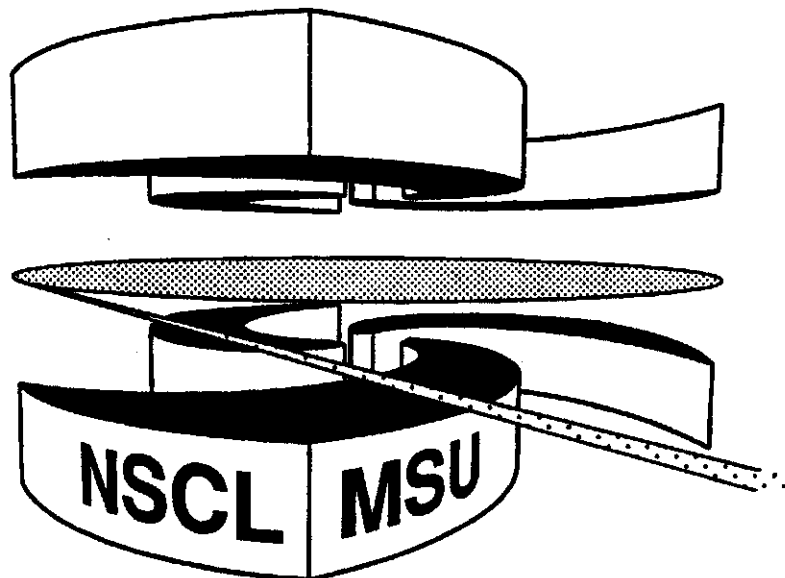


**MICHIGAN STATE  
UNIVERSITY**

**National Superconducting Cyclotron Laboratory**

**EFFECT OF CLUSTER FORMATION ON ISOSPIN  
ASYMMETRY IN THE LIQUID-GAS  
PHASE TRANSITION REGION**

**L. SHI and P. DANIELEWICZ**



**MSUCL-1161**

**JUNE 2000**

# **Effect of Cluster Formation on Isospin Asymmetry in the Liquid-Gas Phase Transition Region**

L. Shi and P. Danielewicz

*National Superconducting Cyclotron Laboratory and Department of Physics and Astronomy,  
Michigan State University, East Lansing, MI 48824*

(May 2, 2000)

## **Abstract**

Abstract: Nuclear matter within the liquid-gas phase transition region is investigated in a mean-field two-component Fermi-gas model. Following largely analytic considerations, it is shown that: (1) Due to density dependence of asymmetry energy, some of the neutron excess from the high-density phase could be expelled into the low-density region. (2) Formation of clusters in the gas phase tends to counteract this trend, making the gas phase more liquid-like and reducing the asymmetry in the gas phase. Flow of asymmetry between the spectator and midrapidity region in reactions is discussed and a possible inversion of the flow direction is indicated.

PACS. 25.70.Mn - Projectile and target fragmentation.

PACS. 25.70.Pq - Multifragment emission and correlations.

PACS. 21.65.+f - Nuclear matter.

One interesting possibility in heavy-ion collisions at intermediate energy is the occurrence of a liquid-gas phase transition. Many recent papers addressed this possibility from different perspectives [1–7]. Following an elementary consideration, it is easy to envisage a first-order phase transition in infinite nuclear matter. Müller and Serot first pointed out the importance of isospin for the liquid-gas phase transition [5]. The additional isospin degree of freedom relaxes the system and makes the transition of second-order. Isospin observables could generally be used to extract a variety of information from heavy-ion collision, see for example the review by Li *et al.* [8,9]. Some recent data analyses tried to explore isospin observables and to relate them to a possible occurrence of the phase transition [10].

