Michigan State University

National Superconducting Cyclotron Laboratory

EXPLORING THE NUCLEAR PION DISPERSION RELATION THROUGH THE ANOMALOUS COUPLING $\gamma \rightarrow \gamma' \pi_O$

ALEXANDER VOLYA, KEVIN HAGLIN, SCOTT PRATT, and VLADIMIR DMITRIEV

MSUCL-1102

JUNE 1998

Exploring the nuclear pion dispersion relation through the anomalous coupling $\gamma \to \gamma' \pi_0$

Alexander Volya, Kevin Haglin and Scott Pratt

National Superconducting Cyclotron Laboratory, Michigan State University, East Lansing, Michigan 48824-1321, USA

Vladimir Dmitriev

Budker Institute of Nuclear Physics, Novosibirsk, 6300900 RUSSIA

(December 12, 1997)

We investigate the possibility of measuring the pion dispersion relation in nuclear matter through the anomalous coupling in the reaction $\gamma \to \gamma' \pi_0$. It is shown that this reaction permits the study of pionic modes for space-like momenta, $|\mathbf{k}_\pi| > \omega_\pi$. If the pion is softened in nuclear matter due to mixing with the delta-hole state, significant strength for this reaction is expected to move into the space-like region. Competing background processes are evaluated, and it is concluded that useful insight can be obtained experimentally, but only through a difficult exclusive measurement.

I. INTRODUCTION

Pions are expected to interact strongly with nuclear matter due to the mixture of pionic and delta-hole states. The mixture is especially strong when the momentum of the pionic mode approaches 300 MeV/c. In this momentum range the energy of the pionic mode, $(k^2 + m_\pi^2)^{1/2}$, crosses the energy of the delta-hole mode, $(k^2 + M_\Delta^2)^{1/2} - M_N$, and the energy of the pionic branch should lower due to level repulsion. This topic drew a great deal of attention in the late 1970s when it was thought that the pionic mode might be pushed below zero energy at sufficient nuclear density, which would result in pion condensation. An extensive review of pionic excitations and condensation in nuclear matter was published by Migdal [1], and recently, kaon condensation has been the subject of several investigations [2]. Since such novel behavior requires higher densities, the discussions are often in the context of neutron stars [3], although relativistic heavy ion collisions were also once proposed as a mechanism for producing sufficient density for condensation.

Despite theoretical efforts in this area, experimental evidence of large in-medium corrections to the pion dispersion relation is sparse. The most promising information is from recent charge-exchange measurements from light ions scattered off heavy nuclei. The cross-sections appear enhanced for channels where pion-exchange is expected to dominate. This enhancement is consistent with the lowering of the energy of the pionic branch, which reduces the amount by which the exchanged pion is off-shell [4]. The interpretation of this experiment suffers only from the fact that the probe is hadronic and must traverse the surface of the heavy nucleus before interacting. Heavy-ion collisions, which can produce matter at three to four times nuclear density, were expected to create environments where the dispersion relation was extremely distorted. However, experimental signals, such as measuring pion spectra [5] or dilepton pairs [6,7], of pionic properties in the interior of these regions proved difficult to extract.

In this paper we propose using high-energy photons to excite pionic excitations. The anomalous coupling of a neutral pion to two photons, which is responsible for the decay of the π^0, can be used to excite a pionic mode in a heavy nucleus. The photon provides a clean probe for entering and exiting the interior of the nucleus. Unfortunately, this has the same drawback as the charge-exchange experiment mentioned in the previous paragraph — only space-like excitations can be investigated. However, in-medium effects are expected to lower the energy of the pionic branch, perhaps to the point that it crosses into the space-like region, $|\mathbf{k}_\pi| > \omega_\pi$. Thus, the reaction $\gamma \to \gamma' + \pi^0$, which is kinematically forbidden in free space, is allowed in the nuclear medium. Furthermore, the measurements of such a branch would provide direct evidence of in-medium correction to the pion dispersion relation.

Our paper is organized into five parts. The next section briefly reviews the in-medium corrections to the pion dispersion relation. The following section shows the contribution to the cross section from the anomalous coupling to the π^0, outlines the procedure one must follow to map out the pion dispersion relation, and presents a discussion of how gauge-invariance constrains the cross-section to disappear at the space-like-to-time-like boundary. Due to this constraint, the contribution from the anomalous coupling is reduced in the region of interest, which allows background processes to pose a major problem. The bulk of the paper is comprised of estimates for background processes which are presented in section IV, where non-pionic delta-hole channels are shown to provide most of the background. Given the possibility for using free-electron lasers to create high-energy photon sources with 100% polarization, we also discuss the use of polarization measurements to eliminate background. We find that the signal will only stand significantly above the background if the final π_0 is detected in the final state with an energy such that the target nucleus is in it's original ground state. We conclude that an experimental investigation is challenging but tenable.

