# Enhancement of nuclear Schiff moments and time reversal violation in atoms due to soft nuclear octupole vibrations

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(September 24, 2002)

## Abstract

Parity and time invariance violating ($\mathcal{P},\mathcal{T}$-odd) nuclear forces produce $\mathcal{P},\mathcal{T}$-odd nuclear moments, for example, the nuclear Schiff moment. In turn, this moment can induce electric dipole moments (EDMs) in atoms. We estimate the contribution to the Schiff moment from the soft collective octupole vibrations which exist in many heavy nuclei. We found that these vibrations give practically the same contribution as that of the static octupole deformation calculated earlier. This confirms a suggestion that the soft octupole vibrations can replace the (controversial) static octupoles in development of the collective Schiff moments. The values of atomic EDM predicted for $^{223,225}$Ra and $^{223}$Rn are enhanced by factors up to $10^3$ compared to experimentally studied spherical nuclei $^{199}$Hg and $^{129}$Xe.

PACS: 32.80.Ys,21.10.Ky,24.80.+y

## I. INTRODUCTION

A discovery of the static electric dipole moment (EDM) of an elementary particle, atom or atomic nucleus would reveal [1] the simultaneous violation of the invariance with respect to spatial inversion ($\mathcal{P}$) and time reversal invariance ($\mathcal{T}$). The best limit on nucleon-nucleon interactions violating parity and time invariance ($\mathcal{P},\mathcal{T}$-odd forces), as well as quark-quark $\mathcal{P},\mathcal{T}$-odd interactions, has been obtained from the measurement of the atomic EDM in $^{199}$Hg [2] calculated in Ref. [3]. According to the Schiff theorem [1,4,5], the nuclear EDM is screened by atomic electrons. The EDM of an atom with closed electron subshells is induced by the nuclear Schiff moment [6] that is defined as a mean square radius of the dipole charge distribution with the contribution of the center-of-charge subtracted,

$$\mathbf{S} = \frac{e}{10}\left[\langle r^2\mathbf{r}\rangle - \frac{5}{3Z}\langle r^2\rangle\langle\mathbf{r}\rangle\right]. \tag{1}$$

Here $\langle r^n\rangle \equiv \int \rho(\mathbf{r}) r^n d^3r$ are the moments of the nuclear charge density operator $\rho$. The vector operators $\langle \mathbf{r}r^2\rangle$ and $\langle\mathbf{r}\rangle$ are due to the $\mathcal{P},\mathcal{T}$-odd contribution to the charge density while $\langle r^2\rangle/Z = r^2_{\rm ch}$ is the nuclear mean square charge radius. Due to the rotational invariance, the existence of the non-zero Schiff moment requires a non-zero nuclear spin $\mathbf{I}$,

$$\mathbf{S} = S\,\frac{\mathbf{I}}{I}. \tag{2}$$

In a search for nuclear structure mechanisms of the enhancement, it was suggested in Ref. [7] that some actinide nuclei have a level of opposite parity and the same spin close to the ground state, and this may enhance the level mixing and the resulting nuclear EDM. In Ref. [6] a similar suggestion was put forward for the enhancement of the Schiff moment $\mathbf{S}$. It was noticed in Ref. [8] that, in contrast to $\mathcal{P}$-odd $\mathcal{T}$-even forces, the parity doublets present at a static pear-shaped deformation can be directly mixed by $\mathcal{P},\mathcal{T}$-odd forces. This mixing might be enhanced because of close intrinsic structure within the doublet, a large value of the static intrinsic Schiff moment and relatively small energy splitting. As shown in Refs. [9,10], nuclei with static octupole deformation may have enhanced collective Schiff moments up to 1000 times larger than the Schiff moments of spherical nuclei.

In Ref. [11] it was suggested that the soft octupole vibrations observed in some regions of the nuclear chart may produce an enhancement similar to that due to static octupole deformation. This would make heavy atoms containing such nuclei with large collective Schiff moments attractive for future experiments in search for $\mathcal{P},\mathcal{T}$-violation. Below we perform the first estimate of the Schiff moment generated in nuclei with the soft octupole mode and show that the result is nearly the same as in the case of the static octupole deformation.

