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## Abstract

Inelastic scattering data of $^{11}$Li from hydrogen at 68$A$ MeV have been interpreted as the excitation of a state in $^{11}$Li at $E_x$ = 1.3 MeV, with an assignment of $J^\pi = \frac{3}{2}^+$. Analysis of those data in a distorted wave approximation assuming transitions to three candidate8 obtained in a (0 t 2)$\hbar\omega$ shell model suggests an alternative nuclear shakeoff mechanism.

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The understanding of the structures of the light halo nuclei in terms of a spherical shell model has presented a challenge because of the very loose binding of the halo nucleons, which extend the matter distributions of those nuclei to large radii. The measurements of transfer reactions leading to excited states of $^{11}$Li by Bohlen *et al.* [1] suggests excited states in $^{11}$Li at 2.47, 4.85, and 6.22 MeV. There have been few shell model calculations presented of halo nuclei: e.g. those of Poppelier, Wood, and Glaudemans [2] for several $0p$-shell neutron-rich halo nuclei, those of Warburton and Brown [3], and those of Sagawa, Brown and Esbensen [4].

The measurements of $^{9,11}$Li scattering from hydrogen taken by Moon *et al.* [5] at 60 and 62 MeV/nucleon and by Korsheninnikov *et al.* [6,7] at 68 and 75 MeV/nucleon provide elastic and inelastic proton scattering data from $^{9,11}$Li in the inverse kinematics. These data not only provide energy spectra, by which comparison may be made with model predictions, but also cross section data with which to test the wave functions. The inelastic scattering data at $E_p = 68$ MeV of Korsheninnikov *et al.* [7] shows a peak at $E_x = 1.3$ MeV in the spectrum. A coupled-channel optical model analysis of the differential cross section obtained from this peak suggested that the excitation is dominantly $L = 1$. For the ground state, $J^\pi; T = \frac{3}{2}^-; \frac{5}{2}$, and so an assignment of $J^\pi = \frac{3}{2}^+$ was made (although that analysis also would allow assignments of $\frac{1}{2}^+$ and of $\frac{5}{2}^+$). This is the starting point by which direct comparison with the shell model can be made.

A problem with the conventional optical model analysis of those data lies with the specification of the optical potential. Korsheninnikov *et al.* [6,7] used a phenomenological optical potential of conventional Woods-Saxon (WS) form that fitted the elastic scattering cross section found by Moon *et al.* [5] in an analysis of their 62 MeV data. Such a central mean field specification for the optical potential may be inappropriate for the scattering of protons from a halo nucleus. It may not take into account appropriately the density extending to large radii arising purely from the distribution of the halo nucleons. An alternative prescription is to use a fully microscopic optical potential formed by the folding of the scattering interaction with the nucleon occupancies and single-particle wave functions. When the halo single-particle wave functions are correctly specified that prescription of the optical potential appropriately takes into account the halo distribution.

An analysis of the data of Moon *et al.* [5] has been performed by Crespo *et al.* [8] assuming a few-body model for the $^{11}$Li nucleus and specifically as a three-body system ($^{9}$Li $+n+n$). A Gaussian distribution model and a cluster model separately were assumed for the $^{9}$Li core. Their calculations for the elastic proton scattering from both $^{9}$Li and $^{11}$Li underpredict the measured cross sections and hence they concluded that the microscopic structure of $^{9}$Li was important. Such structure is included naturally in the shell model wave functions we consider herein.

We have performed a complete $(0+2)\hbar\omega$ shell model calculation for the negative parity states of $^{11}$Li, and a restricted $(1+3)\hbar\omega$ shell-model calculation of its positive parity states, using the code OXBASH [9]. A $0\hbar\omega$ calculation of the ground state spectrum of $^{9}$Li was performed as well. The model contained all orbits from the $0s$ up to, and including, the $0f1p$ shell. Hence the restriction on the $(1+3)\hbar\omega$ model space is only the exclusion of the single-particle excitations up to the $0g1d2s$ shell. The interaction used was the WBP interaction of Warburton and Brown [3], while their P(5 − 16)T interaction [3] was used for the calculation of $^{9}$Li in a pure $0\hbar\omega$ shell model. An energy shift was applied to the $2\hbar\omega$ and $3\hbar\omega$ components in each case to account for the neglect of higher $\hbar\omega$ components [10]. The energy shifts were $\Delta_{2\hbar\omega} = -2.00$ MeV and $\Delta_{3\hbar\omega} = -2.23$ MeV for $^{11}$Li. (Those values are obtained from the calculated shift of the $n\hbar\omega$ configurations due to the $(n+2)\hbar\omega$ admixtures.)