II. THE PION DISPERSION RELATION

The delta-hole and nucleon-hole both contribute to the in-medium correction to the self energy of the pion in nuclear matter. The coupling to the nucleon-hole raises the energy of the pionic mode, while the coupling to the delta-hole state significantly lowers the energy, especially for momenta approaching 300 MeV/c. Numerous theoretical works in this direction [5] have focused on the correction to the pionic mode in perturbative pictures [8,1]. Sophisticated treatments of the pion self energy were performed by Xia, Siemens and Soyeur [9] and Korpa and Malfliet [10]. There, self-consistent corrections to the delta's self energy and width were included. We will forgo such lengthy calculations as we wish to discuss how to experimentally observe the resulting dispersion relation, rather than how to better calculate in-medium corrections.

Self energy corrections can be written diagrammatically as shown in Figure 1. Vertex renormalization due to the effective four-point interaction between deltas and nucleons is incorporated through the phenomenological constant g'. Nucleon-hole contributions can also be considered but are much less important for $\mathbf{k}_\pi$ approaching 300 MeV/c.

Assuming an interaction Lagrangian density of the form,

$$\mathcal{L}_{\pi N\Delta} = \frac{g_{\pi N\Delta}}{m_\pi}(\overline{\Delta}^\mu \psi \partial_\mu \pi + h.c.) \,, \tag{1}$$

we obtain the self energy correction for the pion propagator shown in Figure 1.

The delta propagator is assumed to have a Rarita-Schwinger form [11] for all the calculations in this section, in which the width is inserted by substitution $M_\Delta \to M_\Delta - i\Gamma_\Delta/2$ [12].

$$G_\Delta^{\mu\nu}(q) = \frac{i}{\not{q} - M_\Delta}\left[-g^{\mu\nu} + \frac{1}{3}\gamma^\mu\gamma^\nu + \frac{2}{3M_\Delta^2}q^\mu q^\nu - \frac{1}{3M_\Delta}\left(q^\mu\gamma^\nu - \gamma^\mu q^\nu\right)\right] \tag{2}$$

The nucleon propagator in the presence of nucleons filled to the Fermi momentum p_f has a correction given by:

$$G_N(p) = \frac{i}{\not{p} - M_N + i\varepsilon} + 2\pi(\not{p} - M_N)\delta(p_0^2 - E^2(p))\Theta(p_f - |\mathbf{p}|). \tag{3}$$

With the obtained self-energy term a dispersion relation could be inferred by finding a pole of the corresponding "dressed" pion propagator. Using these equations and assuming the coupling strengths, $g_{\pi\Delta N} = 2.0$ and $g' = 0.8$, we obtain the pion dispersion relation presented in Figure 2.

The value of the effective in-medium coupling constant is not precisely known although vertex corrections have been extensively studied. Many parameters used in more sophisticated analyses [9,10] are still rather uncertain. For instance the delta's mass and width as well as the nucleon's mass in matter are somewhat uncertain, as are the coupling constants $g_{\pi N\Delta}$ and g'.

III. PION PRODUCTION THROUGH THE ANOMALOUS COUPLING

We consider production of a neutral pion through an effective photon "decay" inside nuclear matter as shown in Figure 3.

This process can only occur if the resulting pion is space-like,

$$k_\pi^2 = (k_i - k_f)^2 = -2k_i \cdot k_f = -2\omega_i\omega_f(1 - \cos\theta) \le 0 \,, \tag{4}$$

where k_i and k_f are the incoming and outgoing momenta of the photon, and the outgoing photon leaves at an angle θ. An interaction that has a gauge invariant and parity conserving form is $\mathcal{L}_{\gamma\gamma\pi} = \alpha/(2\pi f_\pi)F^{\mu\nu}\tilde{F}_{\mu\nu}\pi^0$, where α is the electromagnetic fine coupling constant, and the pion decay constant is $f_\pi = 93\,\mathrm{MeV}$. The transition amplitude $\tau_{i\to f} = \langle f, \pi|\mathcal{L}_{\gamma\gamma\pi}|i\rangle$ can be expressed in terms of the momenta and polarizations of the incoming and outgoing photons.

