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ANALYSIS OF SPACE-CHARGE EFFECTS IN THE UNIVERSITY OF MARYLAND ELECTRON RING

X. WU, L.V. VOROBIEV, and R.C. YORK

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X. Wu, L. V. Vorobiev, and R.C. York

Michigan State University
East Lansing, MI

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1 Introduction

Evaluations of space-charge effects in the University of Maryland Electron Ring (UMER) are presented. Discussed are the beam dynamics studies of the UMER lattice using COSY INFINITY including turn-by-turn particle tracking and higher-order map and resonance analysis to evaluate the UMER performance under different space-charge conditions.

2 Computer Code

COSY INFINITY[1] is a beam physics code that uses differential algebraic (DA) methods to provide systematic calculations of particle trajectories resulting from passage through arbitrary optical elements. COSY INFINITY allows the computation of dependencies on system parameters and has the ability to compute and use higher-order maps. In 1997, a special version of COSY INFINITY was created at the NSCL to implement a simple (linear) space-charge force model for studies of Final Focus Systems for Heavy Ion Fusion[2]. This modified version of COSY INFINITY was used to simulate and study the UMER lattice assuming a linear space-charge force.

The implemented space-charge model assumes a uniform, isolated beam of elliptical cross-section with semi-major and semi-minor axes of ***a*** and ***b*** respectively, beam current I, and beam Lorentz factors β and γ. The combined electric and magnetic fields generated by the beam space-charge distribution acting on a particle at transverse position (x,y) are given by:

$$E_x = 2F\frac{I}{\beta\gamma^2}\frac{x}{a(a+b)} \;,\; E_y = 2F\frac{I}{\beta\gamma^2}\frac{y}{b(b+a)}$$

Where:

$$F = \frac{1}{2\pi\varepsilon_0 c}$$

The space-charge potential is then given by:

$$\Phi(x,y) = -F\frac{I}{\beta\gamma^2}\left(\frac{x^2}{a(a+b)} + \frac{y^2}{b(a+b)}\right)$$

This space-charge induced potential is used in the modified version of COSY INFINITY to calculate the transfer matrices and maps.

3 UMER Lattice

The ideal UMER lattice without misalignment and magnet errors was used. The lattice tune point without space-charge was chosen to be $\upsilon_x = 7.78$, $\upsilon_y = 7.70$. The initial electron beam emittance was assumed to be 10 π mm-mrad (un-normalized). For the tracking studies, 400 initial particles satisfying the K-V distribution were generated in transverse phase space (x-x' and y-y'). The effective emittances ($4\varepsilon_{x,y}^{rms}$) of these initial particles, where

$$\varepsilon_x^{rms} = \sqrt{<x^2><x'^2> - <x\cdot x'>^2} \;,\; \varepsilon_y^{rms} = \sqrt{<y^2><y'^2> - <y\cdot y'>^2}$$

match the initial beam emittances of 10 π mm-mrad. For this study, the momentum spread Δp/p was chosen to be zero to avoid the complexity of the higher-order chromatic effects.

4 UMER Simulations

4.1 Space-Charge Tune shift

The matched machine functions (β, α) for the UMER lattice for each beam current was determined and a higher-order, one-turn map obtained for particle tracking and resonance analysis. Table 1 gives the resultant matched beam functions for the ideal UMER lattice, and Figure 1 shows the matched beam envelopes for beam currents of 0 and 100 mA. In this analysis, the lattice quadrupole settings were maintained at values appropriate for a tune of υ_x = 7.78, υ_y = 7.70 without space-charge. The average beam size is about 2 mm for a beam current of 0 mA increasing to about 10 mm for the maximum beam current of 100 mA.

Beam Current (mA)	β_x (m)	α_x	β_y (m)	α_y
0	0.12717	0.02304	0.50466	0.04494
1	0.15565	0.02707	0.58452	0.04946
5	0.30226	0.04900	1.05269	0.08185
10	0.52303	0.08324	1.79742	0.13671
20	0.99622	0.15746	3.41177	0.25736
35	1.72242	0.27175	5.89474	0.44373
50	2.45305	0.38686	8.39395	0.63150
75	3.67352	0.57919	12.56907	0.94537
100	4.89518	0.77174	16.74855	1.25960

Table 1. Matched beam conditions with linear space-charge for UMER lattice.

