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MSUCL-1241 **SEPTEMBER 2002**

Proton Elastic Scattering Potentials: Energy and Isospin Dependence

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(Dated: Draft: September 7, 2002)

Volume integrals of the real potentials derived from proton elastic scattering studies have been calculated for data available from the lowest to the highest energy. Because of the spread of the volume integrals at low energies, an average of volume integrals in each one MeV bin was calculated. These average volume integrals show a logarithmic dependence on the beam energy. The data for proton scattering from Ca isotopes at 1044 MeV were analyzed in terms of the optical model and an isospin component of the potential was derived.

PACS numbers: PACS number(s): 25.10.+s, 25.55.Ci

I. INTRODUCTION

Elastic scattering of protons from nuclei has been studied for several decades. The primary goal of these studies was to determine the proton-nucleus interaction. The interaction between the proton and the nucleus is a many-body problem and the potential has many components due to different mechanisms of nuclear reactions. However, the optical model potential has been found to provide an acceptable macroscopic, phenomenological description of the interaction. It essentially reduces the highly complicated description of the many-body proton-nucleus system to the solution of the Schroedinger equation with a complex mean-field potential. It provides good fits to the differential cross section angular distribution for a wide range of target nuclei at different energies. In many instances the optical model analyses have provided interesting information about the energy, target-mass and isospin dependence of the derived parameters.

Accurate elastic scattering data from a large sample of nuclei are available at many bombarding energies up to 60 MeV [1–5]. Most of these data have been analyzed in terms of the standard nuclear optical model parametrization employing a real and an imaginary potential. Generally the Woods Saxon form (defined in Sect. II) was used for both potentials. Occasionally the derivative of these form factors was used when the scattering is not too sensitive to the interior of the nucleus. Many investigators analyzed individual sets of data and obtained a linear dependence of the real potentials on the incident proton energies. However, because these low energy protons do not penetrate deep into the nucleus, the derived potentials are ambiguous: different sets of potentials that are similar in the surface region provide equally good fits to the data. Because of these ambiguities in the potentials, the parameters obtained by different authors are not consistent with each other, and the resulting uncertainty in interpolation between energies and nuclear masses precludes the determination of reliable systematic trends of the parameters.

With the advent of higher energy beams, single-energy proton elastic scattering studies have been made at 100 MeV [6], 156 MeV [7], 185 MeV [8], 200-500 MeV [9], 800 MeV [10] and 1044 MeV [11]. A comprehensive analysis over a limited energy range of 80-180 MeV proton elastic scattering from several targets [13] was also carried out. Most of these data were analyzed in a consistent and uniform manner in terms of a local optical model potential with Woods Saxon form factors. Because of their higher energies these data required the use of relativistic kinematics and a relativistic extension of the Schroedinger equation. At these energies the the Coulomb repulsion is relatively weak and the nucleus is fairly transparent to the incident proton. Thus the proton is able to sample interior regions of the nucleus, resulting in a significant reduction in the ambiguities of the potentials.