$$\tau_{i\to f} = \frac{\alpha}{2\pi f_\pi} 2\epsilon_{\alpha\beta\gamma\delta} k_i^\alpha k_f^\beta \chi_i^\gamma \chi_f^\delta \qquad (5)$$
$$= \frac{\alpha}{2\pi f_\pi}(\mathbf{k}_\pi{}^2 - \omega_\pi{}^2)\sin(\alpha_i - \alpha_f)$$

where k_i and k_f are the initial and final momenta of the photon and χ_i and χ_f are the corresponding polarizations. In the last line of Eq. (5) the transition amplitude is written in terms of the momentum of the pion, and the incoming and outgoing polarization angles, α_i and α_f, as measured relative to the reaction plane. The principle difficulty in obtaining the goals of this study comes from the fact that the amplitude vanishes at the space-like-to-time-like boundary where the square of a pion four-momentum is zero, which is precisely the kinematic region of interest. This is a direct consequence of gauge symmetries involved in coupling two photons to a pseudoscalar. This result may be problematic since we investigate the area that is close to the space-like-to-time-like boundary, and therefore the cross sections obtained are quite small. The rate of this "decay" can be expressed in terms of the matrix element τ [13]:

$$d\Gamma = (2\pi)^4\delta^4(k_f - p_i - k_i)|\tau_{i\to f}|^2 \frac{1}{2\omega_i}\frac{d^3k_f}{2\omega_f(2\pi)^3}\frac{d^3k_\pi}{2\omega_\pi(2\pi)^3} \qquad (6)$$

We will express our answer in terms of the energy and momentum transfer, $\omega_\pi \equiv \omega_i - \omega_f$ and $\mathbf{k}_\pi \equiv \mathbf{k_i} - \mathbf{k_f}$ respectively. Substituting a Lorentzian form in place of the energy preserving δ -function, allows the incorporation of a finite width to the pionic state.

$$\delta(\omega - E) \to \frac{1}{\pi}\frac{2\omega^2\Gamma}{(\omega^2 - E^2)^2 + \omega^2\Gamma^2} \qquad (7)$$

The decay rate into a pionic mode of energy ω_π and momentum $\mathbf{k}_\pi$ can be expressed as:

$$\frac{d\Gamma}{dw_\pi d|\mathbf{k}_\pi|} = \frac{\alpha^2}{(2\pi)^2 f_\pi^2}\frac{|\mathbf{k}_\pi|(\mathbf{k}_\pi^2 - \omega_\pi^2)^2}{2\omega_i^2}\sin^2(\alpha_i - \alpha_f)\frac{\omega_\pi\Gamma}{(\omega_\pi - E_\pi)^2 + \omega_\pi^2\Gamma^2} \qquad (8)$$

To estimate the pion production cross section one must multiply this result by the nuclear volume. Examples of scattering cross sections for the $\gamma \to \gamma' + \pi^0$ process calculated for a Pb nucleus for different incoming photon energies are shown in Figure 4, where the on-shell energy of the pion is assumed to be $225MeV$ and momentum transfer $|\mathbf{k}_\pi|$ is fixed at 275 MeV/c. Figure 5 shows the dependence of the peak location and height with respect to the on-shell energy. If the on-shell energy is not more then 25 MeV less than $|\mathbf{k}_\pi|$, the peak is probably too small to be observed.

The fact that the cross section is inversely proportional to the incoming photon energy suggests the use of lower energies for a greater signal. Lower energies also allow a more confident prediction of the background as very massive and not well understood nucleonic resonances do not contribute. Incoming photons with energies between 400 and 600 MeV are satisfactory for our purposes given the above considerations and the experimental ease with which they can be created. Lower-energy photons would be difficult to deal with since the final photon could be confused with those from nuclear processes such as giant-dipole decays.

For a 5 mm (a radiation length) lead target, one would need on the order of 10^{14} photons to investigate the peak in the region of interest. This estimate was ascertained by requiring

100,000 final-state photons to correspond to pion momenta between 225 and 275 MeV and pionic energies within 75 MeV of the momentum. This number of photons is within the realm of current experimental constraints, although the elimination of background to be discussed in the next section will push the viability of these measurements. The role of the width is important, as for vanishing widths the shape of the differential cross section in Eq. (6) becomes a sharp spike which would be more easily observed. A confident calculation of the width is not trivial, since the mixture of the pion and the delta-hole might represent a significant contribution.

IV. CALCULATING BACKGROUND PROCESSES

We consider two processes that might overwhelm the $\gamma \to \gamma' + \pi^0$ reaction, namely ordinary Compton scattering, that could also proceed through an intermediate Δ resonance, see Figure 6, and reactions that produce a Δ-hole (which decays into πN) as a final state, see Figure 8. These reactions are the same order in α as $\gamma \to \gamma' + \pi^0$. One might expect these background processes to be smaller than the simple π_0 production process since they are further off-shell in the kinematic region we are exploring. However, even though $\gamma \to \gamma' + \pi^0$ might be nearly on-shell, a gauge constraint forces the matrix element to zero at the $k_\pi^2 = 0$ boundary and allows other channels to compete. At the end of this section we will demonstrate that background processes overwhelm the signal, but that by gating on a final-state π_0, one might sufficiently suppress the background. We also report on our investigation of using polarization measurements to project the signal from the background. Unfortunately, our estimate of the background processes is done without the benefit of an experimental measurement of relevant processes in vacuum, e.g. $\gamma + p \to p' + \gamma' + \pi^0$. Certainly such measurements are possible and would greatly increase the confidence with which we present the background.