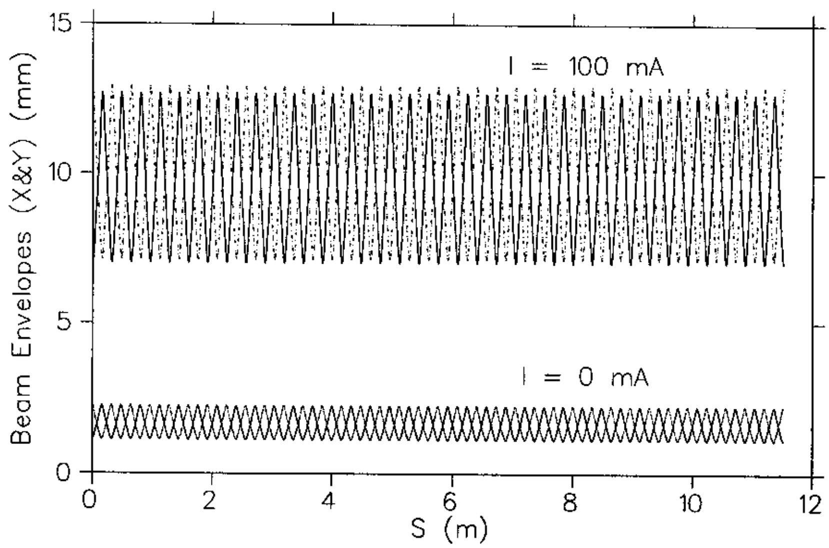

Figure 1. Horizontal (x - solid) and vertical (y - dashed) beam envelopes for beam currents of 0 and 100 mA.

Under the condition of fixed quadrupole strengths, the machine tunes as a function of beam current are given in Table 2. The majority of space-charge tune shift occurs at relatively low beam intensity with the machine tune (υ_x, υ_y) already close to 1.0 for a beam current of only 20 mA. This is caused by the fixed quadrupole strength condition and the beam envelope size increasing with beam current. Figure 2 shows the tune diagram for (υ_x, υ_y) between 0 and 1, and the tune points for currents (35 mA ~ 100 mA) falling within this range.

Beam Current (mA)	υ_x	υ_y
0	7.780	7.700
1	6.526	6.442
5	3.496	3.413
10	2.040	1.977
20	1.075	1.038
35	0.622	0.600
50	0.437	0.421
75	0.292	0.281
100	0.219	0.211

Table 2. UMER lattice tunes vs. beam current.

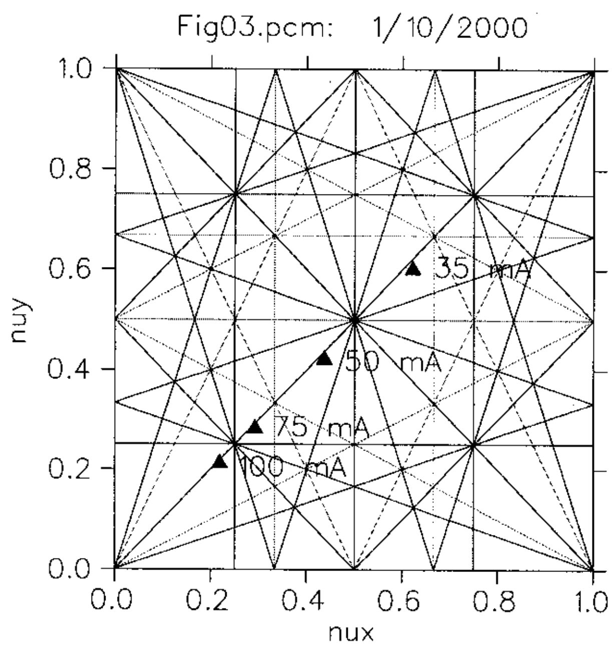

Figure 2. Tune diagram in the range of $\nu_{x,y}$ = 0 to 1. Symbols are evaluated tune points for currents of 35 to 100 mA.

4.2 Turn-by-Turn Particle Tracking

To evaluate the evolution of the transverse phase space, a 5th order, one-turn map was obtained and used to do turn-by-turn particle tracking. Tracking was done for four beam current values (10, 50, 75, and 100 mA) for up to 1000 turns or until significant beam loss occurred.

