submitted to Phys. Rev. C

# The structure of the "island of inversion" nucleus $^{33}$Mg

B.V. Pritychenko[1,2†], T. Glasmacher[1,2], P.D. Cottle[3] R.W. Ibbotson[1‡], K.W. Kemper[3],
L.A. Riley[4], A. Sakharuk[1§] H. Scheit[1,2**], M. Steiner[1], and V. Zelevinsky[1,2]

[1] *National Superconducting Cyclotron Laboratory,*
*Michigan State University,*
*East Lansing, MI 48824*
[2] *Department of Physics and Astronomy,*
*Michigan State University,*
*East Lansing, MI 48824*
[3] *Department of Physics,*
*Florida State University,*
*Tallahassee, FL 32306*
[4] *Department of Physics and Astronomy,*
*Earlham College,*
*Richmond,*
*Indiana 47374*

(Dated: May 17, 2002)

## Abstract

The excitation of the 485 keV state of the neutron-rich "island of inversion" nucleus $^{33}$Mg was measured in intermediate-energy Coulomb excitation. The result of the present experiment suggests that the 485 keV state is a rotational excitation built on the ground state, not a state with different intrinsic structure as proposed previously. If the 485 keV state is indeed a rotational excitation, then the deformation of $^{33}$Mg is similar to that of other nuclei in the island of inversion.

PACS numbers: 25.70.De, 27.30.+t, 23.20.Jc, 21.60.Cs

---

[†] Present address: Plumtree Software, Inc., San Francisco, CA 94111

[‡] Present address: Brookhaven National Laboratory, Upton, NY 11973

[§] Present address: Laboratory for Instructional Technology in Education, Michigan State University, East Lansing, MI 48824

[**] Present address: Max-Planck-Institut für Kernphysik, Postfach 10 39 80, D-69029 Heidelberg, Germany

The "island of inversion" in the neutron-rich isotopes near $^{32}$Mg is a spectacular example of shape coexistence in nuclei, a severe test of our understanding of the mechanism of this phenomenon, and an important challenge for experimental techniques using radioactive beams. In the "normal" configurations of these nuclei, the *sd* neutron shell is full. Only $N - 20$ neutrons are located in the $fp$ shell and the nucleus is either spherical or weakly deformed. An "intruder" configuration can be formed by promoting a pair of neutrons across the $N = 20$ shell closure into the $fp$ shell. These intruder configurations are strongly deformed, but in a naive picture the intruder configuration has a higher energy than the normal configuration because of the energy necessary to promote the neutron pair across the $N = 20$ gap. In "island of inversion" nuclei, which appear to include $N = 20, 21$ and 22 isotopes of Ne, Na and Mg, the normal and intruder configurations are inverted, with the strongly deformed intruder configuration becoming the ground state. The experimental signatures of nuclei in the island include spectra of excited states typical of highly deformed nuclei [1–3], strongly collective electromagnetic transition strengths [2, 4–7], and binding energies that are underpredicted by shell model calculations that only include normal configurations [8–15]. Here we report a measurement of excitation of the 485 keV state in the $N = 21$ isotope $^{33}$Mg by the technique of intermediate energy Coulomb excitation [16] of a beam of this exotic isotope. The result of the present experiment, which is the first measurement of a transition strength in this nucleus, suggests that the 485 keV state is a rotational excitation built on the ground state in this nucleus, and that $^{33}$Mg has a deformation similar to that of other nuclei in the island of inversion. A recent $\beta$-decay study of $^{33}$Mg [17] has suggested that the 485 keV excited state has a different intrinsic structure from the ground state. In this latter study the ground state of $N = 21$ $^{33}$Mg is assigned a configuration with one core neutron excited ($1p$-$1h$) accross $N = 20$ and coupled to the valence neutron, while the structure of the 485 keV excited state is attributed to the coupling of the valence neutron to a ($2p$-$2h$) core excitation. This results in a positive parity for the ground state and negative parity for the excited state in [17]. The study reported here suggests that the 485 keV state is a rotational excitation built on the ground state in this nucleus, and that

$^{33}$Mg has a deformation similar to that of other nuclei in the island of inversion.