Investigators were then able to derive more reliable systematics of the potentials. Of particular interest was the energy dependence of the potentials. The optical model potentials have two basic sources for their energy dependence. First is the intrinsic energy dependence, which is derived from the dispersion relation. Secondly the proton-nucleus potential is non-local. The Fourier transform of this non-local potential to an equivalent local potential naturally leads to an energy dependence. The optical model energy dependence is reflected by the variation of the derived parameters with energy. In many analyses, particularly at low energies, the dependence of the strength, V, is considered. However, because of the correlations between V and the Woods-Saxon geometry parameters, r_0 and a_0, these individual parameters can exhibit spurious energy dependences. On the other hand, it has been found that the volume integral of the potential, J_R, is a well-defined quantity, free of parameter correlations, and thus would provide a reliable measure of the energy dependence of the optical potential. Van Oers *et al.* [8] obtained a linear energy dependence of the real potential volume integrals up to 60 MeV for six target nuclei and a different linear energy dependence from 160 to 200 MeV for ^{40}Ca and ^{208}Pb. This inconsistency was then resolved by determining a logarithmic energy dependence, which provided a good description of p+^{12}C, ^{16}O, ^{27}Al, ^{40}Ca, ^{208}Pb data from 10 MeV to 1000 MeV [14]. It had the form J_R(E) = J_R(0) - βlnE with J_R(0) = 850-930 MeV-fm^3 and β = 142-156 MeV-fm^3. The results gave a zero crossing of the real potential, from attractive to repulsive, at about 500 MeV. Nadasen *et al.* [13] also obtained a logarithmic energy dependence for the energy range of 80-180 MeV, with $J_R(0)$ = 815 MeV-fm^3 and β = 120 MeV-fm^3, which gave an extrapolated zero crossing value of 890 MeV. This global analysis was extended to 1 GeV maximum energy [15] by including newer scattering data at 200, 300, 400 and 500 MeV from TRIUMF [9], as well as the older data at 800 and 1044 MeV. It became apparent that because different investigators analyzed different sets of data, a number of curious, and in some cases inconsistent, features arose from the various analyses. What was lacking was a complete single analysis over the entire range of energies. We have therefore carried out and present in this paper a global review of all proton elastic scattering studies up to 1 GeV. Section II describes the optical model parameter selection and analysis procedure. A new optical model analysis of proton scattering from Ca isotopes at 1 GeV for the determination of the isospin potential is presented in section III. The energy dependence of the potentials is derived in Section IV. Section V contains the results and conclusions of the investigation.

II. OPTICAL MODEL PARAMETER SELECTION AND PROCEDURE

The initial real central potential parameters were taken from the compilation of Perey and Perey [16], which lists potential parameters derived from proton elastic scattering studies up to 1975. More values were obtained from the work of F.G. Perey [17], Becchetti and Greenlees [5], Van Oers [8], Alkhazov *et al.* [18], Kwiatkowski and Wall [6], Igo *et al.* [19], Nadasen *et al.* [13], Woo *et al.* [20] and Hutcheon *et al.* [9]. Most of these optical model analyses have been carried out with the real central potentials of the Woods Saxon form:

$$\mathrm{V(r)} = V_0 / \Big(1 + \exp[(r - r_0 A_t^{1/3})/a_0] \Big)$$

where V_0, r_0 and a_0 define the strength and shape of the potential. Values for V_0, r_0 and a_0 were derived from the analyses of elastic scattering data. The Woods Saxon potential is a spherically symmetric potential which resembles the shape of the nuclear matter distribution. The potential parameters V_0, r_0 and a_0 are not completely independent. They correlate with each other, resulting in continuous ambiguities between them. An increase or decrease in one parameter can be compensated by changes in the other two, resulting in equally good fit to a set of the scattering data. However, two quantities have been found to be free of ambiguities. One is the root-mean-square radius, $r_{<rms>}$, which is the radius of the nucleus averaged over the potential. It basically gives the size of the nucleus and is thus of no interest in the present study. The other is the real potential volume integral, $J_{\rm R}$, which is the spatial integral of the potential, weighted by the strength. This quantity defines the total effective potential for the interaction of the proton with a nucleus at a particular energy. The potential volume integral is not subjected to the V, r_0, a_0 continuous ambiguity. It has been found that different sets of V, r_0, a_0 parameters that provide equally good fits to a set of data all have the same volume integral.

Comparison of analyses of proton elastic scattering from different targets showed that the potential volume integral was proportional to the mass of the target nucleus. This is expected if the potential and nucleon (mass) density distributions have essentially similar radial shapes. Therefore a quantity largely independent of the target mass, namely the reduced volume integral, $J_{\rm R}$/A, where A is the mass number of the target, has been defined. This quantity provides a basis for the determination of the systematics of the potential across the entire periodic table (except for very light few-nucleon systems). The reduced volume integrals of the real potentials calculated using parameters from all available studies of proton elastic scattering, are shown in Fig. 1. They will be discussed in Sect. IV.