For a current of 10 mA, particles were tracked for 1, 100, and 1000 turns. No degradation of the transverse phase space was found. For a current of 50 mA, particles were tracked for 1, 100, and 1000 turns and only a small degradation of the transverse phase space were found. For a current of 50 mA, the transverse phase space after 1000 turns is given in Figure 3.

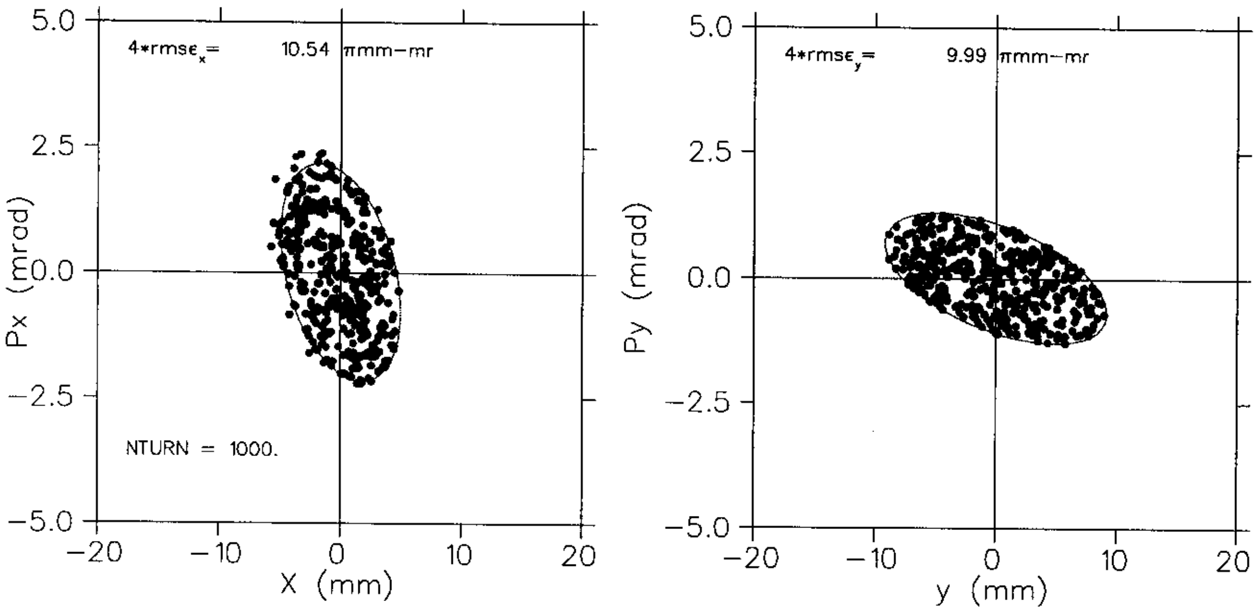

Figure 3. Transverse phase space after 1000 turns for a beam current of 50 mA.

For a current of 75 mA, the particles were tracked for 1, 10, and 78 turns. Significant transverse phase space distortion was seen after only the first turn with a strong increase with turn number. Tracking was terminated after turn 78 due to significant beam loss. Shown in Figure 4 is the phase space after turn 78 for a beam current of 75 mA. An even greater transverse phase space distortion was observed for a beam current of 100 mA. The distortion was significant after only one turn, and the growth rate was much faster with tracking stopped after only 3 turns due to particle loss. Shown in Figure 5 is the phase space after turn 3.

