# Fingerprints of entangled states in reactions with rare isotopes

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## Abstract

We study the presence of entangled states in the detection of nucleon pairs from nuclear decays and in reactions with exotic nuclei, e.g., $^{11}$Li, or $^{6}$He. It is shown that the fingerprints of entangled states in these subsystems are visible in correlation measurements and can be accessed with present experimental techniques. This shows that not only atomic and optical systems, but also nuclear systems serve as important tools to obtain dichotomic outcomes for tests of the Einstein-Podolsky-Rosen paradox.

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## I. INTRODUCTION

In recent years, there has been much interest in the physics of unstable nuclei. Nuclei far from the valley of nuclear stability, with lifetime of a few miliseconds, play an important role in cosmology and astrophysics, as well as in the traditional applications of nuclear physics [1]. Some of these nuclei possess rather unexpected properties, as for example, the two-proton radioactivity, nuclear halos, large deformations, etc. Many theoretical models for these nuclei invoke subtle aspects of basic quantum mechanics. A nuclear halo, for example, is thought to be a simple manifestation of quantum tunneling of loosely bound states. To gain insight into these features, typical experiments involve the measurement of momentum distributions of the fragments, knock-out and stripping reactions, Coulomb excitation, etc.

Rare nuclear isotopes can also be used to study counterintuitive aspects of quantum mechanics. For example, two-proton decay in s-wave states could be used for a test of quantum mechanics versus local realism by means of Bell's inequalities [2]. Since the final state of the two protons can be found in a singlet state, their wave function is spin-entangled. The identification of the spins of the proton in two detectors separated far away would be useful to test the Einstein-Podolski-Rosen (EPR) paradox [3]. In fact, these tests should be performed in different and complementary branches of physics to avoid the loopholes encountered in photon experiments. Correlation experiments with low-energy proton-proton scattering have already been used for studies on the EPR paradox. Two-proton radioactivity would also qualify for this purpose.

Simple quantum entanglements are also visible in peripheral reactions with rare isotopes. We will consider some examples here. To start with, we observe that in Coulomb dissociation experiments of nuclear collisions at intermediate energies ($\gtrsim$ 50 MeV per nucleon), the exchanged photon is almost real, thus the momentum and energy transferred obey the relation $p \simeq E/c$. A single nucleon cannot absorb this photon (the photon momentum is too small, $p \simeq 0$ for $E \simeq 1$ MeV). In order to preserve the energy-momentum condition the photon has to be absorbed by more than one nucleon. The energy can be shared by two nucleons which fly apart in opposite directions. The same happens in nuclear interactions in peripheral collisions, when little energy is transferred to the nucleus. At forward scattering angles the momentum transferred is $|\mathbf{p}-\mathbf{p}'| \simeq p - p'\cos\theta \simeq p - p' \simeq E/v$, where $\mathbf{p}$ ($\mathbf{p}'$) is the initial (final) center of mass energy, $\theta$ is the scattering angle, and $v$ the projectile velocity. For collisions at intermediate energies, $v \simeq c$, and the same argument as for the Coulomb dissociation applies. Experiments which explore this feature have been performed. For example, Coulomb dissociation of $^{11}$Li projectiles (a rare isotope) into a $^{9}$Li nucleus and two neutrons have been done. Since neither $^{10}$Li, nor the di-neutron system are bound, the neutron-neutron correlation in the bound state of $^{11}$Li is the key factor to allow the mere existence of this nucleus. The experiments have concentrated their efforts in understanding this particular aspect of the $^{9}$Li + n + n system.

The singlet and triplet configurations of the neutron-neutron system in $^{11}$Li have different contributions to the total nuclear wavefunctions and a recent experiment [4] has been able to access their relative weights. However, some assumptions on the reaction mechanism were necessary to interpret the results. It is also not clear if different spin configurations in the final state could change the momentum distributions of the fragments. In fact, very little is known about final state configurations in reactions involving unstable nuclei.

In this article we show that one can disentangle the contributions of singlet and triplet spin final states of a two nucleon system in reactions with rare isotopes by measuring momentum correlations. This is a useful result, as a direct measurement of the spin orientations of each nucleon is by far more complicated in such conditions. The application of the method is very general, as it only relies on measured quantities, independent of models for the reaction mechanism. These are the widths and differential cross sections of momentum distributions, which have been obtained experimentally for several reactions involving unstable nuclei.