III. ANALYSIS OF 1044 MEV PROTON ELASTIC SCATTERING FROM CA ISOTOPES

The original analysis of 1044 MeV proton elastic scattering from four Ca isotopes [11] was based on Glauber theory with the purpose of determining neutron and nuclear matter distributions. Thus this study did not provide the volume integrals required for the present investigation. We have therefore carried out an optical model analysis of these data in a formalism consistent with those of lower energy data. The optical model potential of conventional form containing a Coulomb term, a complex nuclear central term, and a complex nuclear spin-orbit term was used. We used the relativistic extension of the Schroedinger equation with relativistic kinematics [12]. Woods Saxon form factors were used for the potentials. Since the proton has spin 1/2, it is advisable to include a spin-orbit potential in the analysis since otherwise systematics in the other components of the potential could be distorted. However, only differential cross section data

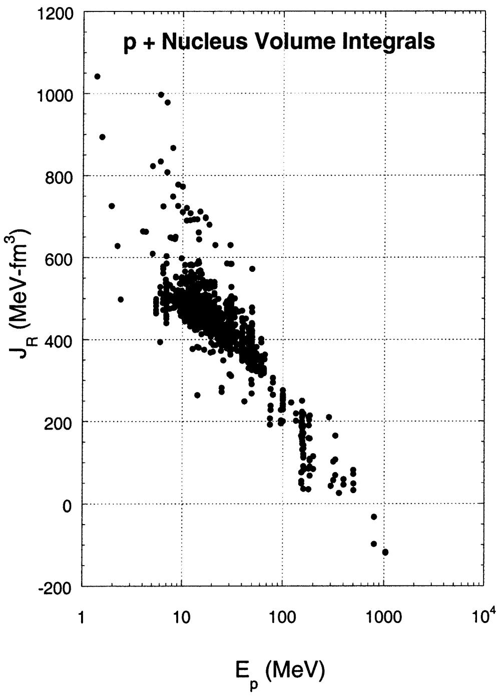

FIG. 1: Proton Volume Integrals versus Energy.

are available at 1044 MeV. In order to include polarization data, needed to fix the spin-orbit potential, we resorted to the 800 MeV analyzing power measurements [19] for all four nuclei. We do not expect the polarization to change much between 800 MeV and 1 GeV. Therefore we transformed the 800 MeV data to 1 GeV by changing the angles of the data by means of the equivalence of momentum transfer, i.e., 2ksin($\theta/2$) values are equal at both 800 MeV and 1 GeV (k is the wave number of the incident proton). This procedure was deemed adaquate for determining a spin-orbit potential at 1 GeV which is sufficiently realistic to constrain ambiguities in the central potential that would otherwise arise from the analysis of cross section data alone.

The code SNOOPY8 [12] was used to carry out the analyses. Starting parameters were obtained from the extrapolations of lower-energy studies. For each angular distribution, first single parameter searches were carried out on all twelve parameters. The optimized fit parameters were then used to carry out all combinations of two-parameter searches. The number of search parameters was continually increased until searches were made on combinations of six parameters. This provided very good fits to the data. An attempt was made to improve the fits were obtained by allowing the normalization of the cross-section data to vary, but only ^{44}Ca preferred a normalization different from unity. Figs. 2 and 3 show the results for ^{48}Ca differential cross section and polarization data. The dots represent the data with the error bars indicated. The solid lines show the optical model fits.