First, we discuss our estimate of normal Compton scattering as illustrated in Figure 6. We consider the intermediate state to be either a delta or a proton. The diagrams with the delta as an intermediate state provide the dominant contribution. The photon is assumed to interact with the baryon via the following couplings [14,15]:

$$\mathcal{L}_{p\gamma p} = e\overline{\psi}\gamma_\mu A^\mu \psi$$
$$\mathcal{L}_{\gamma N\Delta} = e\left\{\frac{G_1}{M_N}\overline{\Delta}^\mu \gamma^\nu \gamma_5 \psi F_{\mu\nu} + \frac{G_2}{M_N^2}\partial^\nu \overline{\Delta}^\mu \gamma_5 \psi F_{\mu\nu} + h.c.\right\} \tag{9}$$

We observed that the contribution from the second term proportional to G_2 is small, and we neglected it in our analyses.The value of $G_1 = 2.63$ has been determined from experiments [16,17]. The regular Compton process was also small, and thus did not warrant including more sophisticated coupling, e.g. through magnetic moments.

The result for the cross section is shown in Figure 7. Again, we have assumed that the photon lost a momentum $|\mathbf{k}_\pi| = 275$ MeV/c. The form would be a sharp peak at low energy if it were not for our replacing the final delta function in the cross-section by a Lorentzian, giving the nucleon an effective width of 25 MeV. Since an on-shell nucleon with 275 MeV/c of momentum has only 50 MeV of energy, there is little contribution in the kinematic region of interest, where ω_π approaches $|\mathbf{k}_\pi|$. Since the kinematics were effectively smeared by the Fermi motion, changing the nucleon's width had little effect.

The primary contribution to the background derives from production of a delta-hole in the final state. Since this is precisely the process that mixes with the pion due to its kinematic proximity to the pionic mode, it is not surprising. Even though the delta-hole should be 50 MeV in energy higher than the pionic branch of the dispersion relation, its contribution is not constrained to go to zero when $|\mathbf{k}_\pi|$ equals ω_π. This lack of a constraint derives from the fact that a spin 3/2 delta and a spin 1/2 nucleon-hole do not necessarily form a pseudo-vector, and couple to $\partial\pi$, but can also couple to $J = 2$. Furthermore, the delta will decay into a nucleon and a pion. If the pion is charged, it can radiate photons readily since it is light and moves quickly. We find that only by gating on the presence of the π_0 and by requiring the target nucleus to be left in it's ground state, can one confidently translate the measurement into information regarding the nuclear pion disperion relation.

The diagrams used for calculating the contributions for a delta-hole being in the final state are shown in Figure 8. In order to maintain gauge invariance, the decay of the delta into pions is included. This decay is also crucial as Bremsstrahlung off the light charged pion is important. The coupling of the electromagnetic field to the delta is accomplished via minimal substitution.

The Lagangian for the delta that results in the Rarita-Schwinger form of the propagator is:

$$\mathcal{L}_\Delta = -\overline{\Delta}^\alpha \left\{(i \not\partial - M_\Delta)g_{\alpha\beta} - (\gamma_\alpha i\partial_\beta + i\partial_\alpha\gamma_\beta) + \gamma_\alpha i \not\partial\gamma_\beta + M_\Delta\gamma_\alpha\gamma_\beta\right\} \Delta^\beta \tag{10}$$

The associated coupling of a delta to a photon, through minimal substitution, is:

$$\mathcal{L}_{\Delta\Delta\gamma} = e\overline{\Delta^\alpha} \left\{g_{\alpha\beta}\gamma^\mu - g^\mu_\alpha\gamma_\beta - g^\mu_\beta\gamma_\alpha + \gamma_\alpha\gamma^\mu\gamma_\beta\right\} \Delta^\beta A_\mu \tag{11}$$

Due to the derivative nature of the $\pi N\Delta$ coupling shown in Eq. (1), minimal substitution requires a four point coupling of the photon to the $\pi N\Delta$ vertex.

$$\mathcal{L}_{\gamma\pi N\Delta} = ie\frac{g_{\pi N\Delta}}{m_\pi}\Delta^\mu A_\mu \psi\pi + h.c. \tag{12}$$

Minimal substitution from the interaction term in Eq. (9), also results in a four-point coupling, which we will neglect since G_2 is set to zero.

The 20 terms necessary for creating a nucleon, a photon and a pion in the final state are shown in Figure 8. A fixed width was used for the delta, which although is not realistic, simplified self-consistency checks regarding gauge invariance and the Ward-Takahashi identity. Since the delta-hole is not far off-shell, the answer should not vary greatly by incorporating an energy and density dependent width.

