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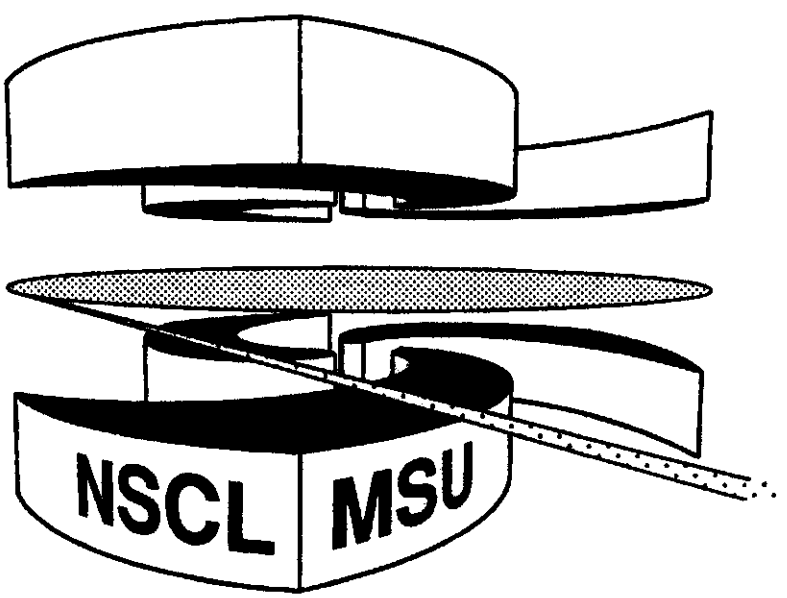

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Energetics of long qua&rotational bands and large amplitude quadrupole collective motion

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Abstract

We show **that** many high spin bands, typically outside the domain of validity of the IBM, manifest the regularities predicted by a simple model of quadrupole vibrations with strong quartic anharmonicity. The examples include bands built on the ground state and on intrinsic excitations decoupled from the quadrupole mode as in isotopes of Rn and Ra. Certain superdeformed bands reveal very similar trends raising the question whether the deformation in these cases is static or dynamic due to the large amplitude collective vibrations.

In spite of the accumulation of experimental information and immense theoretical efforts, at present, nearly half a century after the pioneering work by Bohr and Mottelson [1], we still lack full understanding of soft (transitional) nuclei. The difficulties of microscopic theory are due to the fact that the system under study is a finite self-sustaining drop of superfluid Fermi-liquid in the vicinity of the phase transition to static deformation. Low frequencies and, whence, large vibrational amplitudes and strong collective transition probabilities are the typical features of spectra [2–4] in soft nuclei which imply strong anharmonic interactions between the collective quanta. On the other hand, just because of that one can expect here more or less universal behavior which, similar to Landau theory of macroscopic phase transitions, can be simply parameterized. Currently available spectroscopic data on long collective bands can serve as a testing ground for theoretical approaches. Various models taking into account anharmonic effects were suggested long ago on the phenomenological level [5–7] as well as with the help of estimates based on the microscopic arguments [8–11]. On the other hand, the direct diagonalization taking into account all possible nonlinear terms up to a certain order [7] introduces too many free parameters. By those reasons, recent progress in nuclear spectroscopy turned out to be related mainly to the IBM [12,13] which predicts, in particular, various phase transitions and coexistence regions [14]. However, as a consequence of identifying the interacting bosons with valence fermion pairs, the IBM is invalid for the description of many long high spin bands which go well beyond the IBM cut-off without changing their properties.

The phenomenological model based on a quartic anharmonic vibrator (QAV) was considered in [15]. The model is justified by microscopic estimates [16] which follow from the main features of collective spectra in soft nuclei: dominance of quadrupole degrees of freedom, strong collectivity and adiabaticity. The model is essentially a specific version of Bohr's collective hamiltonian $\mathcal{H}(\alpha, \pi)$ which is expressed solely in terms of collective quadrupole coordinates $\alpha = \{\alpha_{2\mu}\}$ and conjugate momenta $\pi = \{\pi_{2\mu}\}$, or corresponding creation and annihilation operators $d^{\dagger}_{\mu}$ and d_{μ}. Our belief in the existence of such a hamiltonian capable of describing the broad variety of data is based on the fact that as a rule one can establish

a clear correspondence between actual nuclear levels and ideal states of a harmonic vibrator although the specific degeneracies of the harmonic approximation are destroyed. In the zero approximation the model contains no free parameters except for a common energy scale. The simple method suggested first in [10] was used to find an approximate analytical solution of the model and to predict the regularities of energy levels and transition probabilities. As was shown in [17–20], one can find remarkable examples of those regularities in actual data for transitional nuclei. In some cases, as ^{100}Pd, the vibrational bands follow almost exactly the model predictions.