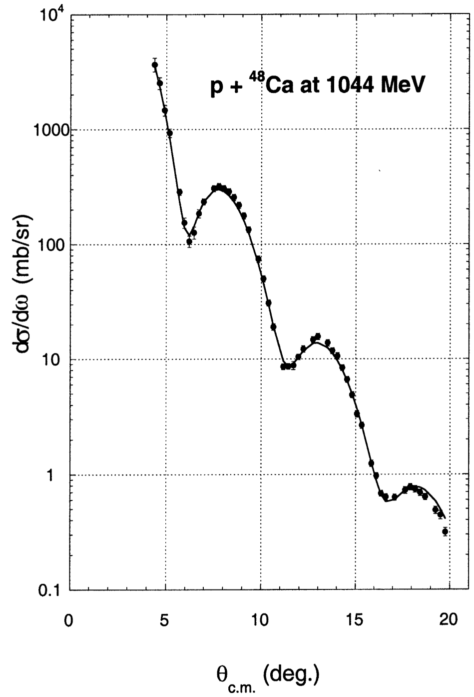

FIG. 2: p + ^{48}Ca differential cross sections (dots) and OM calculations (solid line)

The derived central potential volume integrals are positive, indicating that the repulsive component of the nuclear force dominates at this energy. The volume integrals steadily increased in going from ^{40}Ca to ^{48}Ca. This variation is due to the isospin component of the nuclear potential. Several authors have considered the existence of an isospin component in the central nuclear potential [21]. Lane [22] explicitly showed that the isospin component of the potential is given by J_S(N-Z)/A, where J_S is the strength of the isospin component. Fig. 4 shows the plot of the volume integrals (after subtracting the Coulomb correction term $V_c = 0.4Z/A^{1/3}$) as a function of (N-Z)/A. The linear relationship between J_R/A and (N-Z)/A provides a value of $J_S = 350 \pm 35$ MeV-fm^3. This value agrees well with the values 200-400 obtained by Becchetti and Greenlees [5], 300 $\pm$ 100 by Perey [17], but is much higher than the value of 120 $\pm$ 40 obtained by Kwiatkowski and Wall [6]. The isospin effect arises from nucleon-nucleon interactions. For proton elastic scattering, neutron-rich nuclei have a positive isospin term, which remains essentially constant with energy. Since the total central potential decreases with energy, the isospin component becomes relatively more important as energy increases. In fact, for ^{48}Ca at 1044 MeV we find that almost a third of the potential is due to the isospin effect. This can be understood in terms of the fundamental nucleon-nucleon forces. It is well known that, because of the existence of the triplet state, the proton-neutron force

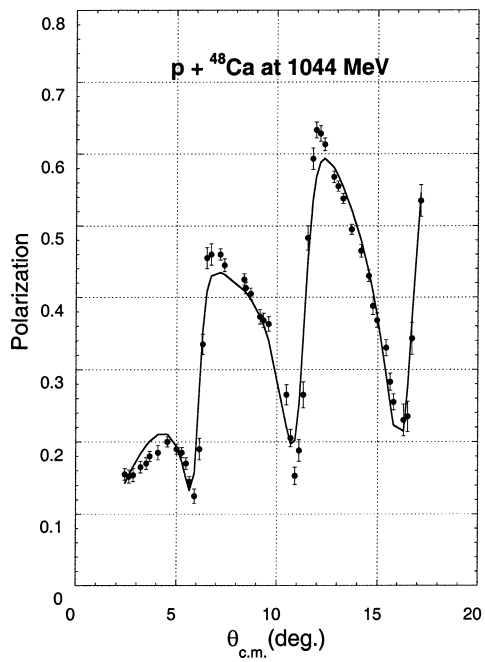

FIG. 3: p+^{48}Ca Analyzing powers (dots) and OM calculations (solid line)