One focus of interest in connection with the phase transition is the midrapidity region in intermediate-energy heavy-ion collisions [10,12,13]. In simulations of semiperipheral collisions, a formation of low-density neck region is observed that likely contributes to the midrapidity [11]. The low-density region in contact with high-density regions (the projectile and target) opens up the possibility for a liquid-gas phase coexistence and phase conversion. In a dynamical simulation with the Boltzmann-Uehling-Uhlenbeck equation, Sobotka *et al.* [11] observed neutron enrichment in the low-density neck region. However, a high  $n/p$  ratio (much higher than in the composite system) was found when counting only free nucleons in the neck region, *i.e.* excluding nucleons in clusters. The paper argued that the symmetric clusters (deuterons and alphas) contributed much to the enrichment of neutron in the neck region. Specific results were purely numerical in nature.

In this paper we shall discuss the isospin asymmetry in the phase transition region in a heavy-ion collision and the effect of clusters on that asymmetry. We will first follow crude statistical arguments, and then construct a model illustrating the same ideas.

In the general discussion, let us first allow no cluster formation in an isospin-equilibrated heavy-ion reaction. For a given temperature and density, a large isospin asymmetry will increase the total energy, which is unfavorable. In a dense phase, the extra energy for maintaining the same asymmetry will be much larger than in a dilute phase. Thus, if a dilute phase is in isospin equilibrium with a dense phase, the asymmetry in the dilute phase

will be larger. For the scenario of a neck region neighbored by a dense region in heavy ion collision, the  $n/p$  ratio in the liquid phase is close to that of the whole system, while  $n/p$  ratio in the gas phase could be much larger than in the composite system. Next, we consider letting the clusters be formed in the gas phase. Then the available phase space for liquid does not change while the phase space for the gas phase increases. The added phase space, which corresponds to clusters, has an  $n/p$  mean value lower than the old phase space for gas. From a statistical equal-partition point of view, partition in the new liquid and gas phase space will drive the whole gas phase more symmetric. If the percentage of clusters is small, however, then there is essentially not much change in the phase space distribution, and asymmetry in the gas phase excluding clusters should not change much.

Now let us build a simple model and show how isospin equilibrates between the two phases. We may start with a two component non-interacting Fermi gas of neutrons and protons, and represent the interaction by an energy density consistent with the empirical nuclear equation of state (EOS). For simplicity we assume no temperature dependence for the interaction energy, and the Coulomb interaction is not considered here. The total free energy of the system is then a sum of the free energies of two non-interacting Fermi gases and of a density-dependent nuclear potential energy. For a single phase at temperature  $T = 0$  and density  $\rho$ , the free energy per nucleon may be written as:

$$f = \mathcal{F}/A = a_1 (\rho/\rho_0)^{2/3} + a_2 (\rho/\rho_0) + a_3 (\rho/\rho_0)^{\sigma-1} + (a_4 (\rho/\rho_0)^{2/3} + a_5 (\rho/\rho_0)) y^2 \quad (1)$$

where  $\rho_0$  is the normal density and  $y$  is the asymmetry parameter,  $y = (N - Z)/(N + Z)$ . For the moment, it is assumed that  $|y| \ll 1$ . The  $(\rho/\rho_0)^{2/3}$  terms come from the non-interacting Fermi gas. The terms  $a_2 (\rho/\rho_0) + a_3 (\rho/\rho_0)^{\sigma-1}$  are associated with a simple parameterization of the nuclear EOS [14–16]. As we are only concerned with the isospin asymmetry in the liquid-gas phase transition, details of the parameterization do not affect our later discussion (though the exact numerical results may change). Given that the interaction generally contributes to the asymmetry energy [9], we adopt a simple parameterization in Eq. (1) for that contribution, of the form  $a_5 (\rho/\rho_0) y^2$ . At  $T > 0$ , the free energy could not be written

in a simple analytic form, but we can still expand the free energy per nucleon about  $y = 0$ . This expansion yields the net free energy of the form:

$$f = \mathcal{F}/A = f_0 + f_y = f_0 + Cy^2 \quad (2)$$

where  $f_0$  and  $C$  are functions of both temperature and density. The second term on the r.h.s. of (2) is due to isospin asymmetry and may be called asymmetric free energy. Since our model is symmetric with respect to proton-neutron interchange, the expansion of the free energy contains no odd powers of  $y$ . In our numerical calculation, we use  $\rho_0 = 0.16 \text{fm}^{-3}$ ,  $\sigma = 2.1612$ ,  $a_2 = -183.05 \text{MeV}$ ,  $a_3 = 144.95 \text{MeV}$ ,  $a_5 = 11.72 \text{MeV}$ , and at  $T = 0$ ,  $a_1 = 22.10 \text{MeV}$ , and  $a_4 = 12.28 \text{MeV}$  ( $a_4 + a_5 \simeq 25 \text{MeV}$  could be obtained from optical potential analysis [17] or from the mass formula [18]). Numerical analysis indicates that a quadratic form in  $y$  is adequate up to almost  $y = 1$  (a similar conclusion has been reached in [19]). Figure 1 shows the calculated the asymmetry coefficient  $C$  as a function of density and temperature in the Fermi gas. The general trend is that  $C$  increases with increasing density and temperature. Therefore, at a given temperature, a dense phase will need more extra energy for maintaining a given asymmetry than a dilute phase.

Now we can consider a system that has two phases of liquid and gas, respectively, in contact with each other. The total free energy will be a sum of the free energies for the two phases. Let us assume that the mechanical and thermal equilibrium has been achieved, and now we only consider the isospin equilibrium between the two phases. Keeping the total asymmetry of the system fixed, we need to vary the asymmetry in the liquid and gas phases to minimize total free energy. This yields the equilibrium condition:

$$C_l y_l = C_g y_g \quad (3)$$

Here  $C_l$  and  $C_g$  denote the asymmetric free energy coefficients in the liquid phase and in the gas phase. At a given temperature, the liquid phase is denser than the gas phase, and the coefficient  $C$  is a monotonously increasing function of density,  $C_l > C_g$ . Thus, the asymmetry in the gas phase  $y_g$  is always larger than that in the liquid phase  $y_l$ . To characterize the

relative asymmetry of the two phases, we may define the isospin asymmetry amplification ratio:

$$R = C_l/C_g = y_g/y_l \quad (4)$$

Figure 2 displays  $R$  vs. temperature for the phases in equilibrium. For our model calculation, the ratio  $R$  stays always larger than 1, which means that the gas phase will always have a higher neutron content than the liquid phase. Notably the amplification ratio is independent of the net isospin asymmetry of the whole system. The ratio  $R$  decreases as temperature increases, so that a large  $n/p$  ratio in the gas phase is easier reached at a low temperature.

In the case of a nonequilibrium process, Eq. (3) is still of a use due to a variational origin of the equation. If a local equilibrium assumption is met, *i.e.* statistical variables are still valid locally, then Eq. (3) tells us the direction of development for the system. The gradient of asymmetry coefficient could result in a net flow of isospin asymmetry, which tries to restore the isospin equilibrium condition Eq. (3). The flow direction is to the steepest decrease of asymmetry coefficient  $C$ . If there is a gradient of density in a nonequilibrium system, hence a gradient of asymmetry coefficient (see Fig. 1), then there could be a flow of isospin asymmetry in the system, with the direction to the low density region.

We know that, if the nucleon density is not too low, the mean field description is quite good. But when the density is low, particle-particle correlations become important, and the validity of a mean field description worsens. One way to incorporate particle correlations is to allow for the formation of clusters in the system (as is done in the BUU calculations [20]). Since clusters are in practice only important for the gas phase, we will only allow clusters there and no clusters in the liquid phase at all. To further simplify the discussion, we shall adopt a droplet model for the clusters (as used by Goodman [4] and many others). We will assume that droplets have the same properties as the liquid phase, that is the same density and asymmetric coefficient; for the present discussion we shall ignore the surface energy term. Suppose the average size of droplets is  $A$ , and asymmetry in terms of average