## II. COLLECTIVE NUCLEAR SCHIFF MOMENT

Consider a nucleus with two levels, ground state $|\text{g.s.}\rangle$ and excited state $|x\rangle$, of opposite parity and close energies, $E_{\text{g.s.}}$ and $E_x$, respectively. Let $W$ be a $\mathcal{P},\mathcal{T}$-odd interaction capable of mixing these unperturbed states. Assuming that the mixing matrix elements of $\mathbf{S}$ and $W$

are real, we can write down the Schiff moment emerging in the actual mixed ground state as

$$\mathbf{S} = 2\frac{\langle \text{g.s.}|W|x\rangle\langle x|\mathbf{S}|\text{g.s.}\rangle}{E_{\text{g.s.}} - E_x}, \tag{3}$$

However, as it was explained in Ref. [6], in the case of mixing of closely lying single-particle states one should not expect a large enhancement. For example, in a simple approximate model, where the strong nuclear potential is proportional to nuclear density and the spin-orbit interaction is neglected, the matrix element $\langle \text{g.s.}|W|x\rangle$ contains the single-particle momentum operator and is proportional to $(E_{\text{g.s.}} - E_x)$ so that the small energy denominator cancels out. As mentioned above, the collective Schiff moments in nuclei with static octupole deformations may be by 2-3 orders of magnitude stronger than single-particle moments in spherical nuclei.

The mechanism generating the collective Schiff moment is the following [9,10]. In the "frozen" body-fixed frame the intrinsic collective Schiff moment $S_{\text{intr}}$ can exist without any $\mathcal{P},\mathcal{T}$-violation. However, in the space-fixed laboratory frame the nucleus has a certain angular momentum rather than orientation, and this makes the expectation value of the Schiff moment vanish in the case of no $\mathcal{P},\mathcal{T}$-violation. Indeed, the intrinsic Schiff moment is directed along the nuclear axis, $\mathbf{S}_{\text{intr}} = S_{\text{intr}}\mathbf{e}$, and in the laboratory frame the only possible correlation $\langle \mathbf{e}\rangle \propto \mathbf{I}$ violates parity and time reversal. The $\mathcal{P},\mathcal{T}$-odd nuclear forces mix rotational states $|I,\pm\rangle$ of the same spin and opposite parity and create an average orientation of the nuclear axis $\mathbf{e}$ along the nuclear spin $\mathbf{I}$,

$$\langle e_z\rangle = 2\alpha\frac{KM}{I(I+1)}, \tag{4}$$

where

$$\alpha = \frac{\langle I-|W|I+\rangle}{E_+ - E_-} \tag{5}$$

is the mixing amplitude of the states of opposite parity, $K = |\mathbf{I}\cdot\mathbf{e}|$ is the absolute value of the projection of the nuclear spin $\mathbf{I}$ on the nuclear axis, and $M$ is the spin projection onto the laboratory quantization axis. The observable Schiff moment in the laboratory frame is then related to the intrinsic moment as

$$S_z = S_{\text{intr}}\langle e_z\rangle = S_{\text{intr}}\frac{2\alpha KM}{I(I+1)}. \tag{6}$$

To estimate the intrinsic Schiff moment, we use the standard description of the surface of an axially symmetric deformed nucleus in the body-fixed frame in terms of the multipole deformation parameters $\beta_l$,

$$R(\theta) = R\,[1 + \sum_{l=1}\beta_l Y_{l0}(\theta)]. \tag{7}$$

In order to keep the center-of-mass at the origin we have to fix $\beta_1$ [20]:

$$\beta_1 = -3\sqrt{\frac{3}{4\pi}} \sum_{l=2} \frac{(l+1)\beta_l \beta_{l+1}}{\sqrt{(2l+1)(2l+3)}} \,. \tag{8}$$