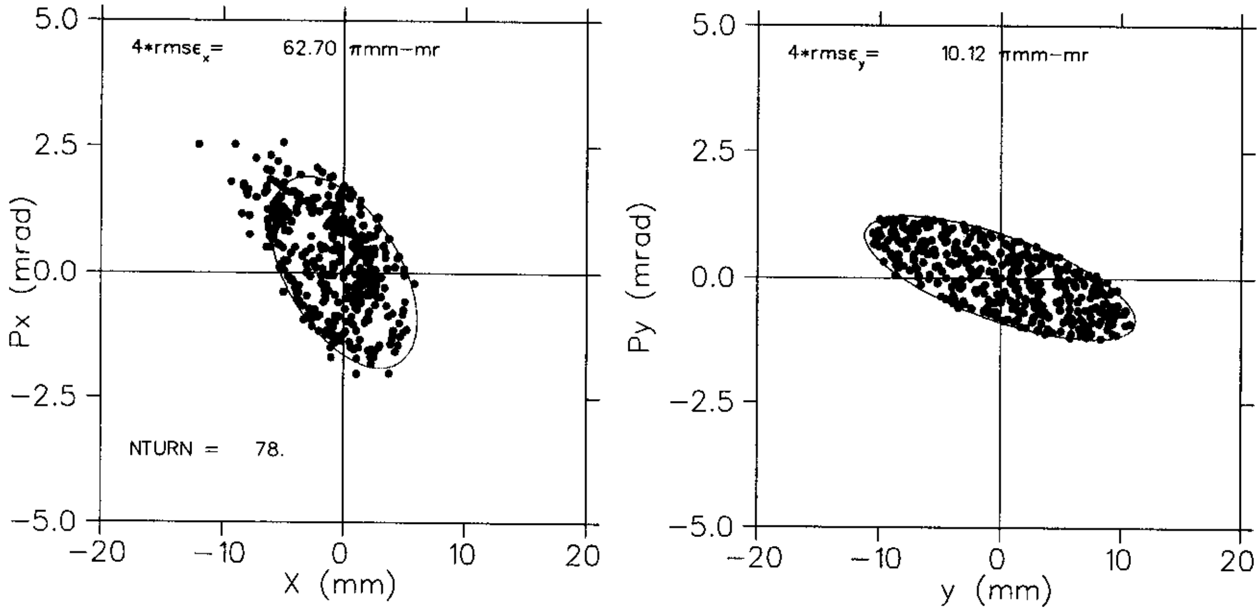

Figure 4. Transverse phase space after 78 turns for a beam current of 75 mA.

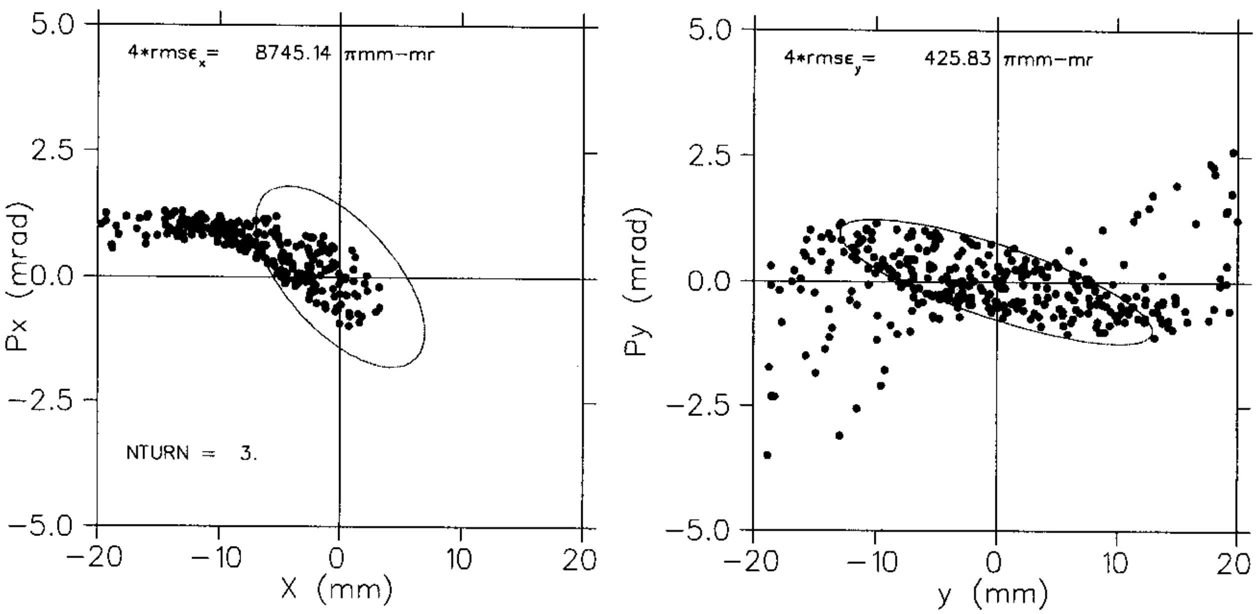

Figure 5. Transverse phase space after 3 turns for a beam current of 100 mA.