The experiment was performed at the National Superconducting Cyclotron Laboratory at Michigan State University. The secondary beam of $^{33}$Mg was produced with a primary beam of 80 MeV/nucleon $^{48}$Ca from the laboratory's K1200 cyclotron. The primary beam was fragmented on a $^{9}$Be production target of thickness 376 mg/cm$^2$ located at the mid-acceptance target position of the A1200 fragment separator [18]. The energy of the secondary $^{33}$Mg beam was 61.8 MeV/nucleon.

A 702 mg/cm$^2$ $^{197}$Au foil was used as the secondary target. The secondary beam slowed significantly in this target, and the beam energy used in the analysis of the $\gamma$-ray cross section was that in the middle of the target, 53.8 MeV/nucleon. The secondary beam was stopped in a cylindrical fast/slow plastic phoswich detector located at zero degrees. Both energy loss in the phoswich detector and time of flight relative to the cyclotron RF signal were used for particle identification. The zero-degree detector subtended the scattering angles of 0° to 2.80° in the laboratory. The number of beam particles detected in the zero-degree detector was $1.7 \times 10^6$.

Photons were detected in coincidence with the zero-degree detector by the NSCL NaI(Tl) array [19]. The $\gamma$-ray spectra measured in coincidence with beam particles identified as $^{33}$Mg in the zero-degree detector are shown in Fig. 1. The upper panel shows the background subtracted spectrum in the laboratory frame. No strong peaks appear, although there is a concentration of counts over the background level between 300 and 600 keV. These counts result in part from the 547 keV $7/2^+ \rightarrow 3/2^+_{gs}$ $\gamma$-ray in the $^{197}$Au target. The lower panel shows the corresponding spectrum in the projectile frame (that is, with an event-by-event Doppler shift). The projectile-frame spectrum includes a strong $\gamma$-ray at 478(5) keV and weaker peaks near 900 and 1400 keV. A 484.9(10) keV $\gamma$-ray was identified with $^{33}$Mg in the $\beta$-decay of $^{33}$Na (although not placed in a level scheme) in Ref. [1] and we assume these two gamma-rays to be the same. A more recent $\beta$-decay study [17] placed this $\gamma$ ray as deexciting a state at 485 keV to the ground state. This $\gamma$ ray was also observed and assigned to $^{33}$Mg in the direct fragmentation study of this nucleus by Yoneda *et al.*

[3]. In the present experiment, the 485 keV $\gamma$ ray was measured to have a cross section of $81 \pm 25$ mb. The quoted error is dominated by statistical uncertainty. Other significant contributions arise from the detector efficiencies and the unknown angular distribution of the $\gamma$-ray. This cross section is integrated over laboratory-frame scattering angles from 0° to 2.80°, the range of angles covered by the phoswich detector.

The two weaker peaks are the 885 keV and 1436 keV $\gamma$-rays in $^{32}$Mg [5], which is populated in the present experiment by the stripping of a neutron from $^{33}$Mg by the $^{197}$Au target. Knockout reactions with cross sections of this magnitude have been observed in studies of other nuclei with the same experimental method [20, 21]. The identity of these $\gamma$ rays was tested by gating on events that have a lower total energy in the zero-degree detector because the residual $^{32}$Mg nuclei (after stripping of a neutron from $^{33}$Mg) have a lower total kinetic energy than the intact $^{33}$Mg nuclei. Figure 2 illustrates the result of this analysis. The upper frame shows the higher energy gate in which the residual nuclei from neutron stripping should be suppressed. In this frame, the 485 keV $\gamma$ ray is quite visible. In the lower frame, which is the lower energy gate and in which the neutron stripping residuals should be enhanced, the peaks at 885 and 1436 keV are relatively strong.