The abundance of new experimental data makes it desirable to test the model on a broader scale. Here we apply the same approach to the long bands of states with the spin step $\Delta J = 2$ and strong intraband E2 gamma-transitions. As a rule, such bands are termed "rotational" even if the level energies deviate from the $J(J+1)$ rule. In fact, such sequences of states often reveal the same regularities as found earlier in low-spin excitations in transitional nuclei. Below we discuss only the energetics of the bands although the data on transition probabilities, when available, usually fit quite well in the scheme [18,19] and we hope to present them elsewhere. First we demonstrate, Table 1, that the bands with very similar features can be found in different nuclei and different regions of the nuclide chart.

TABLES

TABLE I. Relative excitation energy, $[E(J) - E(J_0)]/[E(J_0+2) - E(J_0)]$

	1	2	3	4	5	6	7	QAV	ROT
	^{100}Pd	^{73}Se	^{132}Ce$_{sd-3}$	^{142}Sm$_{sd}$	^{149}Gd$_{sd-3}$	^{152}Dy$_{sd-1}$	^{155}Dy	R_J°	r_J
$J - J_0$	$J_0 = 0$	9/2		(29)	(57/2)	(22)	13/2		22
4	2.13	2.07	2.07	2.08	2.08	2.07	2.10	2.09	2.09
6	3.29	3.27	3.23	3.23	3.23	3.23	3.31	3.26	3.26
8	4.49	4.51	4.46	4.45	4.46	4.45	4.64	4.50	4.51
10	5.81	5.80	5.77	5.76	5.77	5.75	5.86	5.81	5.85
12	7.15	7.22	7.17	7.13	7.15	7.13	7.16	7.17	7.28
14	8.57		8.66	8.59	8.62	8.58	8.57	8.58	8.79
16			10.23	10.12	10.17	10.11	10.07	10.04	10.38
Δ(%)	0.9	0.6	1.0	0.8	0.8	0.8	1.4		1.7

All examples show the spectra of bands with strong stretched, $\Delta J = 2$, E2 transitions between the adjacent states. The value of J refers to the running spin whereas J_0 indicates the spin of the headband state. The numbers in the table give excitation energies of the states with spin J in units of excitation energy of the first excited member of the band with $J_1 = J_0 + 2$. The first case, the ground state band in ^{100}Pd, was originally found as a pure example of new symmetry in [15,17]. Quite similar spectra of yrast bands are seen in many isotopes of Mo, Pd, Ru and Cd. The second example shows the band built on the ground state $J = 9/2^+$ in ^{73}Se; the ground state band in even-even ^{72}Ge is similar. In the case 3 the spectrum of the superdeformed band (sd-3 in notations of Ref. [21]) in ^{132}Ce is given; the actual intraband transitions are known up to $J - J_0 = 20$ but the value of J_0 is not established; the bands sd-1 and sd-2 are similar. Analogous behavior is revealed in column 4 for the superdeformed band in ^{142}Sm with the tentative spin value $J_0 = 29$; this band is reported up to $J = J_0 + 28$. The superdeformed band in ^{149}Gd, column 5 (the sd-3 band according to [21]), starts tentatively at $J_0 = 57/2$ and goes up to $J = J_0 + 40$ keeping the same character. Many similar bands are known in neighboring nuclei. Column 6 shows the famous superdeformed band in ^{152}Dy (sd-1 [21]) with $J_0 = (22)$ and $J_{max} = (60)$. Finally, the column 7 gives the spectrum of the band in the odd-N isotope ^{155}Dy built supposedly on the excited state 577 keV, $J_0^\pi = 13/2^-$, $J_{max} = (57/2)$. In the same nucleus the three-quasiparticle band with the headband at 3306 keV, $J_0^\pi = 35/2^-$, $J_{max} = (75/2)$, is quite similar.