We assume that the center of the charge distribution coincides with the center-of-mass, so that the electric dipole moment vanishes, $e\langle \mathbf{r} \rangle = 0$, and hence there is no screening contribution to the Schiff moment (no second term in eq. (1)). We also assume a constant density for $r < R(\theta)$. The intrinsic Schiff moment $S_{\rm intr}$ is then [9,10]

$$S_{\rm intr} = eZR^3 \frac{3}{20\pi} \sum_{l=2} \frac{(l+1)\beta_l \beta_{l+1}}{\sqrt{(2l+1)(2l+3)}} \approx \frac{9}{20\pi\sqrt{35}} \, eZR^3 \beta_2 \beta_3 \ , \tag{9}$$

where the major contribution comes from $\beta_2\beta_3$, the product of the quadrupole $\beta_2$ and octupole $\beta_3$ deformations. For $\beta_2 \sim \beta_3 \sim 0.1$ and $Z = 88$ (Ra) we obtain $S_{\rm intr} \sim 10 \, e\,{\rm fm}^3$. The estimate of the Schiff moment in the laboratory frame [$K = M = I$ in Eq. (6)] gives [10]

$$S = S_{\rm intr} \frac{2\alpha I}{I+1} \sim 0.01 \, \frac{I}{I+1} \; e\beta_2 \beta_3^2 Z A^{2/3} \frac{\eta G}{m r_0 (E_+ - E_-)}, \tag{10}$$

where $\eta G$ is the strength constant of the $\mathcal{P},\mathcal{T}$-odd nuclear potential, traditionally introduced with the aid of the Fermi weak interaction constant $G$, $R = r_0 A^{1/3}$, $A$ is the nuclear mass number, and $r_0 \approx 1.2$ fm is the internucleon distance. For the isotope $^{225}$Ra, where $E_+ - E_- = 55$ keV and $I$=1/2, this analytical estimate gives the Schiff moment $S \sim 500$ in units $10^{-8}\eta e$ fm$^3$; for $^{223}$Ra ($E_+ - E_- = 50$ keV, $I$=3/2) $S \sim 1000$. The numerical calculation [10] gives $S$=300 for $^{225}$Ra, 400 for $^{223}$Ra and 1000 for $^{223}$Rn. To indicate the accuracy of the calculation we should note that the difference between the values obtained for the Woods-Saxon potential (presented above) and Nilsson potential is within a factor of 2 [13].

The values we obtained are several hundred times larger than the Schiff moment of a spherical nucleus like Hg ($S = -1.4$). An additional enhancement of the atomic EDM appears due to the greater nuclear charge in Ra and Rn and close atomic states of opposite parity [15,16]. Accurate atomic calculations of the EDM for atoms of Hg, Xe, Ra, Rn, and Pu have been performed in Ref. [14].

$\mathcal{T}$ and $\mathcal{P}$-odd nuclear forces can also produce enhanced collective magnetic quadrupole [17] and octupole [18] moments. Although these moments cannot induce the EDM in the ground state of the closed-shell atoms like Xe, Hg, Rn and Ra, they can contribute to the EDM of metastable excited states or of the ground states in open-shell atoms.

Note that the Schiff moment $S$ in Eq. (10) is proportional to the squared octupole deformation parameter $\beta_3^2$. According to [11], in nuclei with a soft octupole vibration mode the dynamical octupole deformation is of the order of $\langle \beta_3^2 \rangle \sim (0.1)^2$, i.e. the same as the static octupole deformation in pear-shaped nuclei. This means that a number of heavy nuclei can have large collective Schiff moments of dynamical origin. A calculation of these moments is the main subject of the present paper.

## III. SCHIFF MOMENT PRODUCED BY SOFT OCTUPOLE VIBRATIONS

A fully microscopic calculation of the Schiff moment in odd-$A$ actinides, even with the aid of simplifying models, such as the particle plus rotor model [19], is very complicated, requires also the solution of the random phase approximation for the soft octupole mode in adjacent even-even isotopes and contains many uncertainties in the choice of the parameters. For our purpose of estimating the magnitude of the effect we use below simple analytical arguments.