The correlated (C) and uncorrelated (U) measurements of nucleons 1 and 2, with relative momentum $\Delta p = |\mathbf{p}_1 - \mathbf{p}_2|$ are defined by

$$C(\Delta p) = \int P\left(\mathbf{p}_1;\mathbf{p}_1+\Delta\mathbf{p}\right) d\mathbf{p}_1 d\Omega_p\ , \qquad U(\Delta p) = \frac{1}{N}\int P\left(\mathbf{p}_1\right) P\left(\mathbf{p}_1+\Delta\mathbf{p}\right) d\mathbf{p}_1 d\Omega_p\ , \quad (1)$$

where $P(\mathbf{p}_1; \mathbf{p}_2)$ is the probability to measure a nucleon having momentum $\mathbf{p}_1$ in coincidence with the measurement of the other nucleon having momentum $\mathbf{p}_2$. The integration in $\Omega_p$ is over all orientations of $\Delta\mathbf{p}$. $P(\mathbf{p}) = \int P(\mathbf{p}; \mathbf{p}')\, d\mathbf{p}'$ is the probability to measure the momentum $\mathbf{p}$ for one of the nucleons, irrespective of what the momentum of the other nucleon is. $N$ is the total number of particle measured, i.e., $N = \int P(\mathbf{p})\, d\mathbf{p}$.

The correlation function is defined as

$$R(\Delta p) = \frac{C(\Delta p)}{U(\Delta p)} - 1. \tag{2}$$

One can describe each nucleon in the final state by Gaussian wavepackets:

$$\psi_i = \left(2\pi\sigma_n^2\right)^{-3/4} \exp\left\{-\frac{\left(\mathbf{p}_i - (\mathbf{P}_c \pm \mathbf{p}_n)/2\right)^2}{4\sigma_n^2}\right\}, \tag{3}$$

where the average recoil momentum of the core is $\mathbf{P}_c$, and the average momentum of the pair at their center of mass is $\mathbf{p}_n$. The momentum spread of the nucleon wavefunctions in the final state, $\sigma_n$, depends on the reaction mechanism for the specific reaction studied. E.g., for the breakup of $^{11}$Li projectiles at $E_{lab} \simeq 300$ MeV/nucleon, the neutrons acquire a momentum spread of the order of $\sigma_n \simeq 20$ MeV/c.

The spins of low energy nucleons imply that they can be arranged into singlet ($S = 0$) and triplet ($S = 1$) states. The spatial part of their wave function is symmetrized accordingly:

$$\Psi^{(\pm)}(\mathbf{p}_1, \mathbf{p}_2; \mathbf{P}_c, \mathbf{p}_n) = A^{\pm}\left\{\psi_1(\mathbf{p}_1; \mathbf{P}_c, \mathbf{p}_n)\psi_2(\mathbf{p}_2; \mathbf{P}_c, \mathbf{p}_n) \pm \psi_1(\mathbf{p}_2; \mathbf{P}_c, \mathbf{p}_n)\psi_2(\mathbf{p}_1; \mathbf{P}_c, \mathbf{p}_n)\right\}. \tag{4}$$

This wavefunction is entangled and cannot be factorized into individual particle wavefunctions. It will lead unavoidably to interferences which are visible if one observes the complete final state. This non-local entanglement is an example of the EPR paradox. The plus (minus) sign refers to $S = 0$ ($S = 1$) states, $A^{\pm} = \left[2(1 \pm \mathcal{O}^2)\right]^{-1/2}$, where the overlap integral is given by $\mathcal{O} = \int \psi_1^*(\mathbf{p}; \mathbf{P}_c, \mathbf{p}_n)\psi_2(\mathbf{p}; \mathbf{P}_c, \mathbf{p}_n)d\mathbf{p} = \exp(-\mathbf{p}_n^2/8\sigma_n^2)$. It does not depend on $\mathbf{P}_c$. Thus, if $\mathbf{p}_n = 0$ the final wave function, $\Psi$, of the pair is 100% an $S = 0$ state, i.e., $\Psi = \Psi^{(+)}$. For $\mathbf{p}_n \neq 0$, the singlet and triplet states can both contribute to $\Psi$.