proton and neutron numbers in droplets is  $y_d$ . The density of nucleons in clusters may be represented as  $\rho_d = \alpha\rho$  and of free nucleons as  $\rho_f = (1 - \alpha)\rho$ , where  $\rho$  is the density of the gas phase. The asymmetric free energy of the new (free nucleons + droplets) gas phase is:

$$f_y = (1 - \alpha) C_f y_f^2 + \alpha C_d y_d^2. \quad (5)$$

Here, the subscripts  $f$  and  $d$  refer to free nucleons and droplets, respectively. To get the isospin equilibrium condition, we can carry out a similar variation of asymmetry parameters in the liquid, free-nucleon gas, and in droplets, as before, obtaining:

$$y_d = y_l, \text{ and } y_f/y_l = C_l/C_f. \quad (6)$$

As the density of the gas phase is low, we may use the ideal gas EOS  $p = \rho T$  for clusters in a calculation. And adding clusters will necessarily decrease  $\rho_f$  in order to satisfy the mechanical equilibrium condition. However, in Fig. 1 we can see that  $C$  decreases only slightly as density decreases. To first order, we can take  $C_f \simeq C_g$ , so that  $y_f$  is nearly the same as the in old gas phase. Overall, the asymmetry of the new gas phase is:

$$y = \alpha y_d + (1 - \alpha) y_f. \quad (7)$$

This may be compared to the asymmetry for the old gas phase,  $y_g \approx y_f$ , which is much larger than  $y_d = y_l$ . It is clear that the more droplets are added to the gas phase, the more it looks like the liquid phase. The amplification ratio now is:

$$R = y/y_l = \alpha + (1 - \alpha) C_l/C_f \approx \alpha + (1 - \alpha) R_0. \quad (8)$$

where  $R_0 = C_l/C_g \gg 1$ . The case of  $\alpha = 0$  corresponds to no cluster formation in the reaction, and the isospin amplification ratio reaches then the maximum  $R_0$ . The gas phase acquires then the largest possible net asymmetry at a given temperature. On the other hand,  $\alpha = 1.0$  corresponds to the gas phase with only clusters and the same net asymmetry as for the liquid phase.

Figure 3 shows the decrease of the amplification factor  $R$  as a function of the cluster concentration  $\alpha$ . As we add more clusters, the low-density gas phase will need more energy

for the same isospin asymmetry, comparable with that of the liquid phase. As a result, the density and asymmetry in clustered gas will both approach those in the liquid phase.

Short of simple tools to estimate typical relative numbers of free neutrons, free protons and clusters in the gas phase in a reaction, we may seek help from experiments. Different regions of velocity space are generally believed to reveal characteristics of different sources, such as the midrapidity particle source for the low-density neck region. Several intermediate-energy experiments pointed out to a neutron-rich midrapidity source in peripheral heavy-ion collisions [10,13,21]. Sobotka *et al.* [23] measured neutron and  ${}^4\text{He}$  emission from a midrapidity source formed in mid-central  ${}^{129}\text{Xe} + {}^{120}\text{Sn}$  collisions at 40MeV/nucleon. They compared their results with results of the INDRA collaboration for the same system [21,24] and gave a quantitative description of the midrapidity source. About half of the charged particles from this source are  ${}^4\text{He}$  and only 10% are free protons. Similiar results have been obtained in other papers [13,21,24,10]. The number of neutrons is approximately the same as the number of charged particles, or 10 times the number of protons in this source [23]. If we take the average cluster size in the midrapidity as about 5 [25], then the percentage of nucleons inside clusters will be  $\alpha \sim 80\%$ . The  $N/Z$  ratio for the midrapidity source is found to be higher than for the full system [23]. Thus the midrapidity source has  $(N/Z)_{mid} \sim 1.65$  or  $y_{mid} \sim 0.25$  while the system has  $(N/Z)_{sys} \sim 1.39$  or  $y_{sys} \sim 0.16$ . The asymmetry amplification ratio is then  $R \sim 1.5$ . For a mid-rapidity source formed in peripheral heavy-ion collision at similiar energy, a fully consistent comparision of different experiments is not easy. Nevertheless, comparison of the peripheral data from [10,13,22] also suggests a high cluster concentration and a high  $n/p$  ratio for free neutron and proton.