The intrinsic nuclear octupole moment in the body-fixed frame is given by [20]

$$O_{\rm intr} = e\int \rho r^3 Y_{30} d^3r \approx \frac{3}{4\pi} eZR^3\beta_3. \tag{11}$$

The intrinsic Schiff moment $S_{\rm intr}$, Eq. (9), is then

$$S_{\rm intr} = \frac{3}{5\sqrt{35}} O_{\rm intr}\beta_2. \tag{12}$$

This relation allows us to approximately express the matrix elements of the Schiff moment operator in odd-$A$ nuclei with static axially symmetric quadrupole deformation $\beta_2$ in terms of the matrix elements of the octupole operator which can be extracted from the observed probabilities of the octupole transitions in an even-even neighbor.

As a characteristic of the effective deformation parameter $\beta_3$ it is convenient to use the r.m.s. value extracted from the reduced octupole transition probability B(E3) [21],

$$B(E3)_{0\to 3} = |\langle 0|O_{\rm intr}|\rangle|^2 \approx \left(\frac{3}{4\pi} eZR^3\right)^2 \langle\beta_3^{\rm rms}\rangle^2 \tag{13}$$

To find the $\mathcal{P},\mathcal{T}$-odd Schiff moment (3) we need to know the matrix elements of the $\mathcal{P},\mathcal{T}$-odd nucleon-nucleon interaction $W$. To the first order in the velocities $p/m$, this interaction can be presented as [6]

$$\hat{W}_{ab} = \frac{G}{\sqrt{2}}\frac{1}{2m}\left((\eta_{ab}\boldsymbol{\sigma}_a - \eta_{ba}\boldsymbol{\sigma}_b)\cdot\boldsymbol{\nabla}_a\delta(\mathbf{r}_a-\mathbf{r}_b) + \eta'_{ab}\left[\boldsymbol{\sigma}_a\times\boldsymbol{\sigma}_b\right]\cdot\{(\mathbf{p}_a-\mathbf{p}_b),\delta(\mathbf{r}_a-\mathbf{r}_b)\}\right), \tag{14}$$

where $\{\ ,\ \}$ is an anticommutator, $G$ is the Fermi constant of the weak interaction, $m$ is the nucleon mass, and $\boldsymbol{\sigma}_{a,b}$, $\mathbf{r}_{a,b}$, and $\mathbf{p}_{a,b}$ are the spins, coordinates, and momenta, respectively, of the nucleons $a$ and $b$. The dimensionless constants $\eta_{ab}$ and $\eta'_{ab}$ characterize the strength of the $\mathcal{P},\mathcal{T}$-odd nuclear forces; experiments on measurement of the EDMs are aimed at extracting the values of these constants.

Now we consider the interaction $W_{ab}$ between the valence nucleon $b$ and the even-even core containing paired nucleons $a$. Under the assumption that the collective octupole excitation does not involve spins of the individual core nucleons, we will keep only the terms that do not contain spin operators $\boldsymbol{\sigma}_a$. Using the contact nature of the interaction (14) and taking the matrix element over the state of the external nucleon $b$ we can present the operator acting onto the core nucleons in the following form:

$$\hat{W}_a = -\frac{G\eta_{ba}}{\sqrt{2}}\frac{1}{2m}\left(\boldsymbol{\nabla}_a\cdot\Psi_b^\dagger(\mathbf{r}_a)\boldsymbol{\sigma}_b\Psi_b(\mathbf{r}_a)\right). \tag{15}$$

The operator $\hat{W}_a$ contains the octupole component proportional to the operator $\hat{O}_{\rm intr} = r^3Y_{30}$ and responsible for the collective excitation. To extract this component, we project out the octupole operator in the intrinsic frame,

$$\hat{W}_a = C_a \hat{O}_{\text{intr}} + \dots, \tag{16}$$

where the amplitude of this component, the contribution of a core nucleon $a$ to the octupole mode, is

$$C_a = \frac{\langle \hat{O}_{\text{intr}} | \hat{W}_a \rangle}{\langle \hat{O}_{\text{intr}} | \hat{O}_{\text{intr}} \rangle_R}. \tag{17}$$

Here we omitted the electric charge $e$ from the definition of the octupole operator $\hat{O}_{\text{intr}}$ since we assume that it acts both on protons and neutrons (an isoscalar octupole mode). In the collective transition to the excited octupole state the matrix element of the octupole operator $\hat{O}_{\text{intr}}$ is enhanced. Therefore, we assume that we can neglect all terms in the expansion (16) except for the one written down explicitly.