In Table 1, the column labeled by R_J° lists the energy ratios predicted by the QAV model [15,17], see eq. (6) below. Those ratios are universal and do not contain any fitted parameter; for the harmonic vibrator one would have $R_J = (J - J_0)/2$. The relative mean square deviations Δ of data from these values are given in the last row; for all columns the deviations are of the order of 1%. The deviations may increase in the upper parts of the bands but still in many cases they stay on the level of (2-3)%. The model was originally developed for even-even nuclei with pure quadrupole dynamics. Here we apply it also for the quadrupole bands built on an excitation of different nature with its own angular momentum

J_0. We simply assume that this intrinsic excitation is decoupled from the main vibrations and just renormalizes the spin value so that the total vibrational spin is $L = J - J_0$.

The last column (ROT) of Table 1 illustrates what would one obtain for a band of a rigid rotor with the lowest spin $J_0 = 22$, the case which is closest to the predictions of the QAV model. In this case the energy ratios are also universal, $r_J = [J(J+1) - J_0(J_0+1)]/(4J_0+6)$. In the examples of Table 1 the QAV model works better (compare for example the case of ^{152}Dy which corresponds to the same value $J = 22$, – the rotational energies grow too fast). Of course, there are cases where the rotational description may give better agreement.

The hamiltonian of the QAV model can be written down as

$$H = H^{(2)} + H^{(4)} + H' = \sum_{\mu} \left(d^{\dagger}_{\mu} d_{\mu} + \frac{1}{2} \right) + \frac{\lambda}{4} \Big[\sum_{\mu} (d_{\mu} + d^{\dagger}_{\tilde{\mu}})(d^{\dagger}_{\mu} + d_{\tilde{\mu}}) \Big]^2 + H', \qquad (1)$$

where we take into account explicitly the harmonic term $H^{(2)}$ (the frequency is taken as an energy unit) and quartic anharmonicity $H^{(4)}$; all corrections are included into H'. There exists only one fourth order invariant, $\sim (\alpha^4)_{00}$, corresponding to a γ-unstable potential. Therefore various possible forms for the term quartic in coordinates are equivalent; the momentum terms are relatively small in the considered adiabatic limit with the exception of virtual rotation which will be taken into account later. The symmetry of the Hamiltonian $H^{(2)} + H^{(4)}$ corresponds to the group $\mathcal{O}(5)$. Although the total boson number N and the number n of "condensate" boson pairs coupled to $L = 0$ (which are created by the anharmonic interaction and play the role similar to the scalar bosons in the IBM, see [15]) are not conserved, the seniority $s = N - 2n$ (5-dimensional angular momentum) is still an exact quantum number, as well as the 3-dimensional spin L. Looking for the quasirotational bands with $s = L/2$ we need not to bother about additional quantum numbers.

The approximate analytic solution for the eigenspectrum of the hamiltonian (1) can be found [10,15] by means of the canonical transformation which utilizes the idea of the condensate of boson pairs with zero angular momentum. The states are approximately characterized by the boson seniority s and the number $\tilde{n}$ of pair excitations, and the corresponding energies are (the dimension of the vibrator in our case is $D = 5$)

$$E(s,\tilde{n}) = \frac{1}{4}\left(3\omega_s + \frac{1}{\omega_s}\right)\left(s + \frac{D}{2}\right) + 2\omega_s\tilde{n}\left\{1 + \lambda_s\left[3(\tilde{n}-1) + s + \frac{D}{2} + 1\right]\right\} + \mathcal{O}(\lambda_s^2). \quad (2)$$

Here the renormalized frequency ω_s in a sector with a given seniority s is a root of

$$\omega_s^3 - \omega_s = f_s \equiv 4\left(s + \frac{D}{2} + 1\right)\lambda; \quad (3)$$

we need to choose the root starting at $\omega_s = 1$ in the harmonic case of $\lambda = 0$. In the solution of (3) the asymptotic regime $\omega_s \approx f_s^{1/3}$ sets in very rapidly as λ grows. The renormalized coupling constant for overcondensate excitations in eq. (2),

$$\lambda_s = \frac{\lambda}{\omega_s^3} = \frac{1 - 1/\omega_s^2}{f_s}, \quad (4)$$

is numerically small being bounded from above by $1/f_s$ so that the high order corrections in (2) for not very large $\tilde{n}$ can be calculated perturbatively. As the seniority increases, the condensate approximation becomes even better (an analog of $1/N$ expansions in field theory). Neglecting all perturbative corrections, the band $\tilde{n} = 0$ has the spectrum

$$E(s,0) \equiv E_{L=2s} = \frac{1}{4}\left(3\omega_s + \frac{1}{\omega_s}\right)\left(s + \frac{5}{2}\right). \quad (5)$$