In our model calculation, Fig. 3 shows that for the cluster concentration  $\alpha$  as high as 80%, the asymmetry amplification ratio  $R$  will decrease by more than a half when compared with the nonclustered gas phase. This large decrease of  $R$  will largely limit the isospin asymmetry in the gas phase when the asymmetry in the liquid phase is fixed. Sobotka *et al.* [23] extracted the temperature for the midrapidity source as 6–7MeV. For this temperature and the cluster concentration  $\alpha \sim 80\%$ , we can read off from Fig. 3 the corresponding



equilibrium value as  $R \sim (1.9 \sim 2.1)$ . This value is higher than the extracted  $R \sim 1.5$  in the experiment, which means that the system only achieved a partial isospin equilibrium and the asymmetry amplification in the gas phase did not reach its full value.

While this kind of equilibrium consideration generally give some limits for the importance of cluster formation on isospin asymmetry in the liquid and gas phase transition region, the developement of isospin asymmetry in heavy-ion collision is essentially a nonequilibrium process which deserves more thorough investigations than can be comprised in this letter, possibly incorporating simulations. So we shall only give some general discussion of the possible isospin asymmetry developement in the system.

Because of the transient nature of heavy-ion collision, the development of isospin equilibrium could depend on two time scales. One time scale is for the separation of the midrapidity source from the remaining sources, and the other is for isospin equilibration. At high enough energy, the three sources separate quickly before isospin equilibration could set in between sources. The isospin asymmetry is then determined by the reaction geometry and the isospin content of the target and projectile. Isospin equilibration and cluster formation operate only as post-reorganization processes, changing only isospin asymmetry for free nucleon and clusters within individual sources. The large isospin asymmetry for free nucleons could be the result of clusterization in the low-density phase, with clusters taking over the role of the liquid phase, consistently with the arguments by Sobotka *et al.* [11]. From our previous discussion, the  $R$  ratio in Fig. 2 sets an upper limit to the asymmetry of the free nucleons in the midrapidity source.

On the other hand, if the energy is low enough, partial isospin equilibrium will set in before different sources separate from each other, and the reaction scenario becomes more complex. As the two heavy ions collide against each other, initial compression of the participants produces a dense phase in the center, while two spectators remain less dense. As the asymmetry coefficient for the dense phase is larger than for the less dense phase (cf. Fig. 1) at the interfaces between the two spectator regions and participant region, there could be a local density gradient from the center out to the two spectators. From

our arguments following Eq. (3), we know there could then appear an isospin asymmetry flow, and it would be out to the two spectators. As the compression stage ends, the center region begins to expand, and the density drops, the asymmetry coefficient also drops as a result. When the gradient of the asymmetry coefficient changes direction, the flow of isospin asymmetry changes direction too. Cluster formation in the center region counteracts the decrease of the asymmetry coefficient, and thus delays the change of flow direction. Further development of the system separates the three sources, and net isospin asymmetries for different sources do not change after the separation. But clusterization still plays a role changing the isospin asymmetry of free nucleons within individual sources. Since dynamical simulations suggest a much longer expansion time than the compression time, we could expect that the isospin asymmetry flow to the midrapidity region dominates. This could give rise to an enhanced asymmetry in the midrapidity region. The experiments also suggest a neutron-rich midrapidity source, which is consistent with the present picture.