Transition elements were calculated for specific linear polarizations, α_i and α_f of the incoming and outgoing photons. The scattering plane of the photons defines the angles. For our signal, $\gamma \to \pi_0\gamma'$, the dependence is proportional to $\sin^2(\alpha_i - \alpha_f)$ as shown in Eq. (5). Since the final states illustrated in Figure 8 are three-body states with large widths from the delta decay, the final three particles could be assumed to be on-shell. Numerical integrations were performed over all final-state variables except for the photon polarizations and the energy and momentum lost by the photon, ω_π and $|\mathbf{k}_\pi|$.

The cross sections for the background are shown in Figure 9. The upper graph shows the background for charged pions, while the lower graph presents the background for the case where a neutral pion is created. The creation of a charged pion overwhelms the signal by more than an order of magnitude. This strength comes from the radiation off the light, fast-moving, charged particle.

Polarization does not significantly ameliorate the background as shown in Figure 9. The signal is proportional to $\sin^2(\alpha_i - \alpha_f)$ which has the same shape as part of the spin-2 delta-hole state.

By gating on neutral pions, one can reduce the background by an order of magnitude as shown by comparing the upper and lower panels of Figure 9. Although this background is now comparable to the signal, it is still possible that a charged pion could be created and undergo a charge exchange with the medium, resulting in a neutral pion.

For the reasons discussed above, the best hope of extracting the signal would be to experimentally verify that the nucleus remained in it's ground state. This could be accomplished by observing the outgoing π_0 with an energy such that the nucleus would be constrained to it's ground state. This effectively eliminates both the delta-hole and nucleon-hole background from consideration. If the incoming nucleon in Figure 8 returns to it's original state within the Fermi sea, such a process can be considered as a contribution to the pionic state, rather than as a separate background process.

This exclusive process of creating a π_0 without exciting the nucleus would certainly reduce the overall size of the signal shown in Figures 4 and 5. The magnitude of this reduction can be considered as the probability for the pion to escape the nucleus. Careful estimates of a pion's width in the medium have been performed, with the conclusions that the width is in the 10-30 MeV range for pions with momenta less than 300 MeV/c [10]. Using the expression for the mean free path, $\lambda = v/\Gamma$, one can expect that the mean free path is on the order of two to five Fermi.

An estimate of the escape probability is shown in Figure 10 as a function of the mean free path λ. The escape probability is the average, $\langle \exp(-x/\lambda) \rangle$, where the average is performed over all originating points within a 7.0 Fermi sphere, and over all possible directions, and x is the distance the pion must travel to escape the sphere. By viewing Figure 10 one can see that absorption will reduce the signal by approximately an order of magnitude. Combined with the difficulty in detecting all three photons (two from the decay of the π_0), the exclusive measurement becomes difficult. If lead is the target, the first excited state is at 2.6 MeV, meaning that the photons must be measured to better than 1.0 MeV to eliminate all excited states from consideration. Of course, even the reduction of the number of excited states to a small number would greatly reduce the unwanted delta-hole and nucleon-hole backgrounds.

V. CONCLUSIONS

The subject of in-medium hadron masses has been historically inconclusive. The subject of heavier mesons such as the rho, is perhaps of even greater interest due to the connection to chiral symmetry restoration. The pion's in-medium properties are unique in that the dispersion relation may dip down into the space-like region at normal nuclear density. This permits the use of simple scattering experiments, such as the process proposed here, to investigate pionic modes.

The prospects for investigating the dispersion relation hinge on detection of neutral pions to eliminate the background contributions. Although detection of the neutral pion can be accomplished with the same apparatus used to measure the scattered photon, the reduction in statistics might make the experiment untenable. It should be pointed out that theoretical efforts can be improved, especially for the understanding of finite-size effects for the pionic modes in the nucleus. An optical model calculation would be welcome.

Although the experimental challenges are significant, this line of investigation does offer the possibility for definitive evidence in the search for in-medium hadronic properties. The implications of such evidence would extend beyond the context of finite nuclei, to infinite nuclear matter and neutron stars.

ACKNOWLEDGMENTS

This work was supported by the National Science Foundation, grant no. PHY-9513900.

---

[1] A. B. Migdal, Rev. Mod. Phys. **50**, 107 (1978).

[2] D.B. Kaplan and A.E. Nelson, Phys Lett. **B175**, 57 (1986).

[3] G.E. Brown, C-H. Lee, M. Rho and V. Thorsson, Nucl. Phys. **A567**, 1 (1994).

[4] V. Dmitriev. Nucl. Phys. **A577** 249c (1994).

[5] C. Fuchs, L.Sehn, E. Lehmann, J. Zipprich and A. Faesler lanl preprint, nucl-th/9610027.

[6] C. Gale and J. Kapusta, Phys. Rev. **C35**, 2107 (1987).