The probabilities in eq. 1 can be written as

$$P(\mathbf{p}_1;\mathbf{p}_2) = \int \frac{d\sigma}{d\mathbf{p}_n d\mathbf{P}_c} \left\{ \left|\Psi^{(+)}(\mathbf{p}_1,\mathbf{p}_2;\mathbf{P}_c,\mathbf{p}_n)\right|^2 + \mathcal{M}\left|\Psi^{(-)}(\mathbf{p}_1,\mathbf{p}_2,;\mathbf{P}_c,\mathbf{p}_n)\right|^2 d\mathbf{P}_c d\mathbf{p}_n \right\} \quad (5)$$

where $\mathcal{M}$ is the mixing parameter, determining the relative contribution of the triplet state, and $d\sigma/d\mathbf{P}_c d\mathbf{p}_n$ is the differential cross section for the process. The experiments in peripheral collisions with unstable nuclei show that the momentum distributions are only slightly shifted from $\mathbf{P}_c = 0$ and that the shift is much smaller than the width of the recoil momentum distribution. Thus, the correlation function is rather independent of the details of $d\sigma/d\mathbf{P}_c d\mathbf{p}_n$. However, it is important to study the dependence of the correlation function on $\mathbf{p}_n$. We thus replace $\mathbf{P}_c = 0$ in the formulas above and perform the integrals to obtain the corelation function $R(\Delta p; p_n)$. The result can be obtained analytically. For pure singlet (+), or pure triplet (-), states one gets

$$R^{(\pm)} = \frac{2\left[g(x) \pm 1\right]\left[1 \pm \mathcal{O}^2\right]}{1 + 2\mathcal{O}^4 + g(x)\mathcal{O}^2 \pm 8h(x)\mathcal{O}^{5/2}} - 1, \quad (6)$$

where $g = \sinh(x)/x$, $h = \sinh(x/2)/x$, and $x = p_n \Delta p / 2\sigma^2$.

The results for $R^{(+)}$ (upper figure) and $R^{(-)}$ (lower figure) are shown in figure 1, as a function of the variables $\Delta p/\sigma_n$ and $p_n/\sigma_n$. One sees that the properties of the correlation functions in the singlet and triplet states are completely different. When the differences of the average momenta of the nucleons is small $\Delta p \ll \sigma_n$, the correlation function is negative only for the triplet state. It is -1 at $\Delta p = 0$ for the triplet state, whereas it is close to zero for the singlet state. While for the former case the correlation function crosses zero at two points, it does not have a null point for the singlet case.

In figure 2 we show the case for which the final state is an admixture of triplet and singlet states, as a function of the mixing parameter $\mathcal{M}$, which appears in eq. 5. The calculation also leads to closed analytical expressions. The dotted, dashed, and solid curves are obtained for $\mathcal{M} = 0.1$, 0.5, and 0.9, respectively. As one increases $\mathcal{M}$ the fingerprint of triplet states in the final wavefunction becomes more and more visible. This means that measurements of pair correlation functions should be able to discriminate the triplet and singlet configurations in the final wavefunction.

The traditional idea of diproton radioactivity is due to the pairing effect. Two protons form a quasiparticle (diproton) under the Coulomb barrier and this facilitates penetration. In a more formal description, one has a system with two valence protons in the same shell and coupled to $J^\pi = 0^+$. The method described above is directly applicable to determine the spin mixing of final states in low-energy two-proton nuclear decay for $0^+ \longrightarrow 0^+$ transitions. In this case the final spin wave function of the pair equals that of the initial wave function. In particular, when singlet states are identified, spin-spin coincidence experiments will generate dichotomic outcomes for each single measurement. These can be tested using Bell's inequalities for spin-spin correlations along different spin-axes. Since not all the particles can be measured, one would have to use a softer version of the Bell's inequalities, as developed by Clauser and Horne [5]. In the case of peripheral nuclear reactions this is not so simple. But, the entanglement of the final wavefunction will also have noticeable consequences, as we show next.

Let us assume for simplicity the emission of a nucleon pair in a reaction involving halo nuclei. For weakly bound nucleons one can assume that the reaction suddenly detaches the pair from the core and that their intrinsic wavefunction is likely to be an entangled state. Their wavefunction can be written in the form $\exp(i\mathbf{p}_1.\mathbf{r}_1)\exp(i\mathbf{p}_2.\mathbf{r}_2) \pm \exp(i\mathbf{p}_2.\mathbf{x}_1)\exp(i\mathbf{p}_1.\mathbf{r}_2)$. This wavefunction leads to destructive, or constructive interferences. The cross section for the reaction process will be given by

$$\sigma(\mathbf{P}, \mathbf{q}) = f(\mathbf{P}, \mathbf{q}, R)\left[1 \pm \cos\left(qR\right)\right]. \tag{7}$$

