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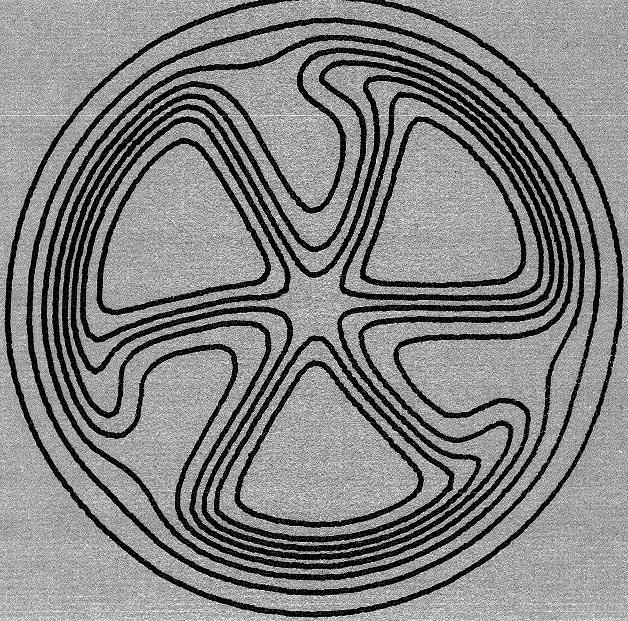

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Prediction of Weak-Coupling Structure from a Shell-Model Basis*

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Mixed-configuration shell-model wave functions predict the sort of two-nucleon transfer phenomena which has been interpreted as evidence for weak coupling to excited 0^+ states.

We present in this note calculations with shell-model wave functions for 0d,1s-shell nuclei which indicate than an apparent weak-coupling phenomenon recently observed experimentally in the 0f,1p shell is, indeed, attributable to weak coupling and, moreover, may be a general feature of the structure of light-and medium-mass nuclei.

The experimental observations we refer to have demonstrated that in several instances,[1,2] the (p,t) reaction on odd-mass fp-shell nuclei populates both the ground state and an excited state of the residual nucleus with pure L=0 angular distributions. This establishes, of course, that both the residual states have J^π equal to the J^π of the target state ($7/2^-$ in the examples studied), but the significance of the observations lies in the absence of any general selection rule which would limit the angular momentum transfer in such cases to <u>only</u> L=0. The explanation of these phenomena thus must arise from specific nuclear structure properties of the states involved.

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The purity of the L=0 transitions to the ground states presumably originates from the dominance of J=0 pairing in forming nuclear ground states. The last, "odd", nucleons of the odd-mass nuclei A+3 and A+1 are pictured as weakly-coupled spectators to the 0^+ ground-state wave functions of the adjacent even-mass nuclei A+2 and A, and the transition between the A+3 and A+1 $J^\pi=7/2^-$ ground states is viewed, essentially, as the pickup of the same correlated J=0 pair by which the A+2 and A ground states are connected. This picture seems empirically verified by the approximate equality of the cross sections of the A+3→A+1 and A+2→A (p,t) ground-state transitions. Weak coupling in such a context is more or less implicitly assumed in all approaches to nuclear structure, just as it is in the very familiar analogous situation involving the "single-particle" states of a nucleus A+1 and the 0^+ ground state of nucleus A.

The question of the extent to which a weak-coupling picture of nuclear structure can be generalized beyond the ground-state 0^+ wave functions is less well settled, except for some nice examples around ^{208}Pb which involve either nearly unique collective states or very pure single-particle states.[3,4] While the simplicity and intuitive appeal of weak-coupling analyses have prompted a considerable number of attempts to formulate interpretations of nuclear structure in the $A \lesssim 100$ region in terms of weak coupling, generally to first excited 2^+ states of even-mass nuclei, the applications of these theories have not been conclusively successful.[5-9] The clearest experimental inference for

such weak-coupling behavior has hitherto come from inelastic scattering studies in the sd-shell.[10]