4.3 Map and Resonance Analysis

To understand the basis of the beam-current-driven degradation of the transverse phase space, a resonance analysis was done. A semi-log plot of the higher-order resonance strengths (2^{nd} to 5^{th}) as a function of beam current is given in Figure 6. The significant increase of resonance strengths with beam current is consistent with the increased transverse phase space distortion with beam intensity.

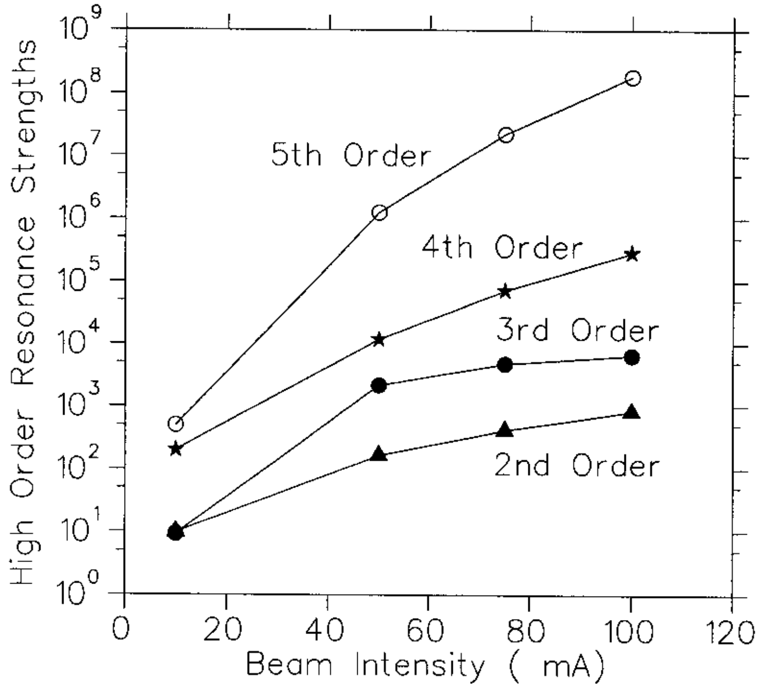

Figure 6. Higher-order resonance strengths as a function of beam intensity.

5 Lattice Tune Adjustment Effects

In the previous analysis, the quadrupole settings were held fixed regardless of beam current. As a result, the machine tune varied with beam current ranging from $\nu_x = 7.78$, $\nu_y = 7.70$ for zero beam current to $\nu_x = 0.22$, $\nu_y = 0.21$ for the maximum beam current of 100 mA. To reduce the tune variation, the quadrupoles were adjusted for different beam currents to provide an indication of whether the transverse phase space dilution was resonance-driven or whether it is an artifact space-charge-driven higher-order geometric terms.

The lattice quadrupoles were adjusted to achieve a tune point of $\nu_x = 0.45$, $\nu_y = 0.42$ for all four beam currents as shown in Figure 7. An additional tune point of $\nu_x = 0.62$, $\nu_y = 0.60$ was achieved for all beam currents excepting the 100 mA case for which a tune of only $\nu_x = 0.467$, $\nu_y = 0.455$ could be achieved with these tune points given in Figure 8.