where $\mathbf{q} = \mathbf{p}_1 - \mathbf{p}_2$, $\mathbf{P}$ is the recoil momentum of the center-of-mass of the pair and $R$ is a parameter which describes the initial spatial localization of the nucleon pair. All other features of the reaction mechanism are included in the function $f(\mathbf{P}, \mathbf{q}, R)$. Thus, for $qR \ll 1$ one should be able to see a destructive interference for triplet final states and constructive interference for singlet final states. In fact, this was clearly seen in an experiment dedicated to study the existence of the $^5$H nucleus [6]. The transfer reaction $^1$H($^6$He,$^2$He)$^5$H was studied by detecting two protons emitted from the decay of $^2$He. The energy correlation of the two protons was measured, as shown in figure 3. If we apply eq. 7 to describe the relative energy distribution of the two protons, we get

$$\sigma(E_{pp}) \propto \begin{cases} \sqrt{E_{pp}}\left[1+\cos\left(\sqrt{m_N E_{pp}/\hbar^2}R\right)\right] \ , & \text{singlet} \\ \sqrt{E_{pp}}\left[1-\cos\left(\sqrt{m_N E_{pp}/\hbar^2}R\right)\right] \ , & \text{triplet} \\ \sqrt{E_{pp}} \ , & \text{uncorrelated} \end{cases} \tag{8}$$

In figure 3 we show the result of eqs. 8 together with the experimental data of ref. [6]. The solid, dashed, and dotted curves correspond to singlet, triplet, and uncorrelated states of the proton pair, respectively. We used $R = 4.6$ fm as the size of the source region where the protons originate. The agreement with the experiment is quite poor, especially for large relative energy of the pair. This is expected since we have neglected final state interactions (e.g., Coulomb interaction). In ref. [6] this spectrum was described theoretically by using the Landau-Smorodinskii's effective range approximation [7] for the protons with a scattering length of $a_{pp} = -7.806$ fm. However, it is important to notice that the most important ingredient is the assumption of a singlet or of a triplet state for the protons. As one easily sees from figure 3 the spectrum is completely different in each situation. When no entanglement exists, there is no interference and the spectrum also changes dramatically.

The examples discussed above involve nucleon pairs interacting with one or more particles. Entanglement measures for general multiparticle systems are still under dispute [8]. However, if the state of the whole system is a pure state, and the full system is being regarded as divided into two subsystems, a convenient measure of entanglement is

$$P_n = Tr\left\{\rho_n^2(t)\right\} = Tr_n\left\{\left[Tr_c\left(|\Psi><\Psi|\right)\right]^2\right\}, \tag{9}$$

where $\rho_n$ is the reduced density operator of the nucleon pair subsystem, $N$, and $|\Psi>$ is the wavefunction of the total system. Under these conditions, it can be shown [9] that this quantity yields the same value no matter for which subsystem it is calculated, i.e., $P_c = P_n \equiv P$ (the traces in eq. 9 can be permuted). $P$ is called the "purity" since it takes on its maximum value 1 if the subsystem is in a pure state. The more $P$ deviates from 1 the larger is the error that would occur in treating the system as an entangled state. To illustrate this point, let us consider the fragmentation of $^{11}$Li projectiles, as obtained in the experiment [4].

The analysis of experiment [4] was based on Hansen's wounded wavefunction model [10]. This method assumes that the collision is so fast that a piece of the initial wavefunction is scraped-off in the collision. This allows to calculate the frozen momentum configuration of the initial state by a Fourier transform of this wavefunction. One therefore assumes a 100% entanglement of the remaining constituents of the nucleus. In the experiment [4] this was used in the case of a knock out reaction, where a neutron was removed from $^{11}$Li. The measurement of the momentum distributions of the neutron and the $^{9}$Li fragment was used to deduce the properties of $^{10}$Li. The nucleus $^{11}$Li is usually called a Borromean nucleus, meaning that both the n-n, as well as the $^{10}$Li, subsystems are not bound. We now show that it is exactly this property which is responsible for the successful interpretation of the experiment [4]. We use eq. 9 for the subsystems $^{10}$Li (c) and n-n (N).