At weak anharmonicity this spectrum is close to $E_L = \alpha L + \beta L^2$ which was suggested by Ejiri *et al.* [22] and reproduced in the appropriate limit of the IBM. In the strong anharmonicity limit, or for high s and L, we come to the universal prediction

$$E_L = 2^{-8/3}3\lambda^{1/3}(L+7)^{1/3}(L+5), \quad (6)$$

which defines the ratios of energies within the band with no parameters at all, see the column R_J° in Table 1 ($L = J - J_0$). We can refer to the limit (6) as that of the $\mathcal{O}(5)$ symmetry for a quartic quadrupole oscillator; the corresponding hamiltonian contains effectively the kinetic energy and the quartic potential well (no quadratic term). For very high spins, eq. (6) gives $E_L \propto L^{4/3}$, as in one of the versions of the model of variable moment of inertia [23]. The $\mathcal{O}(5)$ limit appears in the examples of Table 1 with very high accuracy.

The analysis of many quasirotational bands implies that the deviations from the pure limit (6) are mainly accounted for by the term $\propto L(L+1)$ of "virtual rotation" [17,16]. It is

necessary to stress that in typical soft nuclei this term is responsible only for a part of the whole dependence $E(J)$ and therefore it is not a standard rotational energy of well deformed nuclei. The strength of the virtual rotation term is smaller than a normal inverse moment of inertia. The virtual rotation is closer to the rigid body rotation implying that the rotational energy related to the superfluid mode is mostly included into the anharmonic part. In soft nuclei, the addition of a cubic anharmonic term of variable strength does not improve significantly the quality of description (for example, for a sequence of Xe isotopes [19,20]). As a rule, the cubic term is small because of approximate particle-hole symmetry around the Fermi surface (the analog of the Furry theorem in QED). Below we ignore possible cubic corrections. With a virtual rotation strength, introduced as a correction to the energy ratios, $R_J = R_J^\circ + \sigma L(L+1)$, we have a one-parameter description of the bands. This description successfully used earlier can now be applied to the new data.

The Ra and Rn isotopes were extensively studied in search for octupole deformation [24,25]. The Gammasphere results [26] along with the previous data show the long bands of positive and negative parity in ^{218,220,222}Rn and ^{222,224,226}Ra. First we consider the ground state bands (positive parity) where $J_0 = 0$, $J = L$. Table 2 shows the values of $\sigma_J = (R_J^{\rm exp} - R_J^\circ)/[J(J+1)]$ for each spin J.

TABLE II. Virtual rotation parameters σ_J for positive parity bands.

J	^{218}Rn	^{220}Rn	^{222}Rn	^{222}Ra	^{224}Ra	^{226}Ra
4	-0.0039	0.0061	0.0158	0.0312	0.0440	0.0518
6	-0.0032	0.0087	0.0205	0.0402	0.0575	0.0689
8	-0.0029	0.0092	0.0216	0.0428	0.0617	0.0749
10	-0.0030	0.0088	0.0211	0.0432	0.0623	0.0762
12	-0.0030	0.0082	0.0199	0.0427	0.0614	0.0755
14	-0.0030	0.0076	0.0184	0.0420	0.0600	0.0738
16	-0.0029	0.0072	0.0171	0.0411	0.0584	0.0717
18	-0.0028	0.0066		0.0402	0.0567	0.0695
20	-0.0028	0.0060		0.0393	0.0551	0.0674
22	-0.0029				0.0535	0.0654
24	-0.0029				0.0520	0.0636
26	-0.0029				0.0505	0.0619
28						0.0604
σ	-0.0030	0.0076	0.0192	0.0403	0.0561	0.0677