There is no experimental information on spins and parities in $^{33}$Mg. However, we can use experimental data from other nuclei in the island of inversion to suggest spins and parities in $^{33}$Mg. The ground state spin of $^{31}$Na has been experimentally determined to be 3/2 [22]. With only 11 protons, Na is well down in the *sd* shell, so it is very likely that the negative parity orbits of the *fp* shell are not available to protons in the ground states of Na isotopes, even at the large deformations characteristic of the island of inversion. Hence, we can make a strong argument that the parity of the ground state of $^{31}$Na is positive. Calculations of single-particle energies in the deformed shell model [5, 23] support this point of view. Both $^{31}$Na and $^{33}$Na are in the island of inversion, and it is likely that they have similar deformations. Therefore, the odd proton is likely in the same single particle state in the ground states of both nuclei, so that the ground state spins and parities are the same as well. By this line of reasoning, we can argue that the ground state of $^{33}$Na likely has $J^{\pi} = 3/2^{+}$.

Nummela *et al.* [17] measured the log *ft* value of the $\beta$-decay of the $^{33}$Na ground state to the ground state of $^{33}$Mg to be 5.27(27). This log *ft* value indicates an allowed transition with no change in parity and either no change in spin or a change of one unit. We conclude from this that the ground state of $^{33}$Mg probably has positive parity and a spin of 1/2, 3/2 or 5/2.

For the present study, the issue of the nature of the 485 keV state is central. Given the significant cross section for observing the 485 keV $\gamma$ ray in the present experiment, we arrive at the conclusion that it is either a very strong $E1$ transition or a strong $E2$ transition characteristic of a rotational excitation built on the ground state. The $\beta$-decay study of Nummela *et al.* [17] provides constraints on the nature of the 485 keV state. They did not observe direct population of the 485 keV state via $\beta$-decay at all, but they were able to set a lower limit for the log *ft* value for the 485 keV state of 6.6. In contrast to the case of the ground state, the $\beta$-decay to the 485 keV state is not allowed. We cannot exclude a parity change from the $^{33}$Na ground state or a spin change as large as two units. However, the possibilities for the spin and parity of the 485 keV state are somewhat constrained by the present reaction because only $E1$ and $E2$ excitations can yield cross sections as large as that observed here.

Nummela *et al.* [17] explain the large discrepancy between the log *ft* values for the ground state and 485 keV state by proposing that the latter state has negative parity. Under these circumstances, the 485 keV state would be populated in the present reaction via an $E1$ excitation. Nummela *et al.* suggest $J^\pi = 3/2^+$ for the ground state of $^{33}$Mg and $J^\pi = 3/2^-$ for the 485 keV state, although the data do not exclude $J^\pi = 1/2^+$ and $5/2^+$ for the ground state and $J^\pi = 1/2^-, 5/2^-$ and $7/2^-$ for the 485 keV state.

On the other hand, it is possible that the 485 keV state is a rotational excitation built on the ground state, and that it is excited via an $E2$ transition. Under the rotational scenario, the 485 keV state would have a spin one unit greater than that of the ground state, since a two unit excitation at such a low energy would give a moment of inertia that is unphysically large. However, this constraint leads to a dilemma. The $\beta$ transition to the ground state is allowed, so we would expect that the $\beta$ transition to the 485 keV state (which has the same parity and microscopic structure as the ground state in the rotational scenario) would be allowed as well, unless the spin change from the ground state of $^{33}$Na to the 485 keV state in $^{33}$Mg is at least two units. If the ground state of $^{33}$Na has $J^\pi = 3/2^+$, then the only $^{33}$Mg $J^\pi$ assignments that would satisfy the $\beta$-decay data in the rotational scenario would be $5/2^+$ for the ground state and $7/2^+$ for the 485 keV state.

To obtain the electromagnetic matrix elements for populating the 485 keV state in the rotational scenario we use the coupled channels code ECIS88 [24], which takes into account both electromagnetic and nuclear contributions to scattering reactions. Barrette *et al.* [25] extracted optical model parameters from their study of the $^{17}$O+$^{208}$Pb reaction at 84 MeV/nucleon; we adopt these parameters for the present study. The code's standard axially symmetric deformed rotational form factor was used. The analysis matches the integrated cross section measured for laboratory frame scattering angles 0° to 2.80° to that calculated using ECIS for the same range of scattering angles.

