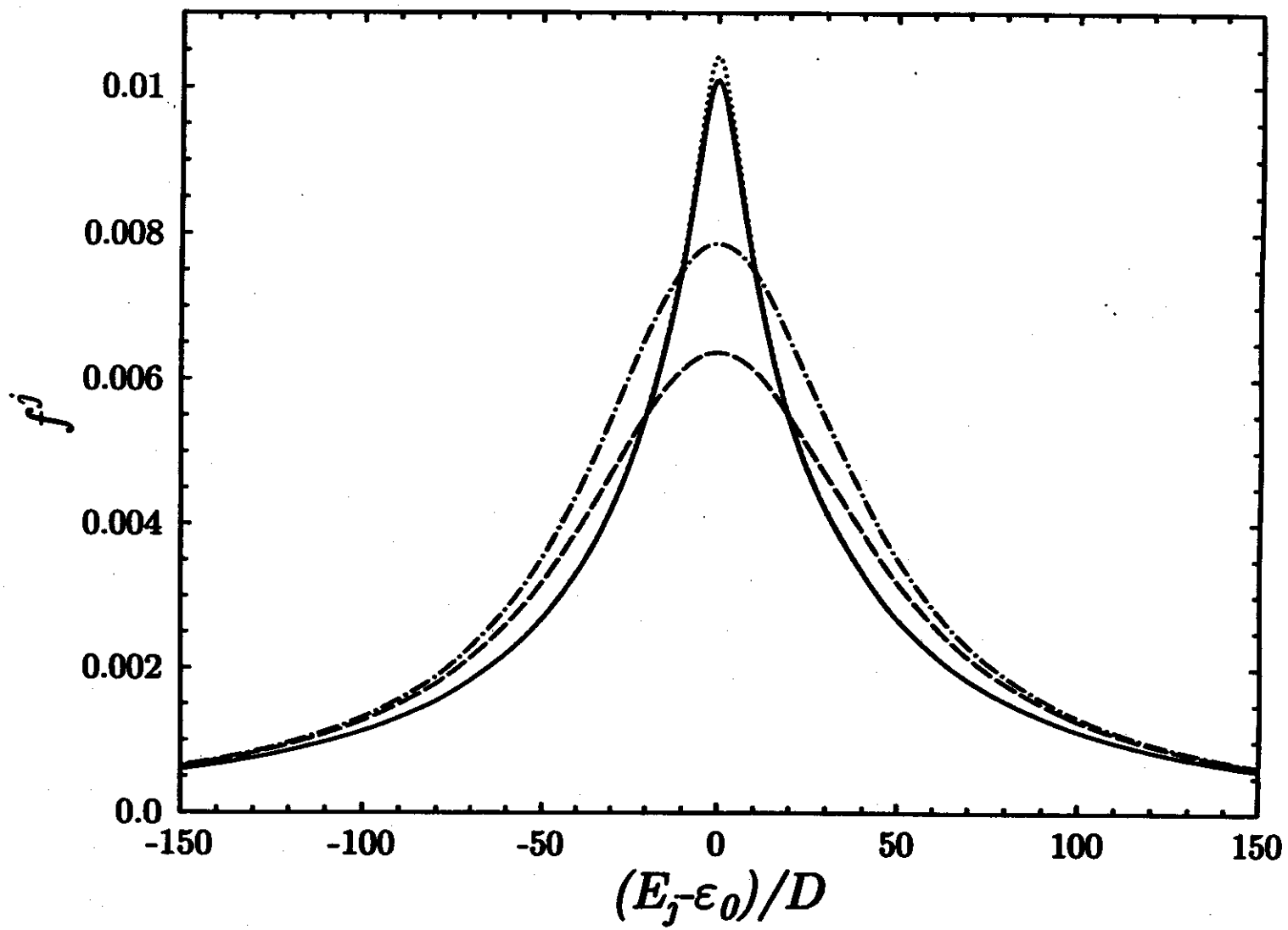
Figure captions

Figure 1. The strength function $\tilde{f}^j$ as a function of energy for the values of parameters $\Gamma_s/D = 100$ and $|\gamma_0 - \gamma_{ev}|/D = 90$. The solid curve gives the exact numerical solution of eq.(25), the dotted line corresponds to the approximation (26), the dash-dotted curves show the Breit-Wigner approximations (29).