For calculating the projection constant $C$ we assume that the amplitude of the octupole vibrations is small compared to the nuclear radius so that the operator $\hat{O}_{\text{intr}} = r^3 Y_{30}$ is acting effectively only within the nucleus, $\hat{O}_{\text{intr}} = 0$ for $r > R + \delta$ where $\delta \ll R$ is some small distance. This allows us to introduce the normalized "octupole" state const $\cdot\, r^3 Y_{30}$ and consistently define the projection procedure. In this case

$$\langle \hat{O}_{\text{intr}} | \hat{O}_{\text{intr}} \rangle_R = \int_0^R r^6 Y_{30}^2 d^3 r = R^9/9, \tag{18}$$

and

$$\langle \hat{O}_{\text{intr}} | \hat{W}_a \rangle = \frac{G\eta_{ba}}{\sqrt{2}} \frac{1}{2m} \int \Psi_b^+(\mathbf{r}) \boldsymbol{\sigma} \Psi_b(\mathbf{r}) \cdot \left( \boldsymbol{\nabla} r^3 Y_{30} \right) d^3 r = \frac{9\sqrt{7}}{40\sqrt{2\pi}} \frac{G\eta_{ba}}{m} R^2 F. \tag{19}$$

Here we introduced the expectation value over external nucleon orbital

$$F = \frac{\langle r^2 (5 n_z^2 \sigma_z - \sigma_z + 2 n_z (\boldsymbol{\sigma} \cdot \mathbf{n})) \rangle}{(3/5) R^2}, \tag{20}$$

where $\mathbf{n} = \mathbf{r}/r$ is the unit vector, $n_z = \cos\theta$. Note that $\langle r^2 \rangle \approx 3R^2/5$, therefore $F \sim 1$. In the case of $^{223,225}$Ra, assuming that the unpaired neutron is in the (asymptotic) Nilsson orbital 1/2[631] with $\sigma_z$=+1 or $-1$, we would get $|F| \approx 0.3$. For a pure $ls$-state $|lm\pm\rangle$ with $m_l = m, s_z = \pm(1/2)$, this factor is given by

$$\langle lm \pm | F | lm\pm \rangle = \pm 2 \frac{l(l+1) - 3m^2}{(2l+3)(2l-1)} \frac{\langle r^2 \rangle}{(3/5) R^2}. \tag{21}$$

For the realistic Nilsson wave functions with the quadrupole deformation around 0.15 or 0.2, the spin-orbital coupling is important, and the wave function of the valence neutron is a superposition of several $|l, m \mp 1/2, \pm 1/2\rangle$ terms. Although the last item in the numerator of the expression (20) contains the matrix elements with $\Delta s_z = \pm 1$ and is sensitive to the interference of the $\sigma_z = \pm$ components that might be constructive or destructive, the quantity $|F|$ is typically between 0.1 and 0.6.

Combining equations above we obtain the effective "octupole" $\mathcal{P}, \mathcal{T}$-odd operator

$$\hat{W}_{\rm oct} = \frac{\langle \hat{O}_{\rm intr}|\hat{W}_a\rangle}{\langle \hat{O}_{\rm intr}|\hat{O}_{\rm intr}\rangle_R}\hat{O}_{\rm intr} = \frac{81}{40}\sqrt{\frac{7}{2\pi}}\,\frac{G\eta_{ba}F}{mR^7}\,\hat{O}_{\rm intr}. \tag{22}$$

Using Eq. (13) for the proton octupole contribution and summing over proton and neutron contributions to the octupole excitation $|{\rm oct}\rangle$, we obtain the expression for the weak matrix element,

$$\langle {\rm g.s.}|W|{\rm oct}\rangle = \frac{81}{40}\sqrt{\frac{7}{2\pi}}\,\frac{G\eta_b F}{mR^7}AR^3\frac{3}{4\pi}\beta_3^{rms}, \tag{23}$$

where $\eta_b = \frac{Z}{A}\eta_{bp} + \frac{N}{A}\eta_{bn}$, $b = n$ for the external neutron or $b = p$ for the external proton.