[7] C. L. Korpa and Scott Pratt, Phys Rev Lett. **64**, 1502 (1990).

[8] T. Ericson and W. Weise, *Pions and Nuclei*, Carendon Press Oxford (1988).

[9] L. Xia, P. J. Siemens, M. Soyeur, Nucl. Phys. **A578**, 498(1994).

[10] C.L. Korpa and R. Malfliet, Phys. Rev. C**52**, 2756 (1995).

[11] B. J. Read, Nucl. Phys. **B52**, 565 (1973).

[12] M. Jetter and H. W. Fearing, Phys. Rev. **C51**,1666 (1995).

[13] Claude Itzykson and Jean-Bernard Zuber "Quantum Field Theory", McGraw-Hill Inc.1980.

[14] L. Tiator, H. J. Weber and D. Drechsel, Nucl. Phys. **A306** , 468 (1978).

[15] H. F. Jones and M. D. Scadron, Ann. Phys. **81**,1 (1973).

[16] F. de Jong, K. Nakayama, V. Herrmann and O. Scholten, Phys. Lett. **B333**, 1 (1994).

[17] H. Garcilazo and E. M. de Goya, Nucl. Phys. **A562**, 521 (1993).

$$\Pi(k_\pi) = \frac{\chi(k_\pi)}{1-g'\chi(k_\pi)/\mathbf{k}_\pi{}^2} \quad , \quad \chi(k_\pi) =$$

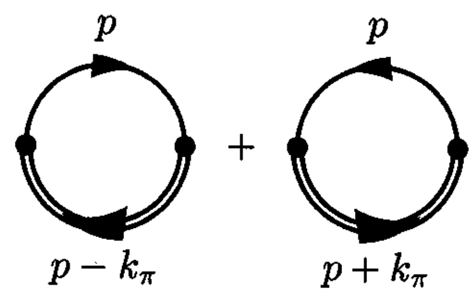

FIG. 1. Delta-hole contributions to the pion self energy.

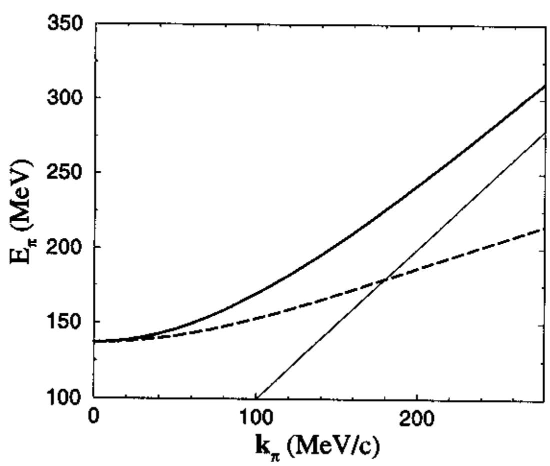

FIG. 2. The pion dispersion relation is shown for both the vacuum case (solid line) and with an effective coupling $g_{\pi N\Delta}$=2.0 and g'=0.8 (dashed line) in Figure 1. The thin strait line shows the space-like-to-time-like boundary.

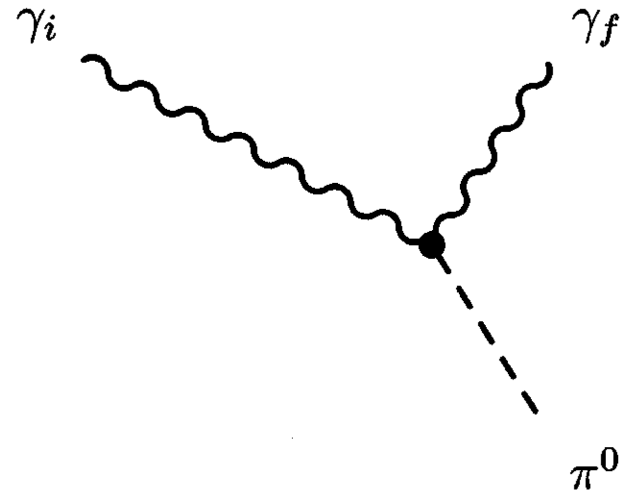

FIG. 3. Pion production diagram.

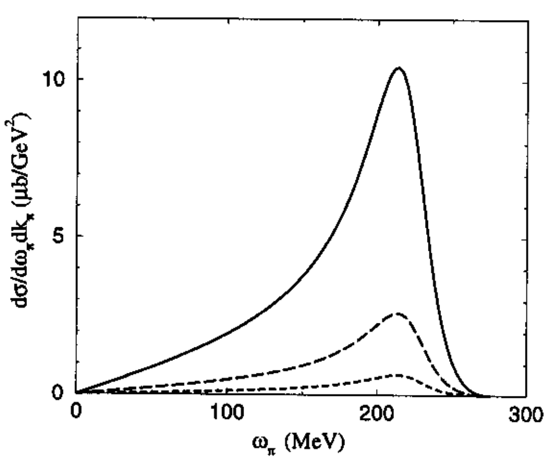

FIG. 4. Neutral pion production through the anomalous coupling $\gamma \to \gamma' + \pi^0$. The on-shell energy is 225 MeV, the momentum transfer $|\mathbf{k}_\pi|$ is 275 MeV/c, and the pion's width is 50 MeV. Cross sections are shown for three incoming photon energies: 500 MeV (solid line), 1.0 GeV (long dashes) and 2.0 GeV (short dashes).