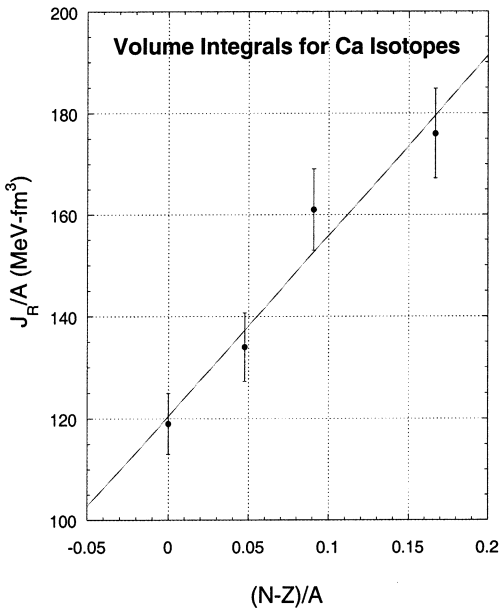

FIG. 4: Volume Integrals for Ca Isotopes at 1044 MeV versus (N-Z)/A. The solid line is a linear fit to the data.

is three times as strong as the proton-proton and neutron-neutron force. Thus the potential for proton scattering from a neutron-rich nucleus is strongly enhanced.

IV. ENERGY DEPENDENCE OF THE REAL VOLUME INTEGRAL

Several theoretical attempts have been made to derive the energy dependence of the empirical real potential. Brueckner *et al.* [23] proposed a description of the dispersive nature of nuclear matter, which gave the correct magnitude of the potential at zero energy. Lipperheide and Schmidt [24] used the dispersion integral, but the real potential was too strong at high energies and did not change sign as indicated by the scattering data. The non-local energy-independent potential of Perey and Buck [25] extended in energy by Engelbrecht and Fiedeldey [26] was in reasonable agreement with experimental results up to about 150 MeV. Passatore noted strong disagreement between the experimentally determined potentials and those calculated from the dispersion relation for the energy range 100-500 MeV [27], but the slope agreed with the empirical results above 500 MeV [28]. Using both the intrinsic energy dependence resulting from the dispersion relation and that due to non-locality, he reformulated the calculations to obtain a logarithmic energy dependence of the potentials up to 1 GeV. As shown below, our results confirm Passatore's predictions.

All the calculated volume integrals, J_R/A, were ordered in terms of increasing energy. Since these are the reduced volume integrals, they should be largely independent of the target mass. Therefore no consideration was given to the mass of the target in determining the systematics of these volume integrals. The volume integrals determined from all known proton elastic scattering studies are plotted as a function of beam energy in Fig. 1. It is observed that the values at low energies have a large spread, ranging from ∼200 to ∼1000 MeV-fm^3. This is basically due to the fact that the incident proton cannot get into the nuclear interior because of Coulomb repulsion effects and a short nuclear mean free path. Thus the proton only samples the surface region of the nucleus. Therefore it is not clear which of these potentials represents the true mean-field interaction between the proton and the nucleus.

As the beam energy increases, the spread of the volume integrals decreases. This is a consequence of the ability of the proton to sample a larger region of the nucleus and experience almost the total average nuclear potential. As this trend continues, the derived volume integrals fall within a cone-shaped region, converging towards single values at energies above 100 MeV. Even the single values at the higher energies are not always completely consistent with each other. These differences may be due to different methodologies of analysis as well as differences in scattering data, particularly in the absolute cross-section normalization. However, there is no apriori reason not to accept the results of any of the studies.

The large spread in the low energy values precludes an accurate determination of the energy dependence, unless one bins the data over some appropriate energy interval to determine an average volume integral for each energy bin. Since investigations were carried out in small energy intervals at low energies, we averaged all results in 1 MeV intervals. For energies below 10 MeV, nuclear structure effects, core polarization, compound nuclear scattering and other reaction machanisms influence and mask the assumed pure potential scattering. Thus the empirical values of J_R/A do not necessarily reflect the actual energy dependence of the real central potential. Therefore we decided to omit results below 10 MeV in the determination of the energy dependence. The average volume integral at each energy is plotted as a function of beam energy in Fig. 5. There is still some spread in these values, particularly at the lower energies. However, the overall pattern of the data clearly indicates a logarithmic dependence of the volume integrals on the incident energy. We have made a least squares fit to the data, which provides a dependence of the volume integrals on incident energy of the form:

$$J_R(\mathrm{E}) = J_R(0) - \beta \ln \mathrm{E}$$

with $J_R(0) = 880 \pm 44$ MeV-fm^3 and $\beta = 138 \pm 7$ MeV-fm^3. The zero crossing of the real potential is at ~600 MeV, in good agreement with the earlier results of phenomenological optical model studies [15] and impulse-approximation calculations [29]