The transtion from the body-fixed frame to the laboratory frame for the vector Schiff moment gives, as in Eq. (4), an extra factor $KM/I(I+1)$ which for the ground nuclear state, $K = M = I$, is equal to $I/(I+1)$. Collecting all factors we obtain the Schiff moment due to the octupole collective vibrations:

$$\mathbf{S} = \frac{2I}{I+1}\frac{\langle {\rm g.s.}|W|{\rm oct}\rangle\langle {\rm oct}|\mathbf{S}|{\rm g.s.}\rangle}{E_{\rm g.s.} - E_{\rm oct}}, \tag{24}$$

$$S = 0.025F\,\frac{I}{I+1}\;e\beta_2(\beta_3^{rms})^2 ZA^{2/3}\frac{\eta_a G}{mr_0(E_{\rm g.s.} - E_{\rm oct})}. \tag{25}$$

The ratio of this dynamical (vibrational) octupole contribution $S_{\rm soft}$ to the static octupole contribution $S_{\rm stat}$ in the deformed case (10) is equal to

$$\frac{S_{\rm soft}}{S_{\rm stat}} = 2.5F\frac{(\beta_3^{rms})^2}{(\beta_3^{static})^2} \approx \frac{(\beta_3^{rms})^2}{(\beta_3^{static})^2}. \tag{26}$$

This means that the dynamical deformation due to low-frequency octupole vibrations produces the same effect as the static octupole deformation.

When doing numerical estimates we may assume $\beta_3^{rms} = \beta_3^{static} = 0.1$ [9–11]. This gives numerical values (in the units of $e\cdot{\rm fm}^3$) for the Schiff moments of $^{223}$Ra and $^{225}$Ra, $S = 700$ and 400, correspondingly; $S = 1000$ [22] for $^{223}$Rn; and in the interval 100 - 1000 for $^{223}$Fr, $^{225}$Ac and other nuclei with the low octupole mode, $E_{\rm oct} - E_{\rm g.s.} < 100$ keV, i.e. close to the values [9,10] obtained in the numerical calculations for the static octupole deformation. However, our value for $^{239}$Pu, where $E_{\rm oct} - E_{\rm g.s.} = 470\,{\rm keV}$, is $S \sim 80$, or even smaller if we take into account that in the ground state of a harmonic oscillator the vibrational amplitude is proportional to $\omega^{-1/2}$ so that $\langle\beta^2\rangle \propto 1/\omega$ as it is approximately the case for both quadrupole and octupole low-lying vibrations in nuclei [23]. This is much smaller than the estimates $S$(Pu)=300$\cdot S$(Hg)$\approx$ 420 presented in Ref. [11].

The work [11] also claims that the contribution of the octupole vibration to the $^{199}$Hg Schiff moment is about half of its shell model value. Strictly speaking, our formula is not applicable for this case since $^{199}$Hg, probably, has no quadrupole deformation. However, for an estimate we can take $\beta_2 = 0.1$ assuming that a similar contribution may appear due to interaction with soft quadrupole vibrations. Now we can make a comparison between $^{225}$Ra and $^{199}$Hg which both have $I = 1/2$. The octupole transition energy in $^{199}$Hg is

3000 keV [11]. Therefore, in the case of $^{199}$Hg we lose one-two orders of magnitude due to the octupole deformation [if $\beta_3^2 \propto (E_{\rm oct} - E_{\rm g.s.})^{-1}$] and one-two orders of magnitude due to the energy denominator that is sixty times greater than in $^{223,225}$Ra. This gives the octupole contribution to the $^{199}$Hg Schiff moment about $400 \cdot 0.001 \sim 0.4$. The shell-model contribution is $-1.4$. This estimate shows that the octupole contribution should not produce any dramatic changes in the $^{199}$Hg Schiff moment.