As seen from Table 2, the virtual rotation correction is a smooth function of J. As a rule, it slightly increases from $J = 4$ to $J \approx 10$ and then decreases. Overall changes of this quantity are not large which allows one to take a mean value of σ_J as an appropriate parameter σ for the description of R_J. This mean value increases monotonously along the isotope chains revealing a smooth transition from quasivibrational to quasirotational bands. The lightest isotope ^{218}Rn has a very small and nearly constant value of σ being a new example of a pure $\mathcal{O}(5)$ symmetry. Fig. 1 shows the quality of the description of the spectrum with the one-parameter expression. A further slight improvement can be reached by fitting σ instead of taking the mean value but, regardless of that, we see that the simple parameterization works quite well. Apparently in all six cases the yrast bands are of the same physical nature as typical quadrupole multiphonon bands in lighter nuclei.

The same nuclei manifest well developed negative parity bands. Starting with $J_0 = 3$, these bands are similar to the positive parity bands. We again assume that the higher states J^- can be described as quadrupole anharmonic vibrations on top of the octupole phonon or small octupole deformation. Since the quadrupole frequency is small, this large amplitude motion dominates the dynamics which is not sensitive to the exact shape at the bottom of the band. Table 3 gives the values of σ_L extracted from the same parameterization where the effective value of the multiphonon quadrupole contribution to the nuclear spin has to be taken as $L = J - 3$.

TABLE III. Virtual rotation parameters for negative parity bands

$J-3$	^{218}Rn	^{220}Rn	^{222}Rn	^{222}Ra	^{224}Ra	^{226}Ra
4	0.0265	0.0186	0.0226	0.0186	0.0183	0.0172
6	0.0316	0.0231	0.0279	0.0249	0.0253	0.0242
8	0.0293	0.0236	0.0291	0.0273	0.0282	0.0274
10	0.0262	0.0226	0.0285	0.0280	0.0294	0.0289
12	0.0234	0.0212	0.0270	0.0280	0.0296	0.0295
14	0.0212	0.0199	0.0249	0.0276	0.0293	0.0295
16	0.0193	0.0186	0.0230	0.0270	0.0287	0.0293
18		0.0174	0.0213		0.0279	0.0288
20					0.0270	0.0283
22					0.0262	0.0277
24					0.0253	0.0271
σ	0.0253	0.0206	0.0255	0.0259	0.0268	0.0271

The average value of $\sigma \approx 2.5 \times 10^{-2}$ is quite stable along this chain. The quality of the overall one-parameter fit of energy ratios is shown by Fig. 2.

In those examples we did not include a possible small harmonic (quadratic in coordinates) contribution $H^{(2)}$ to the collective potential assuming $\lambda \gg 1$. Looking for the influence of this term, we found that spread of values σ_L is minimum in all cases, and consequently, the quality of the one-parameter description is indeed better when the quartic parameter λ, see Eq. (3), is sufficiently large. In fact, at $\lambda \geq 5$, the results are insensitive to a possible harmonic contribution (difference between eqs. (5) and (6)), see Fig. 3.

The similar analysis was performed for various long high spin bands. The general conclusion is that many bands, usually called rotational, have the spectra close to those predicted by the QAV model. Among such cases one can find the bands considered as superdeformed or built on various intrinsic excitations which are almost decoupled from the main vibrational mode. If some of "superdeformed" bands indeed correspond to large amplitude quasivibrational γ-unstable motion, their decay into the normal states could be noticeably enhanced. The full analysis should include intraband and interband transition probabilities as well as the structure of other parts of the spectra. The review of the results together with their systematics will be given elsewhere.

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Figure captions

Figure 1. Energy spectra R_J of ground state bands (positive parity, $J = L$) in isotopes of Rn and Ra as a function of spin J; level energies are expressed in the units of the lowest excitation energies $E(2) - E(0)$; experimental points (crosses) and a one-parameter fit (solid line) with the indicated value of the mean virtual rotation parameter σ. A rigid rotor spectrum is shown by a dashed line for ^{224}Ra.

Figure 2. The same as Fig. 2, for the negative parity bands, $J_0^\pi = 3^-$, in the same nuclei, as a function of $L = J - J_0$.