An ECIS calculation includes two deformation parameters. The "Coulomb deformation" $\beta_C$ denotes the deformation of the proton density in the nucleus, while the "nuclear matter deformation" $\beta_A$ is used to determine the nuclear force contribution to the reaction and involves transition densities for both neutrons and protons. To relate the two deformation parameters, we assume a simple collective model picture in which the deformation lengths $\delta_C = \beta_C R_C$ and $\delta_A = \beta_A R_A$ are equal. An alternative assumption, $\beta_A = \beta_C$ (see e.g. [4]), yields deformation parameters in agreement with the ones quoted below.

To extract the $\beta_2$ values for exciting the 485 keV state (which we assume to have $J^\pi = 7/2^+$) in the rotational scenario, we must know the cross section for directly populating this state in the scattering reaction. However, the scattering reaction would also populate the $J^\pi = 9/2^+$ member of the rotational band, which could then $\gamma$-decay to the 485 keV $7/2^+$ state (although this transition has not been observed in the present experiment). The cross section for the observation of the 485 keV $\gamma$ ray would then include both the cross section for directly populating the 485 keV state and the feeding from the $9/2^+$ state. In our analysis we must take this into account. We perform ECIS calculations for direct excitation of both the $7/2^+$ and $9/2^+$ states and constrain the deformation parameters for excitation of these two states to be equal because they are assumed to be members of the same rotational band. In addition to the uncertainty in the experimental cross section ($\sigma$=81(25) mb)), the uncertainties quoted for the excitation strengths also take into account the possibility that the $9/2^+$ state may decay to the $7/2^+$ state anywhere from 0% to 100% of the time. The error in the measured cross section contributes about 2/3 of the total error quoted below. The results are independent of the energy of the $9/2^+$ state within the quoted total errors.

The ECIS analysis yields the result $\beta_C = 0.52(12)$ and $\beta_A = 0.58(13)$. This result is similar to those found for other nuclei in the island of inversion, $^{31}$Na ($\beta_C = 0.59(10)$ [2]), $^{32}$Mg (the results reported for this nucleus are $\beta_C = 0.52(4)$ by Motobayashi *et al.* [4], $\beta_C = 0.44(5)$ by Pritychenko *et al.* [5] and $\beta_C = 0.61(4)$ by Chisté *et al.* [6]) and $^{34}$Mg ($\beta_C = 0.58(6)$ [7]). The present result for $\beta_C$ in $^{33}$Mg yields $B(E2; 5/2^+ \rightarrow 7/2^+)$=232(107) e$^2$fm$^4$ using the relationships given by the rotor model [26].

We now consider the possibility that the 485 keV state could be populated via an $E1$ excitation. The nuclear force contribution to an $E1$ excitation in the reaction used here is negligible (see e.g. [27]). Therefore, we extract the reduced matrix element $B(E1; 3/2^+_{g.s.} \rightarrow 3/2^-)$ from the experimental cross section using the relativistic Coulomb excitation theory of Winter and Alder [28]. The analysis (described in [16]) gives the result $B(E1; 3/2^+_{g.s.} \rightarrow 3/2^-) = 0.035(10)$ e$^2$fm$^2$, or $5.3(15) \times 10^{-2}$ Weisskopf units (W.u.). As in the analysis using the rotational scenario, the analysis matches the integrated cross section measured for laboratory frame scattering angles 0° to 2.80° to the calculated cross section.

The $E1$ matrix element obtained here is very large. The recommended upper limit given by Endt for this mass region [29] is 0.1 W.u. According to the 1993 compilation of $\gamma$ ray strengths by Endt [29], only four of 1026 $E1$ transitions in the A=21-44 mass region are as strong as or stronger than the $E1$ strength determined here for $^{33}$Mg. These include the 4170 keV→2425 keV ($3/2^- \rightarrow 1/2^+$) transition in $^{21}$Na ($7(2) \times 10^{-2}$ W.u.), the 8655 keV→4935 keV ($1/2^+ \rightarrow 3/2^-$) transition in $^{29}$Si ($5(2) \times 10^{-2}$ W.u.), the 4343 keV→ground state ($3/2^- \to 1/2^+$) transition in $^{29}$P ($3.8(5) \times 10^{-2}$ W.u.), and the 7781 keV→6428 keV ($1^- \to 2^+$) transition in $^{34}$S ($4.5(12) \times 10^{-2}$ W.u.).