For $^{9}$Li, the ground and first-excited state (3.14 MeV in our model calculation) agree with the experimental assignments. The calculated energies and spin-parity assignments for $^{11}$Li are 0 ($J^\pi;T = \frac{3}{2}^-;\frac{5}{2}$), 1.49 ($\frac{3}{2}^-$), 1.83 ($\frac{3}{2}^+$), 1.87 ($\frac{1}{2}^-$), 2.68 ($\frac{1}{2}^+$), and 3.25 MeV ($\frac{5}{2}^+$), of which the second, third, and fourth states are likely candidates for the excitation of $^{11}$Li in the inelastic proton scattering experiment [7]. The calculation by Poppelier *et al.* [2] had the excited $\frac{3}{2}^-$ state at 21.96 MeV. Their result also showed an excited $\frac{5}{2}^+$ state at 2.68 MeV, a

$\frac{1}{2}^-$ state at 4.58 MeV, and a $\frac{3}{2}^+$ state at 3.13 MeV. It is important to note that all excited states in $^{11}$Li are broad continuum states, and their energies in our shell model are accurate to about 1 MeV.

The ground state wave function of $^{11}$Li in our model is 62.71% $|0\hbar\omega\rangle$ + 37.29% $|2\hbar\omega\rangle$. This wave function contains a substantial admixture of $2\hbar\omega$ components, of which 19.62% come from the pure $(0d)^2$ configurations and a further 10.02% arise from the pure $(1s)^2$ configurations.

Our methods of analyses of the elastic and inelastic proton scattering data follows those we used in analyses of the elastic and inelastic scattering data from 200 MeV protons on $^{12}$C [11] and on $^{6,7}$Li [12]. Those analyses are based upon an effective nucleon–nucleon ($NN$) interaction in coordinate space that has been obtained from an accurate mapping of the ($NN$) $g$ matrices of the Paris $NN$ interaction [13] for infinite nuclear matter obtained from solving the Bruckner-Bethe-Goldstone equations [14]. That complex interaction is both energy and density dependent. Folding the effective interaction with the target density matrix elements then yields energy dependent, complex and nonlocal nucleon–nucleus ($NA$) optical potentials in which is contained the density dependence required to describe well, without renormalisations, both elastic and inelastic scattering data [11]. The latter have been calculated in the distorted wave approximation (DWA) in which the same effective interaction is the transition operator and the distorted waves are obtained from the microscopic optical potentials. The interaction at 65 MeV also has been used in analyses of proton elastic and inelastic scattering from diverse targets [15], wherein very good agreement with cross section and polarization data has been obtained. The code DWBA91 of Raynal [16] has been used to calculate all of the elastic and inelastic scattering cross sections.

Specification of the single particle wave functions is important in analyses of scattering data. This is especially true for the scattering from $^{11}$Li, as that halo nucleus requires single particle wave functions that reproduce the density extending to large radii. Such is not the case for $^{9}$Li. We have used HO and WS single particle wave functions in the calculations. As there are no electron scattering data by which to set the wave functions, the WS parameter values were determined from fits to the longitudinal elastic electron scattering form factors for either $^7$Li [12] (a choice predicated on the similarity of charge) or $^9$Be [17] (a choice predicated on mass). However, for the scattering with $^{11}$Li the WS functions were adjusted to define the halo nature of that nucleus. Specifically we used WS wave functions with a binding energy of 500 keV for the halo neutron orbits, namely the $0p_{\frac{1}{2}}$ orbit and the $0d1s$ and $0f1p$ shells in the complete $(0+2)\hbar\omega$ shell model space. The problem of choosing appropriate radial wave functions for transitions between loosely bound states has been illuminated (and resolved) by Millener *et al.* [18] for the case of $^{11}$Be.