Figure 3. The relative mean square deviation of fitted level energies from the data for ^{218}Rn as a function of the anharmonicity parameter (in units of the harmonic frequency); $\lambda = 0$ corresponds to a harmonic approximation.

FIGURE 1

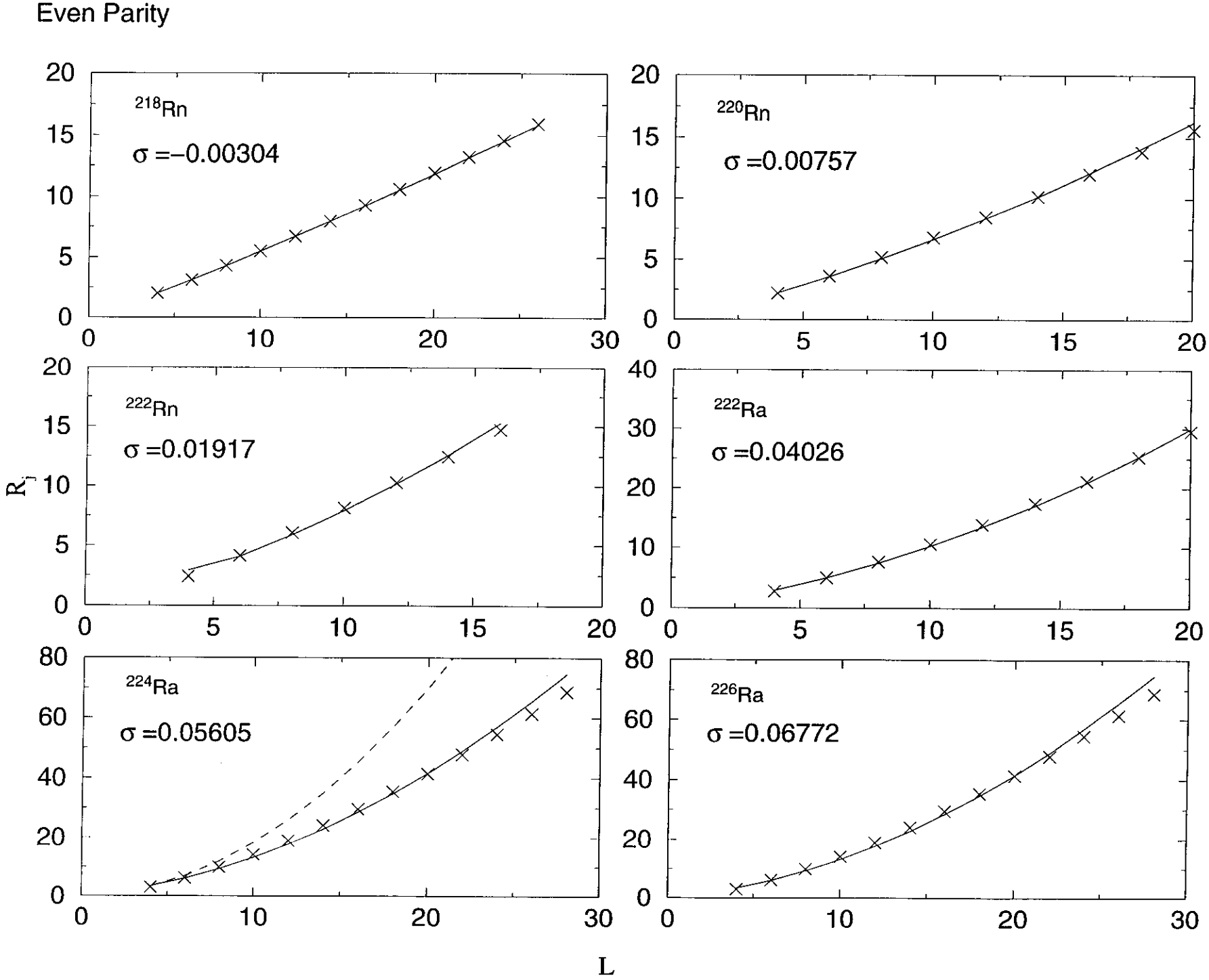
Even Parity
^{218}Rn
σ =–0.00304
^{220}Rn
σ =0.00757
^{222}Rn
σ =0.01917
^{222}Ra
σ =0.04026
^{224}Ra
σ =0.05605
^{226}Ra
σ =0.06772
R_j
L