We use the results of ref. [11], where it was shown that the bound-state properties of $^{11}$Li can be obtained with a three-body model by using phenomenological potentials of the form

$$V_{nn} = S_n e^{-\rho^2/b_n^2}, \qquad V_{n\mathbf{9}} = S_1 e^{-\lambda^2/c_1^2} + S_2 e^{-\lambda^2/c_2^2}, \tag{10}$$

where

$$\boldsymbol{\rho} = \mathbf{r}_n - \mathbf{r}_{n'}, \qquad \boldsymbol{\lambda} = \frac{1}{2}\left(\mathbf{r}_n + \mathbf{r}_{n'}\right) - \mathbf{r}_\mathbf{9}, \tag{11}$$

are the Jacobian variables. The following potential parameters were used: (a) $S_n = -31$ MeV, $b_n = 1.8$ fm, to reproduce the low-energy scattering length and effective range of the n-n system. (b) The set of values $S_1 = -7.0$ MeV, $c_1 = 2.4$ fm, $S_2 = -1.0$ MeV, $c_2 = 3.0$ fm, reproduce the size and the binding of $^{11}$Li ($\sim 0.3$ MeV) quite reasonably.

In eq. 9 one should insert all bound states of the subsystems to obtain the probability that a measurement of the Hansen's wavefunction would give an entangled $^{10}$Li nucleus. However, there are no bound states in these systems. In fact, we notice that the potentials presented above barely bind the n-n and the $^9$Li subsystems. They yield the (negative) scattering lengths, $a_{nn} = -17.4$ fm and $a_{n9} = -10.66$ fm, respectively. These are large values, reflecting almost bound states, with $E \simeq 0$. Thus, one can estimate the value of the purity in eq. 9 by replacing their respective wavefunction by a constant value within $^{11}$Li. This yields the particularly simple expression

$$P \simeq \left(\frac{1}{4\pi < \lambda >^3 /3}\right)^{1/2} \left(\frac{1}{4\pi < \rho >^3 /3}\right)^{1/2} \int d\boldsymbol{\rho} \left\{\int d\boldsymbol{\lambda} \Psi_{^{11}\mathrm{Li}} (\boldsymbol{\rho}, \boldsymbol{\lambda})\right\}^2 , \qquad (12)$$

where $\Psi_{^{11}\mathrm{Li}}$ is the $^{11}$Li wavefunction in terms of the Jacobian variables and $< \rho >= 3.3$ fm and $< \lambda >= 3.1$ fm are the most probable values of the Jacobian coordinates. The $^{11}$Li wavefunction in eq. 12 is obtained from a variational calculation, as in [11]. We get $P \simeq 0.8$ which is very close to unity, thus proving our assertion.

In conclusion, we have shown that entangled states in nuclear decay and in reactions with exotic nuclei is another route to study important problems of basic quantum mechanics interest, as the Einstein-Podolski-Rosen paradox. These studies would be complementary to others performed in atomic physics. We have also shown that, due to the entanglement of final states, simple correlation functions yield compelling evidence of the spin and momentum states of the nuclear subsystems. The forthcoming RIA (Rare Isotope Accelerator) facility will be an ideal laboratory to perform these studies [12].