The results of the calculations made for the elastic scattering of 62 and 68 MeV protons from $^{11}$Li, and of 62 MeV protons from $^9$Li are shown in Fig. 1. Therein the data for 62$A$ MeV [5] and 68$A$ MeV [7] $^{11}$Li scattering from hydrogen are compared in the top panel of Fig. 1 to the result at 62 MeV made using the $^9$Be WS wave functions (solid line), and also to that at 68 MeV (dashed line). The result of calculation of the scattering from $^{11}$Li made at 62 MeV using harmonic oscillators is displayed as the dot-dashed line. It overestimates the cross section significantly, and illustrates the need for specifying the halo density distribution appropriately. The results of the calculations using the WS single-particle wave functions are insensitive to changing the binding energy of the halo orbits to as low as 50 keV. The data for the elastic scattering of 62$A$ MeV $^9$Li from hydrogen [5] and the results of calculations made using the WS wave functions are compared in the bottom panel of Fig. 1, wherein the results obtained using the $^9$Be and $^7$Li sets are displayed by the solid and dashed lines respectively. There the use of the $^9$Be set of WS functions is closer in agreement with the data, although the $^7$Li set provides a reasonable respresentation. In the case of $^{11}$Li, the results using both WS sets are quite similar, and hence only the $^9$Be results are displayed. The excellent agreement with experiment for both nuclei confirms the conclusion by Crespo *et al.* [8] of the need for the specification of the full structure of the $^9$Li core.

The total nuclear elastic scattering cross sections from $^9$Li and $^{11}$Li are 281 and 393 mb, respectively. This is in disagreement with Moon *et al.* [5], who interpreted their experiment as indicating a significant decrease in cross section from $^9$Li to $^{11}$Li. Our result is due to the contributions made from the cross section values at the (unobserved) forward angles for $^{11}$Li and is what should be expected: the larger size of $^{11}$Li leads to a narrower and more intense diffraction pattern.

The data for the inelastic scattering of 68 MeV protons from $^{11}$Li [7] are compared with the results of our calculations in Fig. 2(a) and neither the shapes nor the magnitudes agree. It is a possibility that a problem may lie in our choice of wave functions for the excited states. It is also possible that our approach does not guarantee proper treatment of excitations in the continuum. Also, as the excited (continuum) states are broad, the excitation of many higher lying states may contribute to the inelastic cross section. Hence we propose a simpler model, incorporating the excitation of the continuum as a whole.

The basic process is elastic scattering of the proton from the $^9$Li core [19]. The momentum imparted to the halo in the new center-of-mass (c.m.) system entails a certain probability of breakup into the constituents $^9$Li$+n+n$. Such processes commonly are encountered in atomic physics, where they are referred to as "shakeoff". Another analogy is the recoilless absorption of photons in the Mössbauer effect, where the probability that the struck system remains in its ground state is referred to as the Debye-Waller (DW) factor.

Since the $^{11}$Li halo has no bound excited states, we may calculate the shake–off probability, $P_s$, as unity minus the DW factor. This is equivalent to a (non-energy weighted) sum rule. Assume, for a simple estimate, that the spatial wave function of the ground state with energy $E_0$ may be written as a product of two neutron wave functions (neglecting the center-of-mass corrections and $nn$ correlations), i.e. $|\Psi\rangle = |1s\rangle_1 |1s\rangle_2$ and that the momentum transferred to the $^9$Li core after elastic scattering of a proton is $\vec{Q}$. In the c.m. system after the scattering, the change in momentum of each of the two neutrons is $-\vec{q} = -\vec{Q}/(A+2)$ and that of the core is $+2\vec{q}$. In the sudden approximation, the wave function after the scattering is

$$\left|\vec{Q}\right\rangle = \exp\left(-i\vec{q}\cdot\vec{r}_1 - i\vec{q}\cdot\vec{r}_2\right)|1s\rangle_1 |1s\rangle_2 \ . \tag{1}$$

The DW factor is now $\left|\left\langle\vec{Q}\,|\Psi\right\rangle\right|^2$, the square of the elastic overlap amplitude, which has been evaluated with WS wave functions for a $1s$ state. Noting that the shake–off probability to lowest order in $q$ is $\frac{2}{3}\left\langle r^2\right\rangle q^2$, we adjusted the WS potential to reproduce the experimental root-mean-square radius of 7 fm for the halo and the corresponding binding energy of each neutron is 0.56 MeV.