Figure 2. The relative probability $\tilde{f}(E)$, see eqs. (56-58), of excitation of a simple state through the channel c as a function of energy, E/D, and the evaporation width, γ_{ev}/D. The value of the spreading width, $\Gamma_s/D = 10$ is chosen for illustrative purposes to make the oscillations along the energy axis clearly seen; the oscillations rapidly disappear as γ_{ev} grows.

Figure 3. Elastic cross section σ^{cc} in the channel c as a function of energy. The parameters in part *(a)* are $\gamma_0/D = 10$ and $\Gamma_s/D = 5$; the cross section is shown for different evaporation widths, γ_{ev}/D, which correspond to the values $y = 0.3$ (dots), 0.7 (solid curve) and 1 (dash-dotted curve). The situation with $\Gamma_s > \gamma_0$ is shown, for the same values of y, in part *(b)* where $\gamma_0/D = 5$ and $\Gamma_s/D = 10$; note the different scale for the cross section. Part *(c)*shows the cross section for $\gamma_0/D = 100$ and $\Gamma_s/D = 15$ with $y = 0.5$ (oscillatory curve) and $y = 1$ (thick curve).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0.008
0.006
0.004
0.002
0.0
ρ_j
-150
-100
-50
0
50
100
150
$(E_j-\epsilon_0)/D$