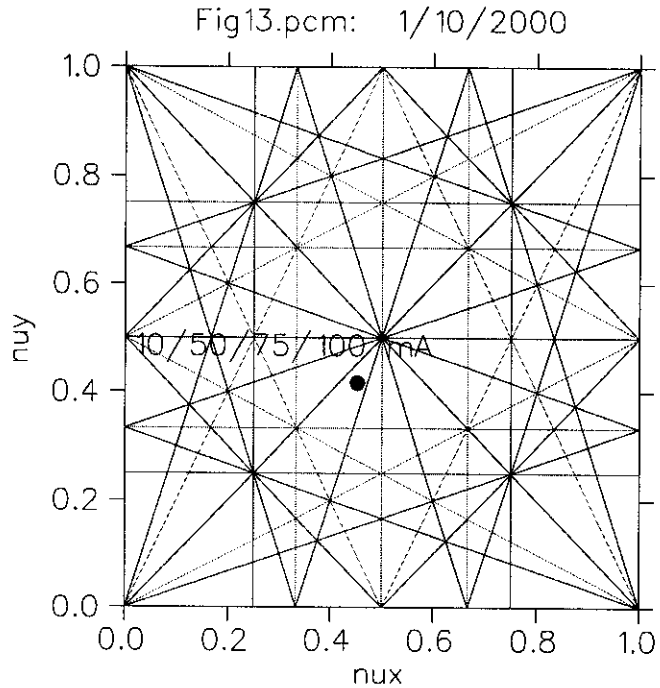

Figure 7. Machine tune diagram over range of 0 to 1. The black circle is the position of the tune point achieved for beam currents of 10 to 100 mA.

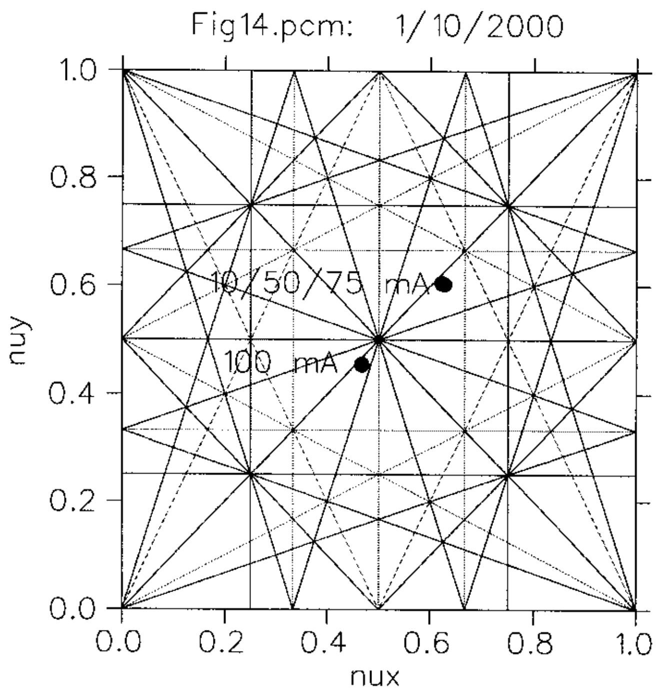

Figure 8. Machine tune diagram over range of 0 to 1. The black circles show the position of the tune points achieved for beam currents of 10 to 100 mA.

COSY INFINITY was again used to obtain 5th order, one-turn maps for turn-by-turn tracking and resonance analysis. The tracking results were virtually unchanged. For example, the maximum turn number achieved for the 75 and 100 mA cases were 80 and 4 respectively whereas the previous analysis achieved 78 and 3 turns respectively. In addition, the resonance strengths were also found to be similar. For example, the resonance strengths as a function of beam current for all three cases discussed for 3rd and 5th order are given in Figure 9 and Figure 10 respectively. Similar results were obtained for the 2nd and 4th order resonance strengths.

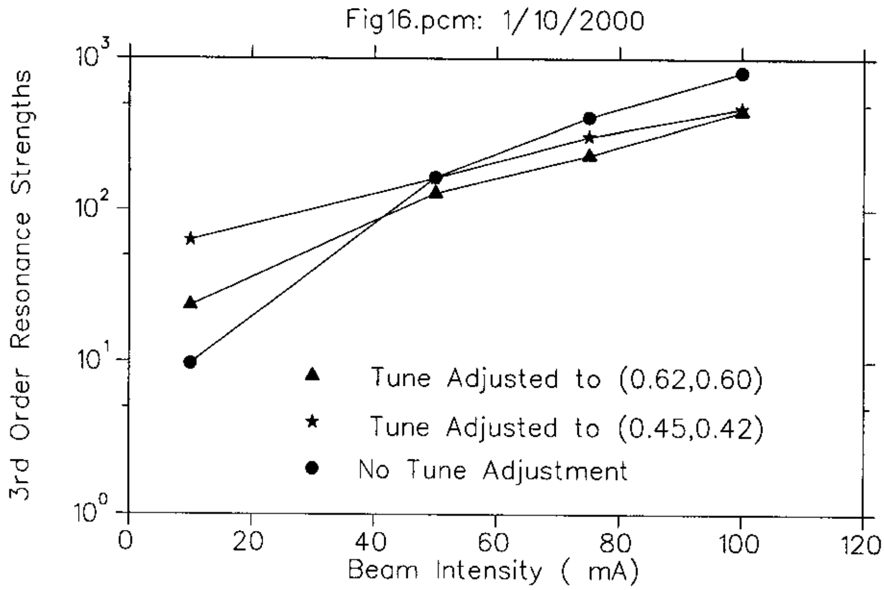

Figure 9. 3rd order resonance strengths as a function of beam current for the three tune conditions evaluated.