The L=0 (p,t) transitions observed to excited states in odd-mass fp-shell nuclei yield a new kind of direct evidence that a weak-coupling analysis can indeed be extended to certain select excited states in the complex medium- and light-mass region of nuclei. The $J^\pi=7/2^-$ excited states in the A+1 nuclei which are populated by the pure L=0 transitions come at energies closely correlated with the energeis of the first excited 0^+ states in the adjacent A nuclei. Moreover, the differential cross sections of the A+3→A+1 (excited $7/2^-$ state) transition are comparable (generally within a factor of two) or those of the A+2→A (excited 0^+ state) transitions, similar to the situation observed for the even and odd ground-state transitions. On this basis, it has been postulated[1,2] that the excited A+1, $J^\pi=7/2^-$ states in question have wave-functions corresponding to $\psi(0_2^+) \otimes \rho_{j=7/2,\ell=3}$, namely a $f_{7/2}$ nucleon coupled to the wave function of the first excited 0^+ state in nucleus A.

Successful nuclear structure calculations for fp-shell nuclei which encompass all six of the relevant states in one of the A, A+2, A+1, A+3 groups in which this weak-coupling phenomenon has been studied to date do not exist. Hence it has not been possible to attempt an interpretation of the observed phenomena with quantitative wave functions. However, the data so far accumulated suggest that this kind of behavior might be quite ubiquitous, and suggest attempts to find it in regions outside the fp shell.

Shell-model wave functions for sd-shell nuclei which have been thoroughly tested and found to be quite successful in accounting for a wide range of nuclear phenomena have been described in the literature.[11,12] We were accordingly prompted to investigate these wave functions to see if the sort of weak-coupling structure experimentally observed in the fp shell might be theoretically predicted in the sd-shell.

In this note we show that, indeed, mixed-configuration shell-model calculations for sd-shell nuclei predict the same sort of (p,t) phenomena as has been observed experimentally in the fp shell. Thus, the shell-model wave functions appear to implicitly contain the sort of correlations whose external manifestations we call "weak coupling". Moreover, with these rather thoroughly validated wave functions in hand, we are able both to test the internal consistancy of the weak-coupling picture by calculating overlaps which cannot be experimentally measured and to make some initial hypotheses as to the nature of the "excited-core" states to which the coupling occurs.

The specific cases we have considered so far are the correlated odd-and even-mass (p,t) transitions $^{31}P \to {}^{29}P$, $^{30}Si \to {}^{28}Si$ and $^{35}Cl \to {}^{33}Cl$, $^{34}S \to {}^{32}S$. We present here the results for the Cl-S case. Shown in Fig. 1 are theoretical differential cross sections for $^{34}S(0_1^+) \to {}^{32}S(0_{1,2}^+)$ and $^{35}Cl(3/2_1^+) \to {}^{33}Cl(3/2_{1,2,3,4}^+)$. The assumed bombarding energy is 40 MeV and the optical model parameters used (see Table I) are ones adapted from the literature of elastic scattering analyses. They have been checked against experimental

data for $^{29}Si \to {}^{27}Si$, where they yield good fits to L=0 transitions.[13] The two-nucleon transfer amplitudes used in computing these cross sections were obtained from $d_{5/2}-s_{1/2}-d_{3/2}$ shell-model wave functions described in the literature.[11,12]

Obviously, the (p,t) predictions single out the third $3/2^+$ state of ^{33}Cl as being associated with the first 0^+ excited state of ^{32}S, just as the ground states of the two systems are clearly associated. The match-ups in relative intensities (see Table II and Fig. 1) and the purity of the L=0 nature of odd odd transitions (see Fig. 1) are extremely similar to what has been experimentally observed for the fp-shell nuclei. The calculated excitation energies are, for ^{32}S, 3.68 MeV, and for ^{33}Cl, 2.31, 3.61 and 4.39 MeV. Hence the match-up in energy between the excited 0^+ state and the $3/2^+$ state with the L=0 distribution is also suggestive of weak coupling. (The calculated excitation energies are in good agreement with the experimental values of 3.78 for ^{32}S and 2.35, 3.98 and 4.11 or 4.47 for ^{33}Cl.[14])

It is known from experiment that the ground state of ^{33}Cl does indeed closely resemble a $d_{3/2}$ proton coupled to the ground-state wave function of ^{32}S. The single-nucleon spectroscopic factor is a measure of the exactness of this picture, and the value of 0.70 calculated for the transition between these states from the shell-model wave functions, a number consistent with experimental values, confirms, as expected, that $^{33}Cl(3/2_1^+) = {}^{32}(0_1^+) \otimes d_{3/2}$ is a very accurate description. (The maximum possible spectroscopic factor is something less than 1.0 since the shell-model wave functions realistically do not predict completely closed $s_{1/2}$ and $d_{5/2}$ orbits for the ground state of ^{32}S.)