In conclusion, we have investigated the isospin asymmetry in the nuclear liquid-gas phase-transition region. In the framework of the two-component Fermi-gas with a parameterized interaction, under the assumption of isospin equilibrium, we found that a neutron enrichment in the gas phase is due to the density-dependent part of the asymmetry energy. Meeting the isospin equilibrium condition, Eq. (3), drives extra neutrons out to the low-density phase. The formation of clusters, which have average asymmetry smaller than the gas phase, will make the gas phase more liquid-like, and counteract the neutron enrichment in the gas phase. The  ${}^4\text{He}$  clusters will be the most important due to their predominance in the neck region [13,10]. Based on the isospin equilibrium requirement, isospin asymmetry flow was suggested if there exists a local density gradient in heavy-ion collisions. Since the midrapidity undergoes compression and expansion, we also suggested a possible change of the direction of the isospin asymmetry flow during the evolution of the system.

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This work was partially supported by the National Science Foundation under Grant

PHY-9605207.

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## FIGURES

FIG. 1. The asymmetry coefficient  $C$  as a function of density and temperature. The lines, from bottom to top, correspond to the temperature of 0, 2, 4, 6, 8, 10 and 12 MeV, respectively.

FIG. 2. The amplification factor  $R$  for the liquid-gas phase transition, as a function of temperature.

FIG. 3. The amplification factor  $R$  as a function of cluster concentration  $\alpha$ . The lines from top to bottom are for the temperatures of 5, 6, 7, 8, 9, and 10 MeV, respectively.