Figure 1

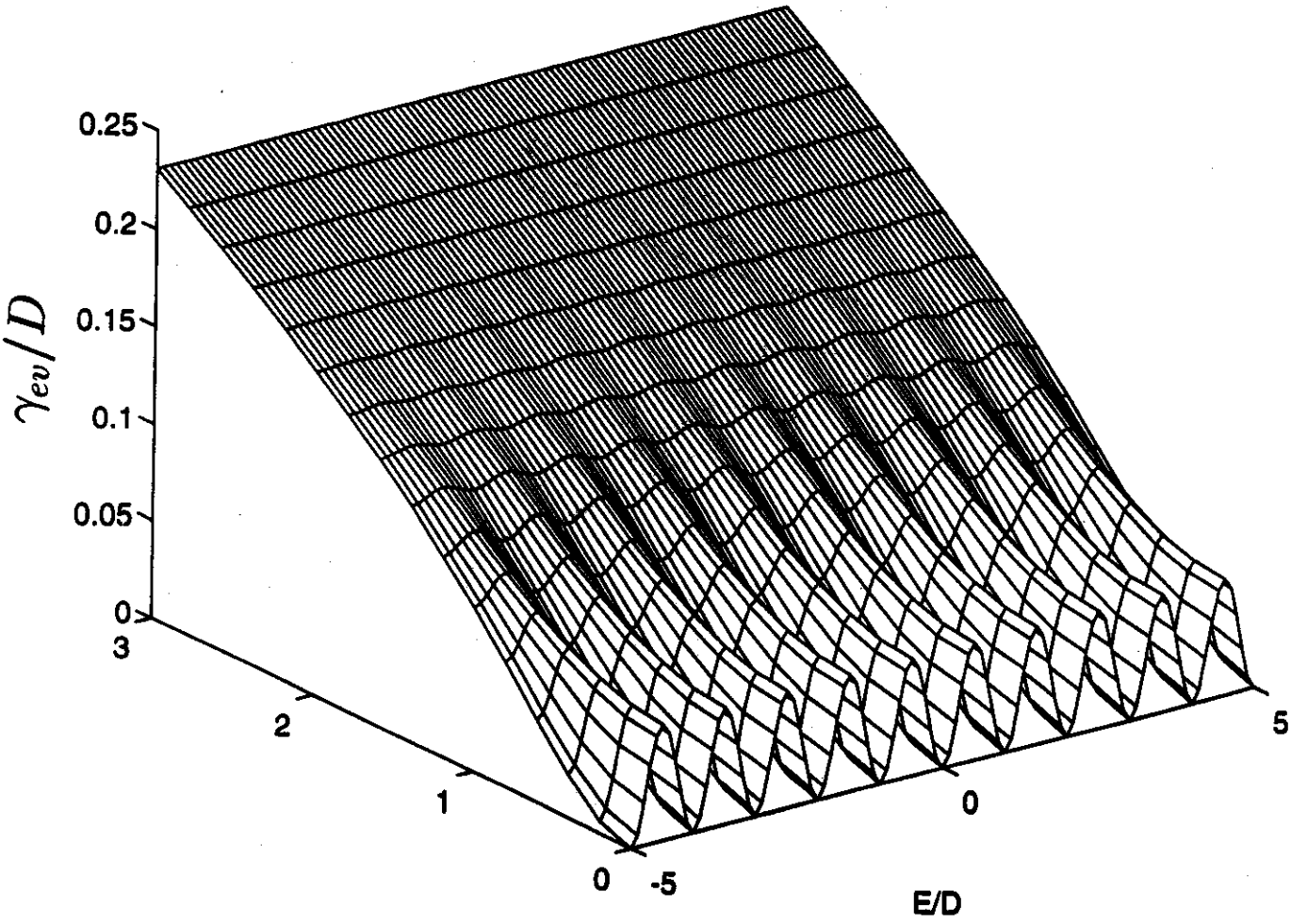

Figure 2