We are able to complete the verification of a weak-coupling explanation of the two-nucleon transfer phenomena in a rather novel way by simply calculating the single-nucleon $d_{3/2}$ spectroscopic factor for the transition $^{32}S(0_2^+)\rightarrow{}^{33}Cl(3/2_3^+)$. A matrix of theoretical spectroscopic factors for the transitions between $^{32}S(0_1^+$ and $0_2^+)$ and $^{33}Cl(3/2_{1-4}^+)$ is presented in Table III. Again it is evident that $^{33}Cl(3/2_3^+)$ has a special relationship to $^{32}S(0_2^+)$. The spectroscopic factor of 0.36 is much the largest such value after the ground states overlap value of 0.70. It indicates that the approximation $^{33}Cl(3/2_3^+) = {}^{32}S(0_2^+) \otimes d_{3/2}$ is also an accurate approximation, although only about 50% as exact as the same type of relation was for the ground states.

Calculations for $^{30}Si\rightarrow{}^{28}Si$ and $^{31}P\rightarrow{}^{29}P$ yield much the same picture. Here, $1/2^+$ states are associated with the ground and first excited 0^+ states. The two-nucleon and one-nucleon overlaps are all only about 75% as good as those of the S-Cl example and the L=0 purity of the odd-odd (p,t) distributions is not significant, because $1/2^+\rightarrow 1/2^+$ transitions require L=0 rigourously.

The most clear-cut of the fp-shell experimental examples of this sort of phenomena appear to involve excited states whose wave functions involve particle-hole excitations, in which sd-shell particles are jumped into fp-shell orbits. The theoretical wave functions we use here involve only sd-shell orbits. However, both

examples we have studied so far do involve minor shell closures, $d_{5/2}$ at ^{28}S and $s_{1/2}$ at ^{32}S. Hence the excited 0^+ states do have a mild "particle-hole" type of structure relative to the ground state. Whether this is an essential element for the manifestation of the weak-coupling structure, or whether the phenomena occurs for essentially all first excited 0^+ states is one of the interesting points for further study.

Of course, the primary need now is to obtain experimental tests of our predictions for two-nucleon transfer. If these are validated, then it would appear that further experimental work to fully delineate the extent of this pleasingly transparent sort of nuclear coupling is in order, and that a more extensive study of the shell-model wave functions, with the aim of projecting out all their "weak-coupling" features, would be valuable.

TABLE I.--Optical Model parameters used in calculating (p,t) cross sections.

	V	W	W'	r_o	a	r'_c	r'_o	a'
proton	45.5		13.0	1.20	0.70	1.25	1.25	0.70
triton	173.9	20.6		1.15	0.72	1.40	1.50	0.82

Table II. Relative integrated cross sections for $^{34}S(p,t)^{32}S(0^+_{1,2})$ and $^{35}Cl(p,t)^{33}Cl(3/2^+_{1,2,3,4})$

$^{34}S \rightarrow {}^{32}S$		$^{35}Cl \rightarrow {}^{33}Cl$	
$E_x(0^+)$	σ_{int}	$E_x(3/2^+)$	σ_{int}
0.00	100.0	0.00	69.7
3.68	13.8	2.31	1.2
		3.61	21.4
		4.39	3.5

Table III. Single-nucleon spectroscopic factors between 0^+ states in ^{32}S and $3/2^+$ states in ^{33}Cl.

	$(3^+/2)_1$	$(3^+/2)_2$	$(3^+/2)_3$	$(3^+/2)_4$
$^{32}S(0^+_1)$	0.70	0.05	0.00	0.00
$^{32}S(0^+_2)$	0.06	0.07	0.36	0.03
$^{32}S(0^+_3)$	0.00	0.01	0.00	0.05
$^{32}S(0^+_4)$	0.00	0.00	0.03	0.01

Figure Caption

Figure 1 Predicted angular distributions for the (p,t) reaction on targets of ^{34}S and ^{35}Cl, leading to 0^+ states and $3/2^+$ states, respectively. A proton energy of 40 MeV is assumed.

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