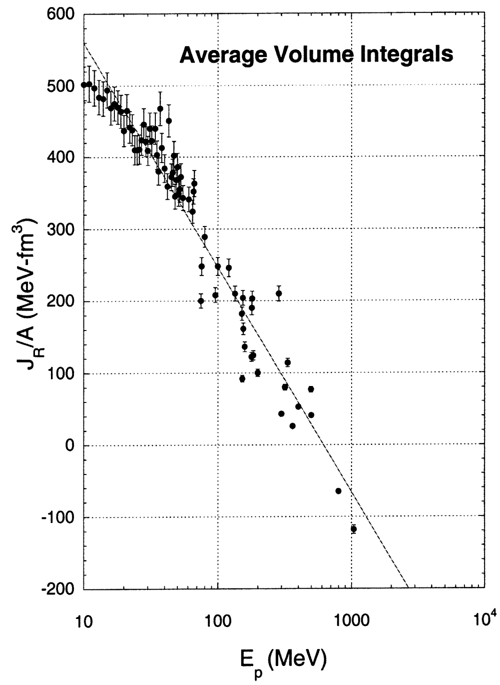

FIG. 5: Volume Integrals for proton elastic scattering averaged over 1 MeV bins. The solid line is a logarithmic fit to the data.

V. SUMMARY AND CONCLUSION

We have carried out a global analysis of all available potential parameters for proton elastic scattering from known studies at all energies. The volume integral of the real potential was calculated for each parameter set. These volume integrals, averaged over 1 MeV bins, show a logarithmic dependence on the proton energy from the lowest energies to 1 GeV. The derived relationship between the volume integrals and the energy describes the real part of the proton-nucleus interaction as a function of beam energy. It may thus be concluded that the attractive mean field dominates the proton-nucleus interaction at low energies. As the energy increases, the repulsive nucleon-nucleon interaction increases in importance. In the energy region around 600 MeV, the two effects balance each other and the net real potential goes to zero. Beyond this region the repulsive component of the potential dominates. Analyses using more flexible radial shapes for the nuclear potential than the simple Woods-Saxon form considered here indicate [15] that the change from attraction to repulsion occurs first in the nuclear interior at a much lower proton energy (between 200 and 300 MeV) while the nuclear surface potential remains weakly attractive up to 800 MeV.

From an utilitarian point of view, this study provides an universal formulation of the proton-nucleus real potential at any energy up to 1 GeV for all target nuclei. Thus the p-nucleus potentials required for global reaction studies can be derived from this formulation. By assuming reasonable radius, r_0, and diffuseness, a_0, parameters, one can determine the strength, V, from the correct volume integral for a particular target nucleus at the appropriate energy.

We have also carried out optical model analyses of 1044 MeV proton elastic scattering from Ca isotopes. This provided an asymmetry potential of the form J_S(N-Z)/A for Ca isotopes at 1044 MeV. The value of ~350 MeV-fm^3 for the isospin strength, J_S, is in agreement with the values determined at lower energies. In the determination of proton potentials for reaction studies, it is important to include the asymmetry potential for neutron-rich nuclei. Therefore, the volume integral of the real potential should be increased by J_S(N-Z)/A. Values of J_S ranging from ~300 MeV-fm^3 to~400 MeV-fm^3 at 1 GeV may be appropriate.

This work has been supported by the U.S. National Science Foundation under Grant Nos. PHY-9971836 (UM-Dearborn), PHY-0140010 (University of Maryland), PHY-9602872 (IUCF).

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