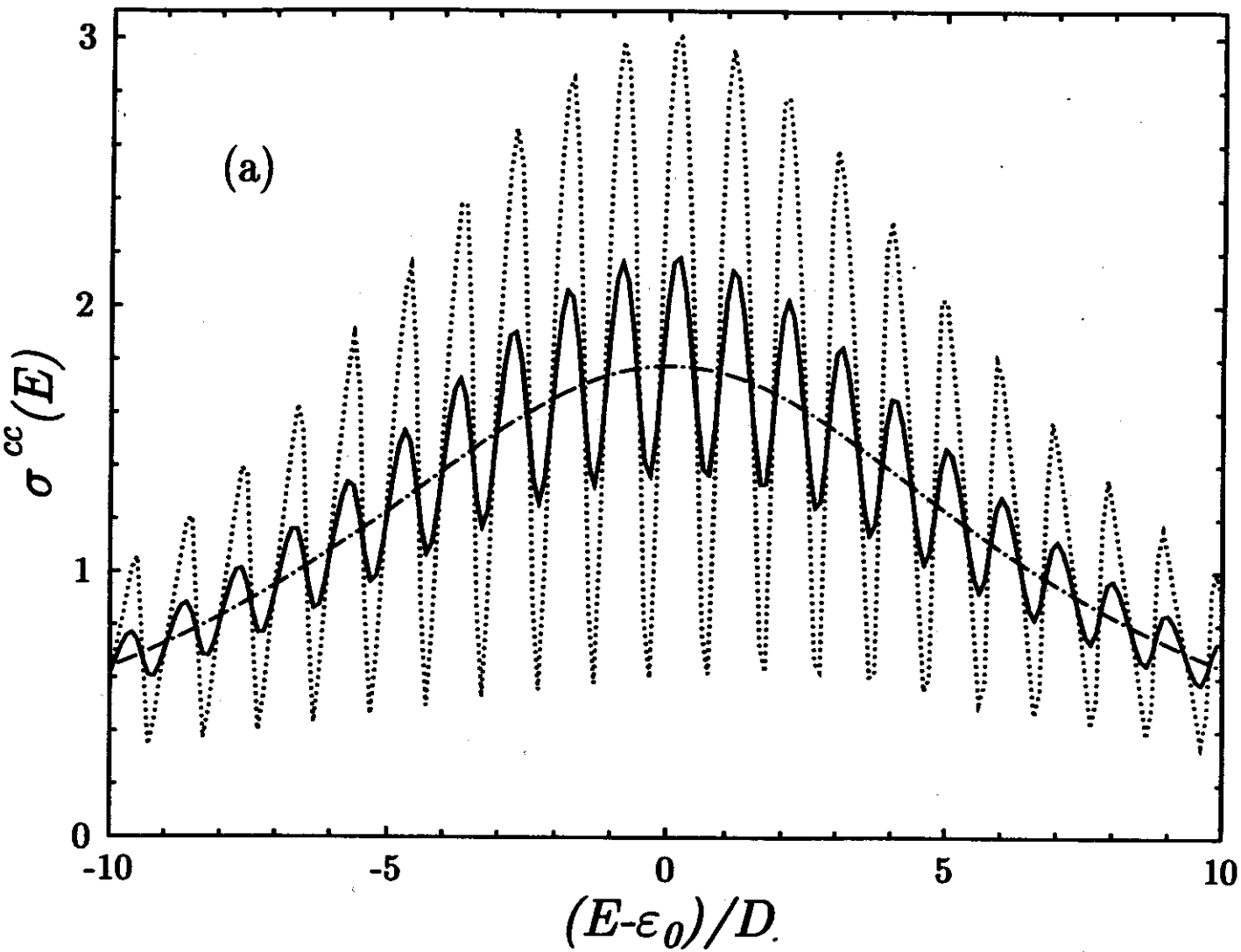

Figure 3a

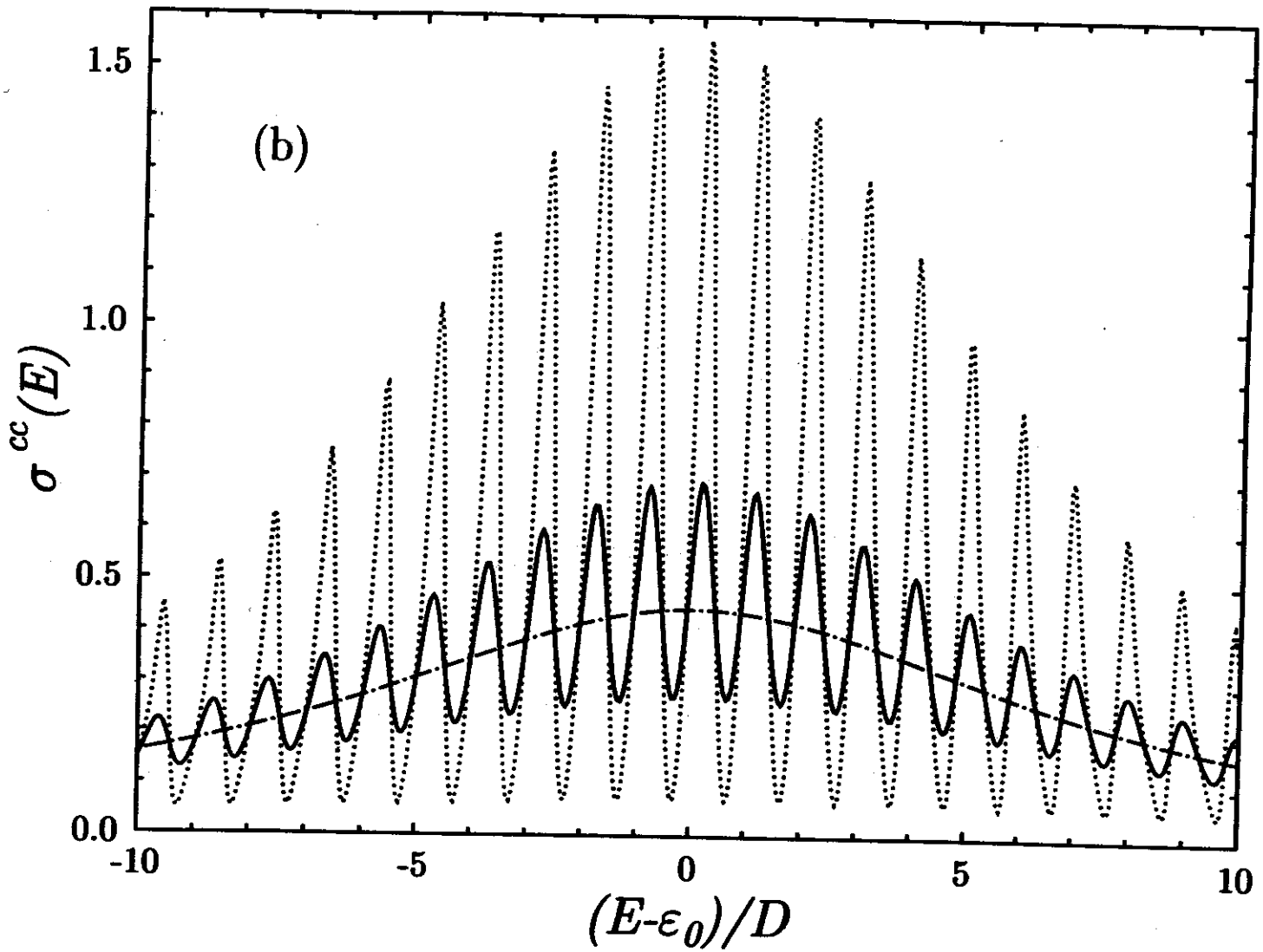