In short, the rotational scenario yields a result which fits neatly with the systematic behavior of $E2$ strengths in the island of inversion. In contrast, the $E1$ analysis gives a result that would be surprising, although it would not quite violate previously established empirical limits. On this basis, we argue that the 485 keV state is more likely to be a rotational excitation. The $\beta$-decay data of Numela *et al.* [17] are consistent with the rotational scenario only if the ground state spin of $^{33}$Mg is $J^\pi = 5/2^+$.

In this case the odd neutron must be located in the [202 5/2] Nilsson orbit. We can examine whether the present results can be understood in the simple picture of a [202 5/2] neutron strongly coupled to the $^{32}$Mg core by performing a calculation using the particle-rotor model. If we take the moment of inertia $I$ and deformation parameter $\delta$ for the $^{32}$Mg core to be those reported in Ref. [5] ($1/2I = E(2^+_1)/6 = (0.8855\text{ MeV})/6$, and $\delta = 0.48$) and the oscillator parameter $\omega = (41\text{ MeV})/A^{1/3} = 12.78$ MeV, then the model gives $E(7/2^+) = 419$ keV, which is not far from the experimental result of 485 keV. The model also yields the electromagnetic matrix element $B(E2; 5/2^+ \to 7/2^+)$=286 e$^2$fm$^4$, which is in good agreement with the experimental result of 232(107) e$^2$fm$^4$. The calculation assumes an effective charge of the valence neutron of $e_{\text{eff}} = (Z/A)_{^{32}\text{Mg}}$. A small variation (10%) of the oscillator frequency $\omega_{\text{osc}}$ reduces the calculated value to $B(E2\uparrow)$=260 e$^2$fm$^4$.

In summary, we have measured the excitation of the 485 keV $\gamma$-ray in $^{33}$Mg via intermediate-energy Coulomb excitation. Our results suggest that the 485 keV state is a $J^\pi = 7/2^+$ rotational excitation built on a $J^\pi = 5/2^+$ ground state, with a deformation comparable to other island of inversion nuclei.

This work was supported by the National Science Foundation under grants PHY-9875122, PHY-9970991, PHY-0070911, PHY-0110253, and the State of Florida.

## REFERENCES

[1] D. Guillemaud-Mueller *et al.*, Nucl. Phys. **A426**, 37 (1984).

[2] B.V. Pritychenko *et al.*, Phys. Rev. **C63**, 011305(R) (2001).

[3] K. Yoneda *et al.*, Phys. Lett. B **499**, 233 (2001).

[4] T. Motobayashi *et al.*, Phys. Lett. B **346**, 9 (1995).

[5] B.V. Pritychenko *et al.*, Phys. Lett. B **461**, 322 (1999).

[6] V. Chisté *et al.*, Phys. Lett. B **514**, 233 (2001).

[7] H. Iwasaki *et al.*, Phys. Lett. B **522**, 227 (2001).

[8] C. Thibault *et al.*, Phys. Rev. **C12**, 644 (1975).

[9] W. Chung and B.H. Wildenthal, Phys. Rev. **C22**, 2260 (1980).

[10] A. Watt *et al.*, J. Phys. G **7**, L145 (1981).

[11] C. Détraz *et al.*, Nucl. Phys. **A394**, 378 (1983).

[12] D.J. Viera *et al.*, Phys. Rev. Lett. **57**, 3253 (1986).

[13] A. Poves and J. Retamosa, Phys. Lett. B **184**, 311 (1987).

[14] E.K. Warburton, J.A. Becker and B.A. Brown, Phys. Rev. **C41**, 1147 (1990).

[15] E. Caurier, F. Nowacki, A. Poves and J. Retamosa, Phys. Rev. **C58**, 2033 (1998).

[16] T. Glasmacher, Annu. Rev. Nucl. Part. Sci. **48**, 1 (1998).

[17] S. Nummela *et al.*, Phys. Rev. **C64**, 054313 (2001).