FIGURE 2

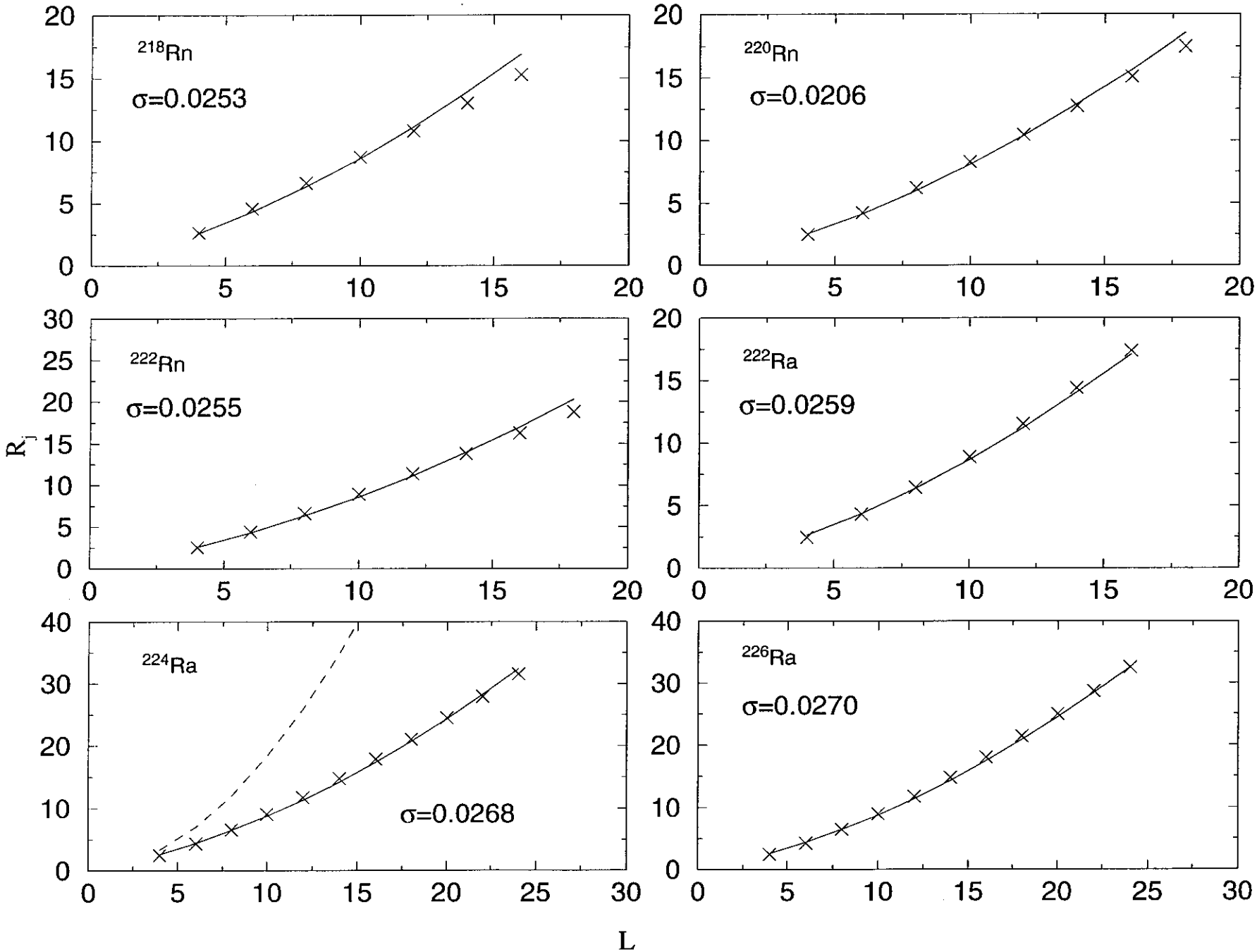
Odd Parity
^{218}Rn
σ=0.0253
^{220}Rn
σ=0.0206
^{222}Rn
σ=0.0255
^{222}Ra
σ=0.0259
^{224}Ra
σ=0.0268
^{226}Ra
σ=0.0270
R_j
L

FIGURE 3