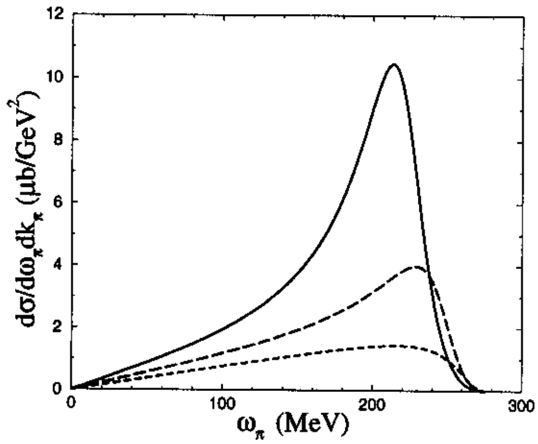

FIG. 5. Neutral pion production through the anomalous coupling $\gamma \to \gamma' + \pi^0$. On-shell energies are 225 MeV (solid line), 250 MeV (long dashes) and 275 MeV (short dashes). The momentum transfer $|\mathbf{k}_\pi|$ is 275 MeV/c, and the pion width is 50 MeV. The incoming photon has an energy of 500 MeV.

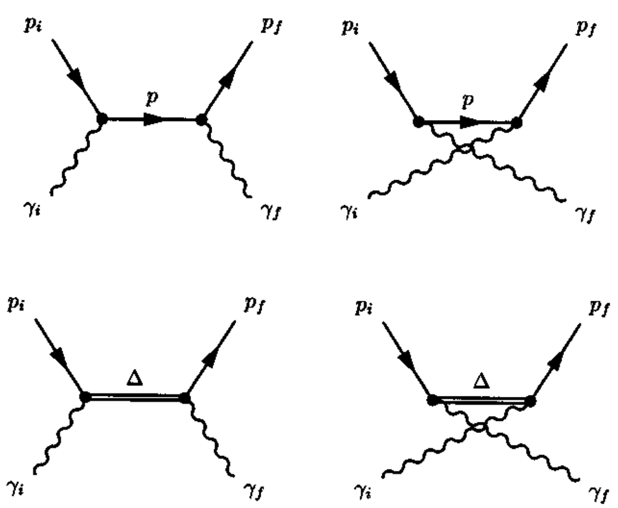

FIG. 6. Feynman diagrams for the background Compton-like processes $\gamma + N \to \gamma' + N'$.

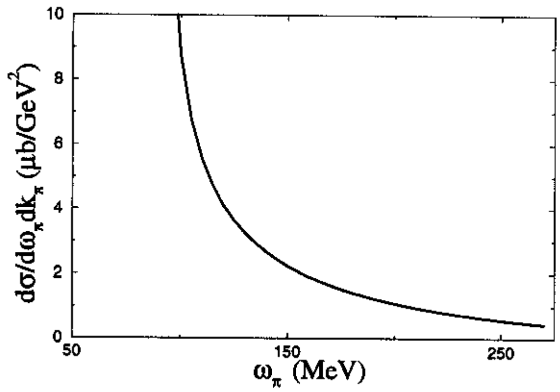

FIG. 7. The contribution from $\gamma + N \to \gamma' + N'$ assuming the outgoing nucleon has a width of 25 MeV. This component is negligible for energy transfers greater than 200 MeV.

