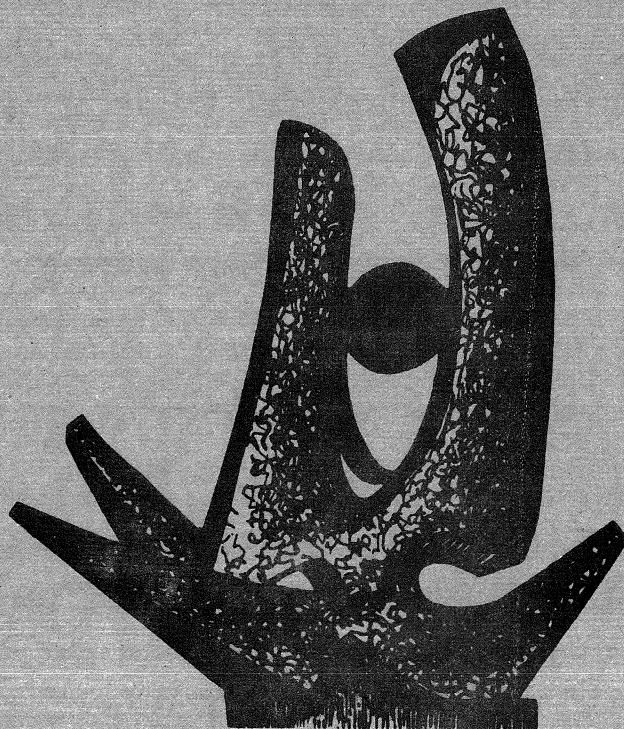


MICHIGAN STATE UNIVERSITY

CYCLOTRON LABORATORY

POPULATION OF LEVELS IN ^{199}Hg FOLLOWING ^{199}Tl DECAY
AND INTERMEDIATE COUPLING CALCULATIONS FOR ^{199}Hg

G. J. MATHEWS, F. M. BERNTHAL, and J. D. IMMELE



MSUCL-128
COO-1779-144

OCTOBER 1974

1111111111

POPULATION OF LEVELS IN ^{199}Hg FOLLOWING ^{199}Tl DECAY
AND INTERMEDIATE COUPLING CALCULATIONS FOR $^{199}\text{Hg}^*$

G. J. Mathews
Departments of Chemistry and Physics and
Cyclotron Laboratory,
Michigan State University, East Lansing, Michigan 48824
and
Department of Chemistry,
University of Maryland, College Park, Maryland 20742

and

F. M. Bernthal
Departments of Chemistry and Physics and
Cyclotron Laboratory,
Michigan State University, East Lansing, Michigan 48824

and

J. D. Immele[†]
Department of Physics and Astronomy,
University of Maryland, College Park, Maryland 20742

[†]Present address: Lawrence Livermore Laboratory, Livermore, California 94550.

Abstract

The electron capture decay of ^{199}Tl to levels in ^{199}Hg has been reinvestigated. Twenty new transitions are assigned to the ^{199}Tl decay scheme and the population of two new levels in ^{199}Hg at 750.4 and 1221.2 keV is confirmed. A quasiparticle-phonon coupling model is applied to low-lying levels in ^{199}Hg . Good agreement with experiment is obtained for the eigenvalues, spectroscopic factors, and electromagnetic transition rates and moments by variation of the $f_{5/2}$ neutron energy and by introducing a phenomenological reduction factor in the coupling matrix element.

KEYWORD ABSTRACT

RADIOACTIVITY ^{199}Tl measured E_γ , I_γ , γ - γ coin, deduced $\log ft$. ^{199}Hg deduced levels, J , π .

NUCLEAR STRUCTURE ^{199}Hg calculated levels, J , π , S , μ , q , $B(E2)$, $B(M1)$, δ , branching ratios. Quasiparticle-phonon coupling.

I. INTRODUCTION

The level structure of ^{199}Hg has been of considerable theoretical and experimental interest because of its apparent amenity to various interpretations. In the core excitation model of de-Shalit¹ and an extension by Kalish and Gal,^{2,3} the one and two phonon 2^+ vibrations of the core are rather strongly mixed among the two observed $3/2^-$, $5/2^-$, doublets which represent core coupling to the ground $p_{1/2}$ neutron. The assumption of small single particle admixture in the core-coupled doublets can apparently account for a number of the decay properties of those states, although problems remain, particularly with regard to the $(5/2)^2 \rightarrow (5/2)^1$ and $(3/2)^2 \rightarrow (3/2)^1$ transitions.² Transfer reaction data⁴ have indicated an apparent fragmentation of the $p_{3/2}$ and $f_{5/2}$ strength throughout the low-lying levels of ^{199}Hg , which suggests in addition that the assumption of small single particle admixture in several of the low-lying "phonon" states is incorrect. Microscopic pairing plus quadrupole calculations^{5,6} employing different treatments of the pairing interaction have met with another difficulty in this region in that they predict a $3/2^-$ ground state. In section IV of this work consideration is given to an intermediate coupling description of the levels in ^{199}Hg . Rather good agreement with experiment is obtained.

In view of the theoretical importance of the relative γ -ray branching intensities from the presumed core-coupled excited states, and the desirability of obtaining a complete picture of the low-energy spectrum

of states in ^{199}Hg , it seemed worthwhile to supplement the early Na(Tl) investigation by Bauer *et al.*⁷ of the 7.4-hour electron capture decay of ^{199}Tl to levels in ^{199}Hg . Subsequent experiments detailed the decay of the $13/2^+$ isomeric state (532 keV, $t_{1/2} = 43$ min) and verified the existence of the $5/2^-$ member of the second particle-core coupled doublet seen in Coulomb excitation work.^{2,8} Confirming many of the results of these later studies, the present work assigns two new states in ^{199}Hg at 750.4 and 1221.2 keV and places 20 new γ -rays in the ^{199}Tl decay scheme.