The differential cross section for inelastic scattering was calculated as the product of $P_s$ and the differential cross section for elastic scattering from $^9$Li. Its total amounts to 20 mb. From Fig. 2(b) it is seen that this calculation agrees well with the inelastic scattering results at 68 MeV [7]. The experimental points with error bars refer to an assumed narrow and symmetric interval around the 1.3 MeV peak. The experimental energy spectrum in [7] shows a pronounced tail towards higher energies, and if the cross sections are rescaled to include this, the agreement is improved. Indeed, an asymmetric shape for the shakeoff is expected from our estimates and also from calculations and measurements of Coulomb shakeoff as is discussed below.

We have not calculated the energy spectrum predicted by our semi-quantitative model but compare instead with the average excitation energy obtained from a ratio of sum rules. The expectation value of the final energy $\left\langle\vec{Q}\,|H|\,\vec{Q}\right\rangle = E_0 + q^2/m$ is equivalent to an energy-weighted sum rule. This shows that the scattering increases the average energy by the amount that would be imparted classically to the two neutrons (an example of Ehrenfest's theorem). The neglected kinetic energy of the core recoil increases the second term by a factor $(1+2/A)$. The ratio of $\left\langle\vec{Q}\,|H|\,\vec{Q}\right\rangle - E_0$ to $P_s$ gives the average excitation energy $E^*$ for the decaying states which, to second order in $q$, must be independent of the momentum transfer. The evaluation with $P_s$ calculated from WS wave functions leads to a slow increase in $E^*$ from 1.6 MeV at 0° to 2.5 MeV at 60° in good agreement with the experimental distribution [6,7] which has its peak at 1.3 MeV and its center of gravity close to 4 MeV.

In the high energy limit, Coulomb excitation must show the same transitions to the continuum as does the nuclear shake–off process if we identify $\vec{Q}$ with the momentum transfer in the (sudden) Coulomb collision. Two calculations of the spectrum of final energies for $^{11}$Li have been carried out with three-body models. Pushkin *et al.* [20] used a simple three-body model with s-wave interactions in the initial state and with plane waves as the final states to derive an analytical expression for the energy distribution. That distribution peaks close to 1 MeV and decreases slowly towards higher energies. A more detailed calculation by Esbensen and Bertsch [21] assumed that the neutron-$^9$Li interaction was dominated by a $p$ state resonance at 0.8 MeV. They also found the main strength to be at a low energy. Also they used the ratio of sum rules to estimate the mean excitation energy to be near to 2 MeV and found that it was not very model-dependent.

In conclusion, there does not seem to be any compelling evidence from the proton scattering experiments of Korsheninnikov *et al.* [6,7] for a 1.3 MeV excited level in $^{11}$Li. The asymmetric energy spectrum observed in those experiments seems to be essentially the result of the elastic scattering from the $^9$Li core leading to shakeoff. Our interpretation of the observed structure as an effect of nuclear–shakeoff is not in conflict with a shell model nor an alternative to it. Dynamic effects of this kind should appear naturally if the transition in the spectrum from bound to continuum states is included explicitly. However, states are predicted in the shell model in the region 1–2 MeV. Such states may be observed and interpreted in single-nucleon transfer experiments on $^{12}$Be.

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## FIGURES

FIG. 1. Elastic scattering of 60 and 68 MeV from $^{11}$Li (top) and $^{9}$Li (bottom). The data of Moon *et al.* [5] (circles) and of Korsheninnikov *et al.* [7] (squares) are compared to the results of the calculations made using the $(0+2)\hbar\omega$ and $(1+3)\hbar\omega$ shell model wave functions. The results for the scattering from $^{11}$Li at 60 MeV used both the $^{9}$Be WS (solid line) and HO single particle wave functions (dot-dashed line). The result at 68 MeV using the WS wave functions is displayed by the dashed line. The result for the scattering from $^{9}$Li using the $^{7}$Li set of WS wave functions is displayed by the dashed line.