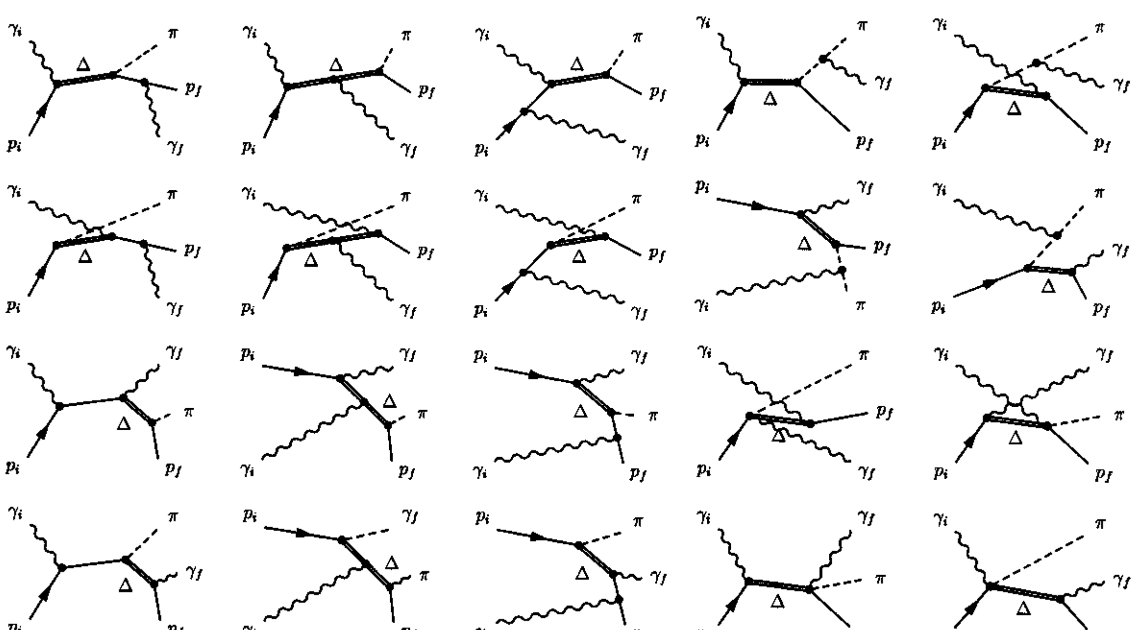

FIG. 8. Matrix elements for calculating background processes where a pion, photon and excited nucleon are in the final state. These diagrams can be thought of as processes where a delta-hole (which subsequently decays) is created.

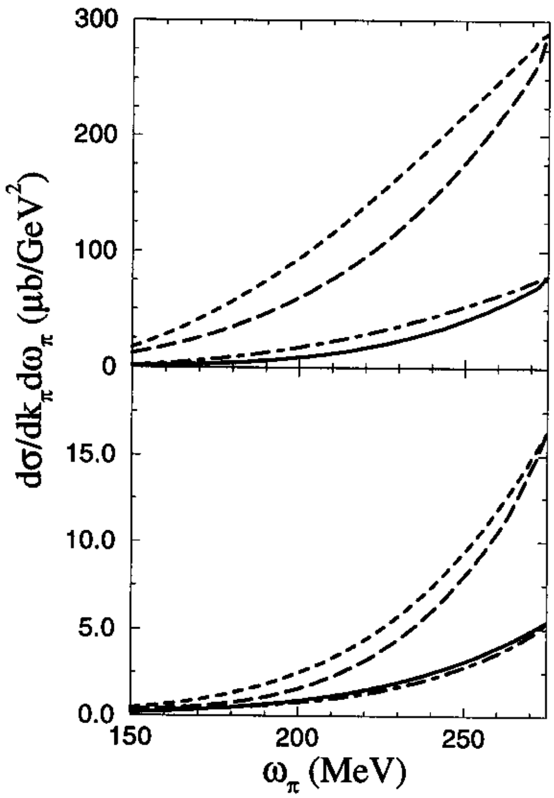

FIG. 9. The cross-section for the background process where a charged pion is created is shown in the upper panel. By requiring the created pion to be neutral, the background is greatly reduced as shown in the lower panel. Polarization projections are shown for $\alpha_i = 0$, $\alpha_f = 0$ (solid line),$\alpha_i = 0$, $\alpha_f = \pi/2$ (short dashes), $\alpha_i = \pi/2$, $\alpha_f = 0$ (long dashes), $\alpha_i = \pi/2$, $\alpha_f = 0$ (dot-dashes).

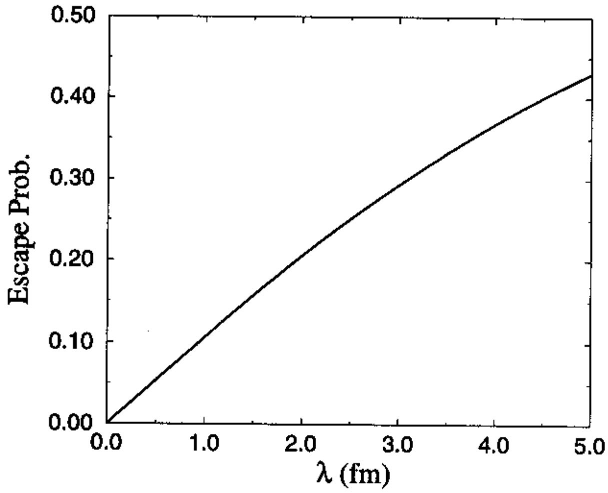

FIG. 10. The escape probability of a pion from a nucleus with the radius of 7 Fermi as a function of the mean free path λ.