Figure 3b

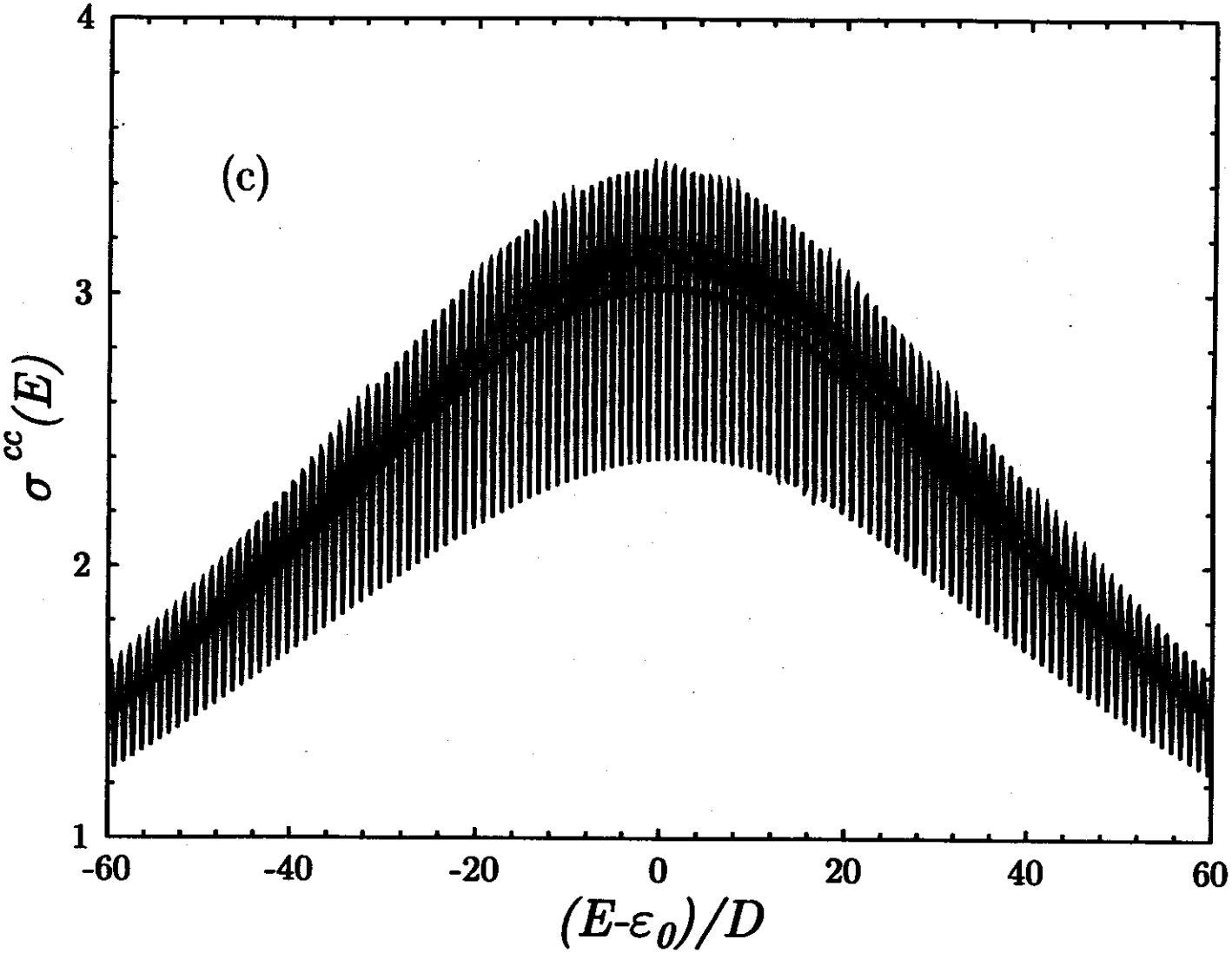

Figure 3c