FIG. 2. Comparison of the inelastic 68 MeV proton scattering data [7] with (a) the results of the calculations made assuming the transition to the $\frac{3}{2}^-$ (1.49 MeV) (solid line), the $\frac{3}{2}^+$ (1.83 MeV) (dashed line), and the $\frac{1}{2}^-$ (1.87 MeV) (dot-dashed line) states, and (b) the result of the calculation assuming the shake-off mechanism (solid line). The dashed line in (b) is the measured inelastic cross section of [7] rescaled under the assumption that the total observed energy spectrum of that experiment represents shakeoff. The elastic scattering of 68 MeV protons from $^{9}$Li is displayed by the dot-dashed line.

Fig 1

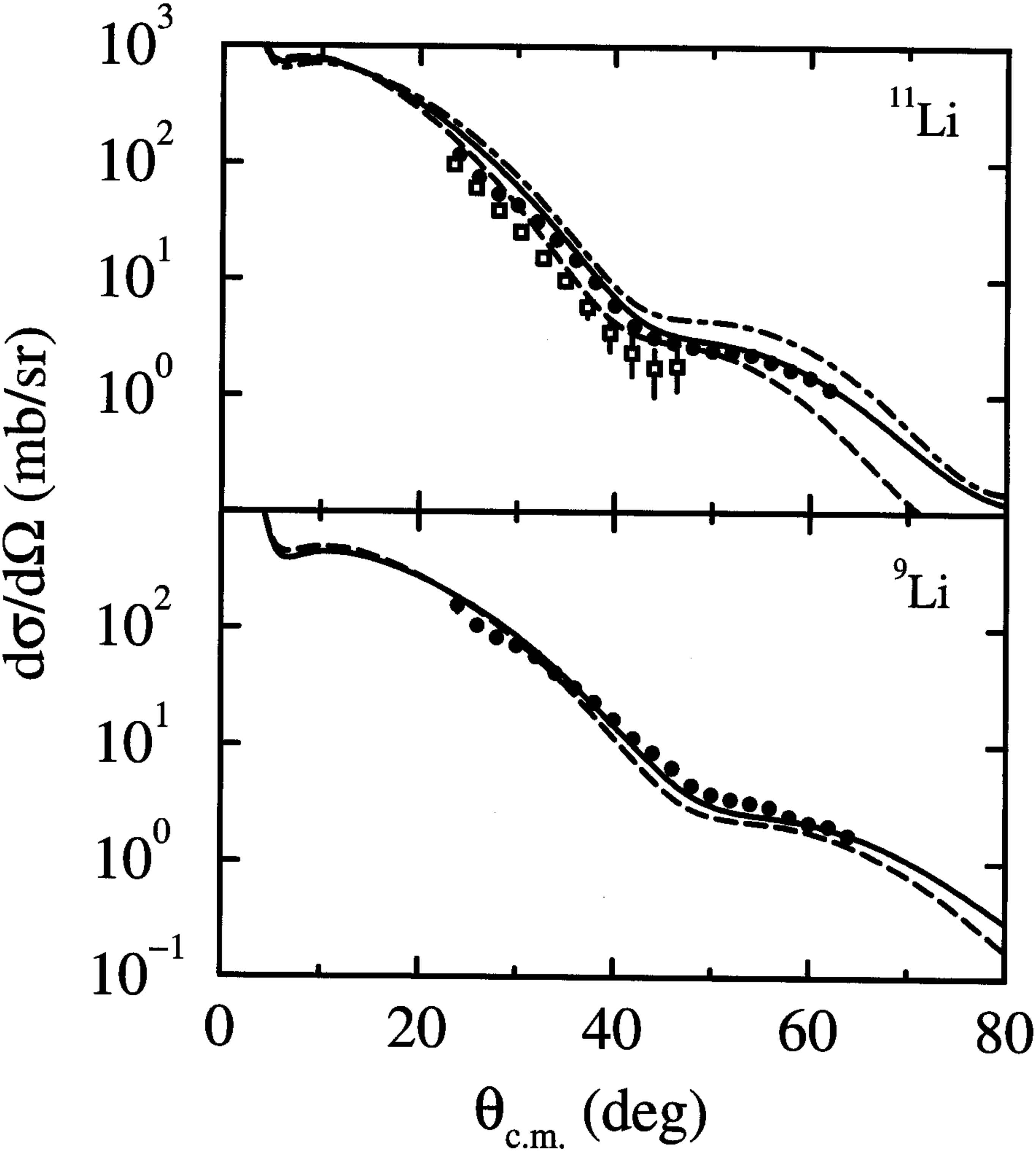
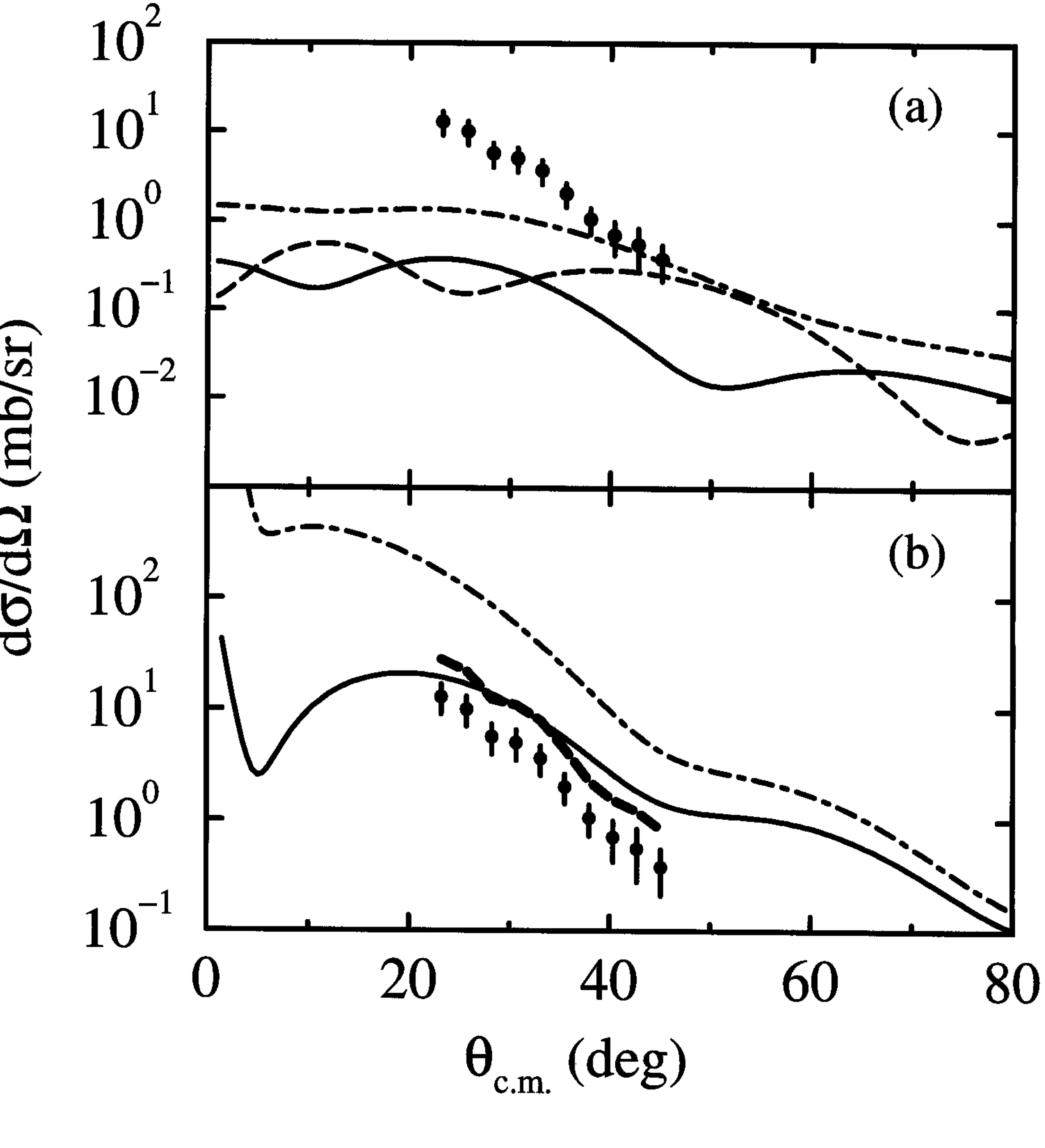


Fig 2



(a)
(b)
$10^2$
$10^1$
$10^0$
$10^{-1}$
$10^{-2}$
$10^2$
$10^1$
$10^0$
$10^{-1}$
0
20
40
60
80
dσ/dΩ (mb/sr)
$\theta_{c.m.}$ (deg)