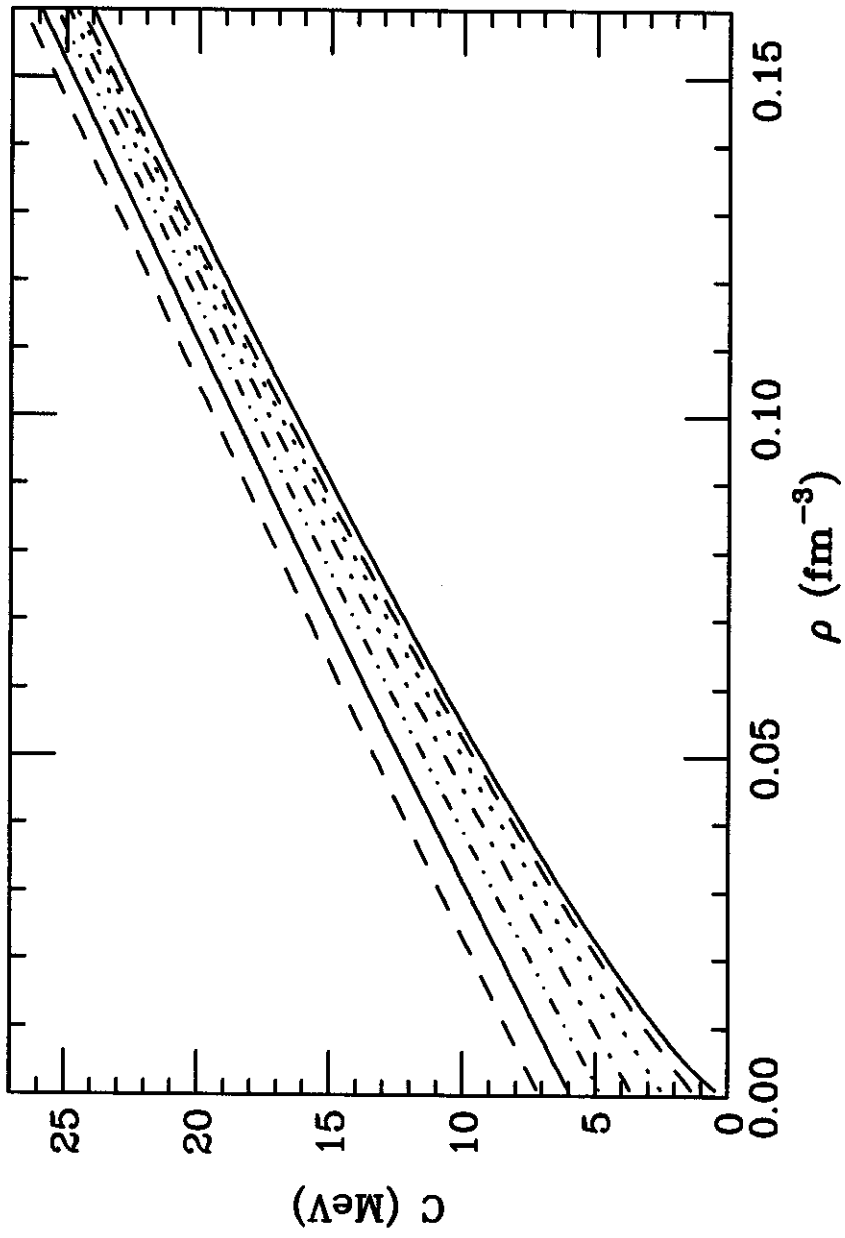


FIG. 1

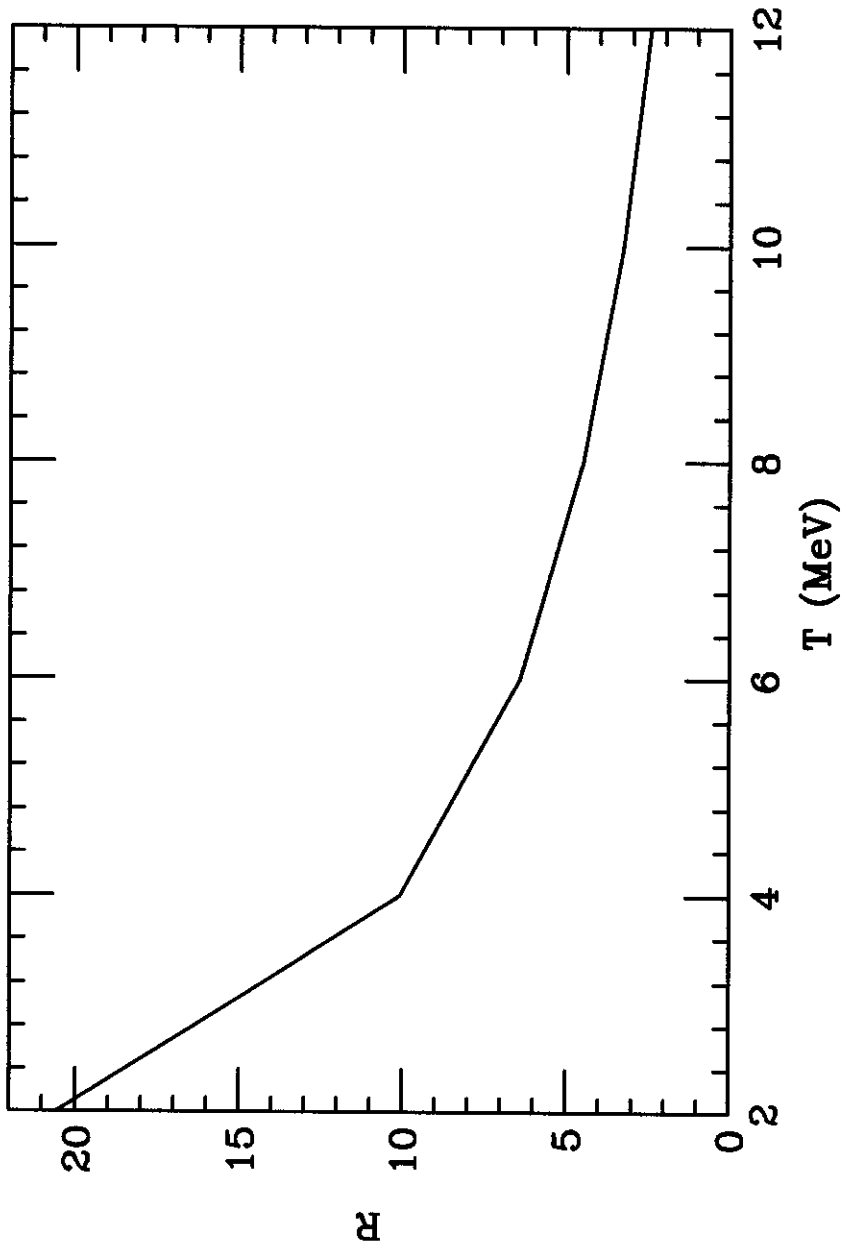