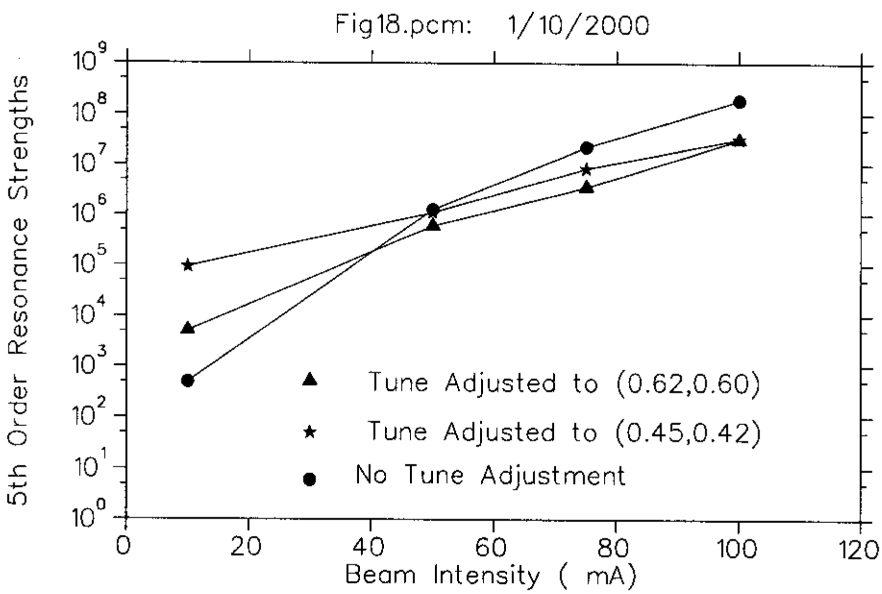

Figure 10. 5th order resonance strengths as a function of beam current for the three tune conditions evaluated.

6 Summary

A space-charge modified version of COSY INFINITY code was used to study the effects of space-charge on the UMER lattice performance. The studies used a linear space-charge model based on the beam-current-dependent matched beam envelopes and the ideal UMER lattice without misalignment and magnet errors. Higher-order lattice maps were used for resonance analysis and for turn-by-turn particle tracking of initial K-V distributions. The beam momentum spread was not included in this analysis, but could be added in later studies.

The results indicate that for low beam intensity (<50 mA) operation, beam-intensity-driven, higher-order geometric aberration terms are small and machine operation should be adequate. For the higher beam intensities evaluated (75 ~ 100 mA), there is a substantial increase in these higher-order terms causing significant beam emittance growth which may limit the turn number. Substantiating the view of an increase in the intensity-driven geometric terms, adjustment of the tune point had only a small effect yielding similar turn-by-turn tracking results.

Further studies are planned using a new, 2D, self-consistent space-charge code[3] developed at MSU and the SIMPSON computer code[4] to verify the result and expand the analysis using a more general (non-linear) space-charge model.

7 References

1 M. Berz, "Users Guide and Reference Manual for COSY Infinity", MSUCL-97, Michigan State University, 1995.

2 X. Wu and R. C. York et al, "Design and Simulation Studies of a One-tenth Scale Final Focus System for Heavy Ion Fusion", IEEE Particle Accelerator Conference, Vancouver, Canada, p. 195, (1997).

3 L. V. Vorobiev and R. C. York, "Numerical Study of the Injection Line for the University of Maryland Electron Ring", MSUCL-1136, Michigan State University, 1999.

4 S. Machida, "Space-Charge Calculations in Synchrotrons", IEEE Particle Accelerator Conference, Washington, DC, p. 3224, (1993).