## ACKNOWLEDGEMENTS

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FIGURES

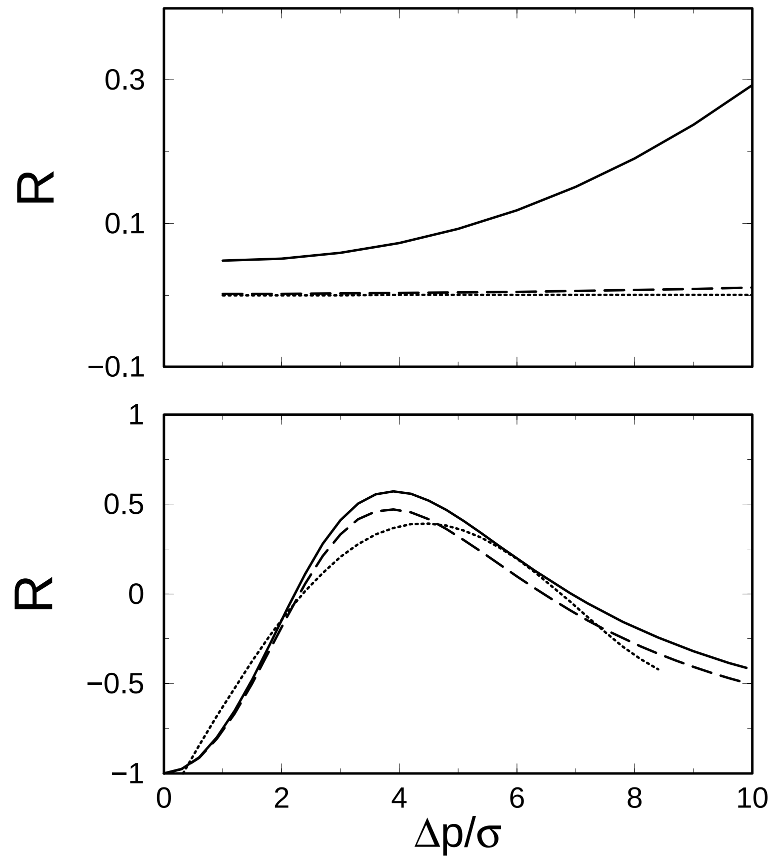


FIG. 1. Correlation functions for different values of the average momentum, $p_n$. The upper (lower) figure is for the singlet (triplet) state. The dotted, dashed, and solid lines correspond to $p_n/\sigma_n = 0.01$, 0.1, and 0.5, respectively.

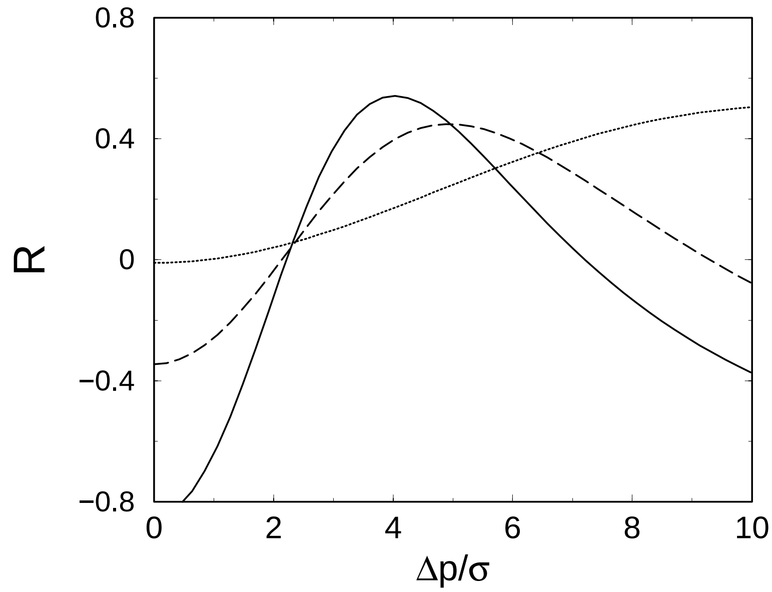


FIG. 2. Correlation functions for $p_n/\sigma_n = 0.5$ and for different admixtures of singlet and triplet states. The dotted, dashed, and solid lines correspond to $\mathcal{M} = 0.1$, 0.5, and 0.9, respectively. $\mathcal{M}$ is the absolute contribution of the triplet state.

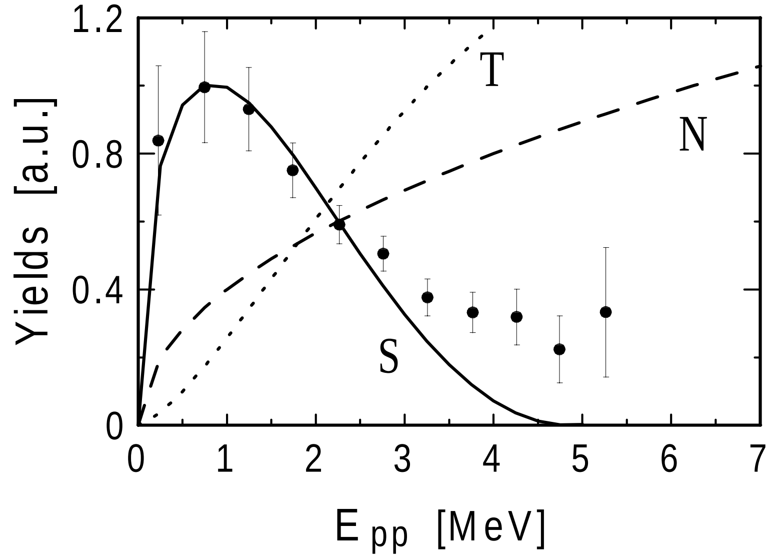


FIG. 3. Relative energy distribution of two protons from the reaction p($^6$He, ppt). Calculations are for triplet (T) and singlet (S) states. The dotted curve assumes no correlation in the pair wavefunction.