FIG. 2



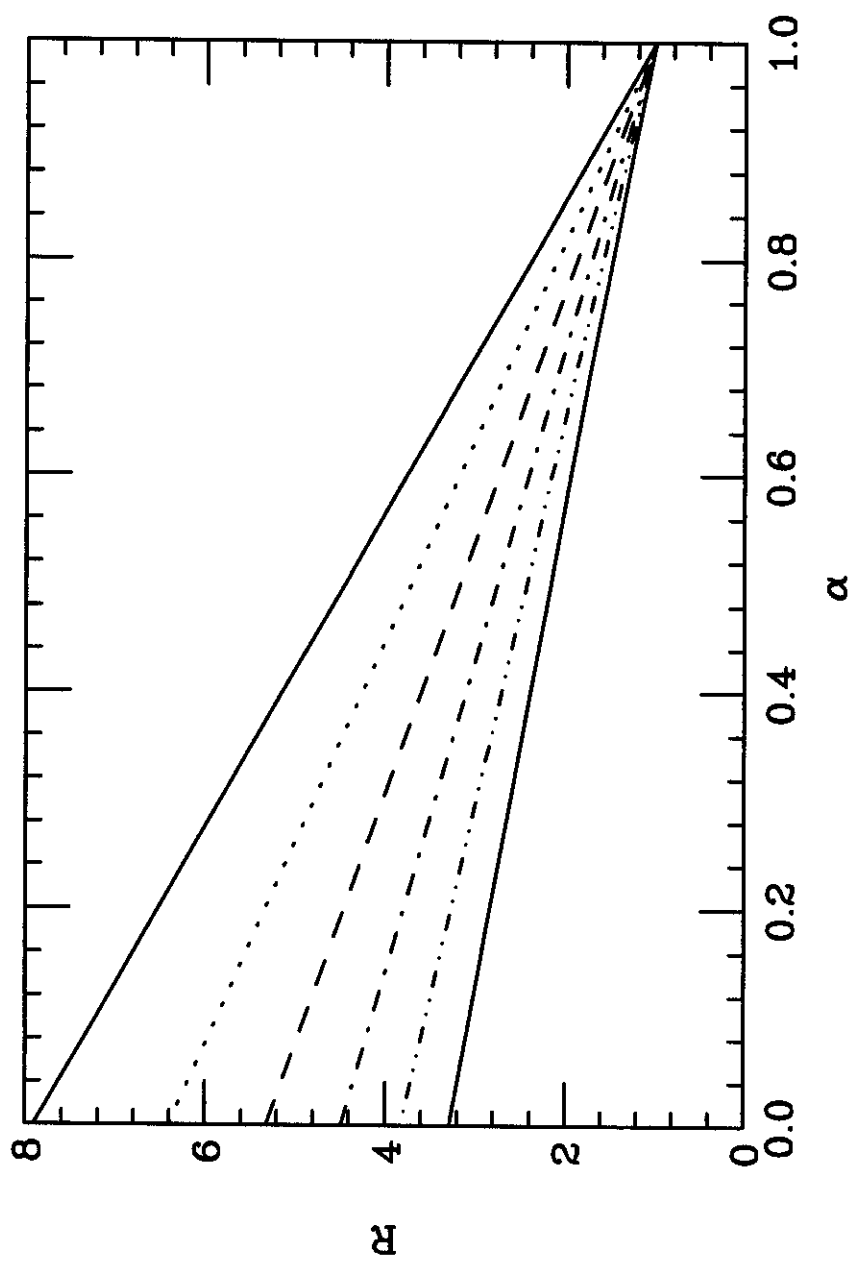


FIG. 3