## IV. FROM NUCLEAR SCHIFF MOMENT TO ATOMIC EDM

Finally, we should calculate the atomic electric dipole moments induced by the nuclear Schiff moments. The atomic EDM is generated by the $\mathcal{P}, \mathcal{T}$-odd part of the nuclear electrostatic potential $\varphi(\mathbf{r})$. The potential produced by the point-like Schiff moment is usually presented in the form [6] proportional to the gradient of the delta-function at the origin,

$$\varphi(\mathbf{r}) = 4\pi \mathbf{S} \cdot \boldsymbol{\nabla} \delta(\mathbf{r}). \tag{27}$$

The natural generalization of the Schiff moment potential for a finite-size nucleus is [12]

$$\varphi(\mathbf{r}) = -3(\mathbf{S} \cdot \mathbf{r}) \frac{n(r)}{B} \ , \tag{28}$$

where $B = \int n(r) r^4 dr \approx R^5/5$, $R$ is the nuclear radius, and $n(r)$ is a smooth function which is 1 for $r < R - \delta$ and 0 for $r > R + \delta$ while $\delta << R$ is some small distance; $n(r)$ can be taken as proportional to the nuclear density. This potential (28) corresponds to a constant electric field $\boldsymbol{\mathcal{E}}$ inside the nucleus that is directed along the nuclear spin, $\boldsymbol{\mathcal{E}} \propto \mathbf{I}$. The interaction $-e\varphi$ mixes electron orbitals of opposite parity and produces EDMs in atoms.

In Ref. [14] we have performed atomic calculations of coefficients $k$ that define the atomic EDM $d$ in terms of the Schiff moments,

$$d = k \cdot 10^{-17} \cdot \left( \frac{S}{e \cdot \mathrm{fm}^3} \right) e \cdot \mathrm{cm}. \tag{29}$$

The factors $k$ rapidly grow with the nuclear charge that leads to an additional enhancement in Ra, $k = -8.5$, and Rn, $k = 3.3$, in comparison with the lighter electronic analogues, Hg, $k = -2.8$, and Xe, $k = 0.38$. Taking some conservative "minimal" values of the Schiff moments for the different methods of calculations (static analytical, static numerical, soft analytical) we obtain the values of the atomic electric dipole moments given in the Table.

| Nucleus | $S$, $10^{-8} e \cdot \mathrm{fm}^3$ | $d$, $10^{-25} e \cdot \mathrm{cm} \cdot \eta_n$ |
|---|---|---|
| $^{225}$Ra | 300 | 2500 |
| $^{223}$Ra | 400 | 3400 |
| $^{223}$Rn | 1000 | 3300 |

This may be compared with typical values of the induced dipole moment predicted for spherical nuclei, $d = 4$ and $d = 0.7$ for $^{199}$Hg and $^{129}$Xe, respectively.

To conclude, we made analytical estimates for the nuclear Schiff moment in nuclei known as soft with respect to the octupole excitation mode. We found a strong enhancement of the average magnitude close to what was found for nuclei having static octupole deformation. The value of the Schiff moment is typically $10^2 \div 10^3$ in units of $10^{-8} e \cdot \mathrm{fm}^3$; it leads to the prediction of the enhancement of the atomic EDM on the level of $10^3$ as compared to spherical nuclei.

A related idea should be explored in the future: it is known that light isotopes of Rn and Ra are spherical but with a soft quadrupole mode and therefore large amplitude of quadrupole vibrations. The spectra of these nuclei display long quasivibrational bands based on the ground state and on the the octupole phonon, with positive and negative parity, respectively. These bands are connected via low-energy electric dipole transitions. This situation, with a nucleus soft with respect to both quadrupole and octupole modes, seems to be favorable for the enhancement of $\mathcal{P}, \mathcal{T}$-odd effects.

The authors are thankful to the Institute of Nuclear Theory, University of Washington, where this work was started, for hospitality and support. V.Z. acknowledges support from the NSF grant PHY-0070911. V.F. acknowledges support from the Australian Rsearch Council.

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