[18] B.M. Sherrill *et al.*, Nucl. Inst. and Meth. **B56**, 1106 (1991).

[19] H. Scheit, T. Glasmacher, R.W. Ibbotson, and P.G. Thirolf, Nucl. Inst. and Meth. **A422**, 124 (1999).

[20] R.W. Ibbotson *et al.*, Phys. Rev. Lett. **80**, 2081 (1998).

[21] B.V. Pritychenko *et al.*, Phys. Rev. **C62**, 051601(R) (2000).

[22] G. Huber *et al.*, Phys. Rev. **C18**, 2342 (1978).

[23] S. Péru, M. Girod, and J.F. Berger, Eur. Phys. J A **9**, 35 (2000).

[24] J. Raynal, Phys. Rev. **C23**, 2571 (1981)

[25] J. Barrette *et al.*, Phys. Lett. B **209**, 182 (1988).

[26] P. Ring and P. Schuck, *The Nuclear Many-Body Problem* (Springer-Verlag, New York, 1980), p. 125.

[27] T. Motobayashi, Nucl. Phys. **A 682**, 345c (2001).

[28] A. Winter and K. Alder, Nucl. Phys. **A319**, 518 (1979).

[29] P.M. Endt, At. Data Nucl. Data Tables **55**, 171 (1993).

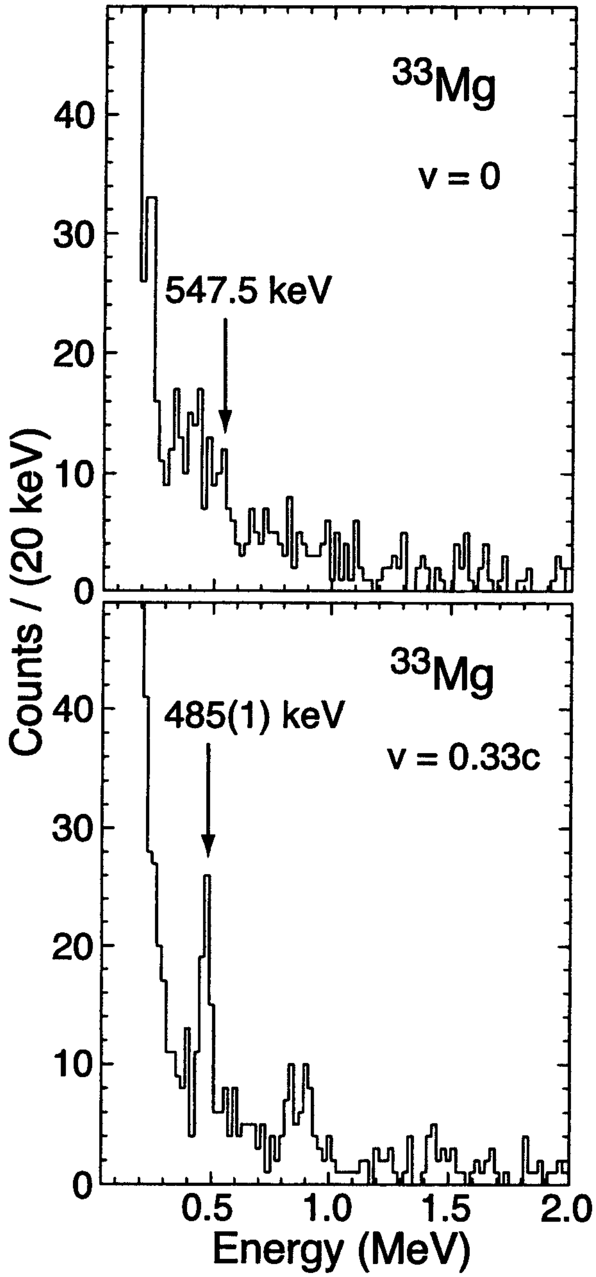

FIG. 1: The $\gamma$ ray spectrum gated on $^{33}$Mg particles. The upper panel shows the laboratory-frame spectrum with counts from the 547 keV $\frac{7^+}{2} \rightarrow \frac{3^+}{2}$ transition in the gold target visible. The lower panel illustrates the projectile-frame spectrum which is adjusted for Doppler shifts.

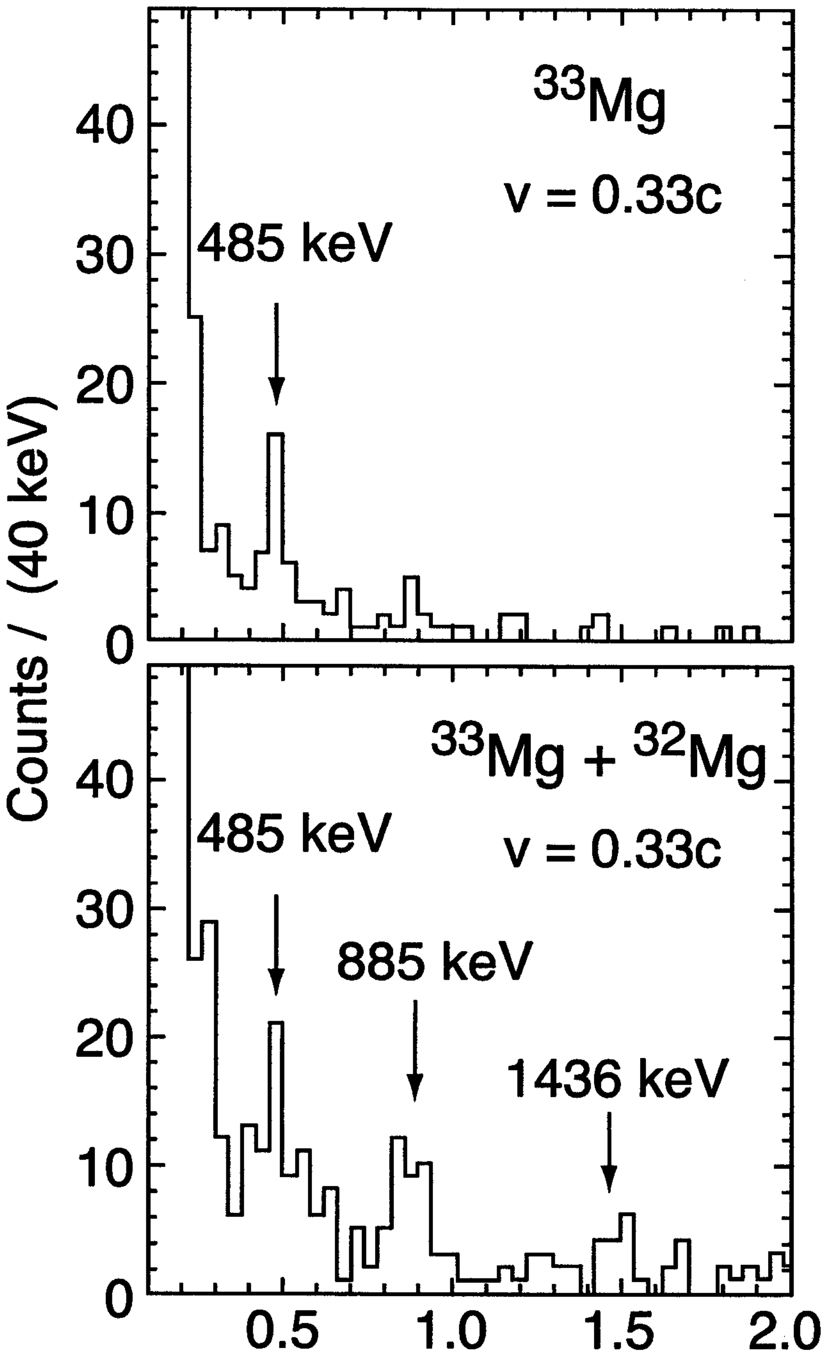

FIG. 2: Energy gates described in the text used to search for $\gamma$ rays in residual nuclei from neutron stripping. The upper frame is the higher kinetic energy gate corresponding to $^{33}$Mg electiles, while the lower frame is the lower kinetic energy gate selecting neutron-stripping reactions.