II. EXPERIMENTAL

Sources of 7.4-hour ^{199}Tl were prepared by the $^{199}\text{Au}(\alpha, 2n)^{199}\text{Tl}$ reaction. Three layers of 12 mg/cm² gold foil (99.99%) were bombarded with 29-MeV alpha particles from the Michigan State University Cyclotron. Several hours were allowed for 5.3-h ^{198}Tl to decay before counting was begun.

Gamma-ray singles spectra were obtained both with a 10.4% Ge(Li) detector (2.4 keV FWHM; 35:1 peak-to-Compton ratio at 1333 keV) and with a 1 cm³ high-resolution spectrometer (650 eV FWHM at 122 keV). The γ - γ coincidence data were obtained with two Ge(Li) detectors positioned at 180° geometry, and in a second experiment, at 90° geometry. The two-parameter 4096 × 4096 coincidence data were stored serially on magnetic tape for later sorting and analysis.

A singles γ -ray spectrum from ^{199}Tl decay is shown in Fig. 1. The ^{199}Tl peaks were assigned on the basis of their decay relative to the known 455.5-keV transition. Assignments to ^{198}Tl and ^{200}Tl were made in a similar way by normalization to the 636.8- and 368.0-keV transitions with reference to the data in Refs. 9 and 10. On the basis of the γ -ray singles data and the relative γ -ray half-lives, the γ -ray transitions listed in Table 1 have been assigned to ^{199}Tl decay. The decay periods for the γ -rays were checked in 10 spectra over a period of 6 half-lives. Two separate experiments were carried out, one with a source relatively rich in ^{200}Tl , the other with ^{198}Tl the dominant contaminant. Analysis of the singles data was carried out with the photopeak fitting routine SAMPO.¹¹ Because of the presence of an unresolvable ^{200}Tl 591.8-keV line, the intensity of the 592.0-keV transition in ^{199}Tl was estimated from coincidence data. Assignment of this line to ^{199}Tl decay was supported by its appearance in the 158- and 471-keV coincidence gates. The 765.7-keV line is so weak that it was impossible to derive a reliable half-life from the singles data, but the line appears quite clearly in the 455- and 247-keV coincidence gates.

Three additional lines for which evidence is seen in the coincidence data can be tentatively assigned. The strongest of these (though not the most certain), the 245.1-keV $(5/2^-)^2 \rightarrow (3/2^-)^1$ transition is indicated with a dashed line in Fig. 3, and a tentative relative intensity limit is deduced from the coincidence data. Somewhat better evidence for a weak 205.6-keV line is seen in the "cleaner" 337-keV coincidence gate (Fig. 2), and in the same gate, the 403.5-keV transition to ground appears,

implying non-negligible feeding between the 414- and 404-keV states via a 10.4-keV transition. A tentative total relative intensity for the 10.4-keV *transition*, based on the coincidence data, is shown in Table 1.

The other coincidence spectra in Fig. 2 confirm the assignment of new levels at 750.4 and 1221.2 keV in ^{199}Hg . Some caution must be exercised in interpreting these data. As the spectra are derived from data which did not include timing information as a third parameter, they could not be corrected for random events in the usual way. In addition, the EC decay of ^{199}Tl with the attendant high x-ray intensity gives rise to relatively intense (x-ray)-(γ -ray) sum lines in the spectra. The intrusion of the 26-hour ^{200}Tl contaminant is also quite clear in many of the spectra. The contaminant lines could be determined from their known decay relationships, and from the variation in their coincidence intensity as a function of time. Nevertheless, some puzzling features remain (e.g. the apparent presence of a line near 158 keV in the 403-keV gate).

III. ^{199}Tl DECAY SCHEME

The γ -ray singles and coincidence data are consistent with the decay scheme proposed in Fig. 3. All of the γ -ray lines that could be associated with ^{199}Tl decay on the basis of half-life and coincidence data are accounted for in the level scheme shown. Also included are low-energy transitions at 36.83, 49.74 and 51.93 keV previously observed in the electron spectroscopy work of Jung and Svedburg.¹² Their

conversion electron data are used as a basis for deducing the conversion coefficients and multiplicities of several transitions below 500 keV, as shown in Table 1. In the normalization, the 158.36-keV ($5/2^- \rightarrow 1/2^-$) transition was assumed to be pure $E2$, with a K-conversion coefficient as calculated by Hager and Seltzer.¹³ In a few cases, multiplicities and mixing ratios are known from the Coulomb excitation γ -ray angular distributions and from γ - γ angular correlations, (cf. the summary in Ref. 14).

The $\log ft$ values for ^{199}Tl decay were calculated from γ -ray intensity balance, with the assumption of 50% feeding to the ^{199}Hg ground state, a datum attributed to Bauer *et al.*^{7,14}

The spin and parity assignments for the ground and first four excited states of ^{199}Hg are consistent with those proposed by Kalish *et al.*⁸ With use of the transfer reaction data of Moyer⁴ and the γ -ray data from this work, it is possible to make further probable assignments for ^{199}Hg levels as follows:

455.5-keV level

A spin and parity $1/2^-$ was assigned to this state first by Jung and Svedburg¹² and subsequently with increased confidence by Bauer *et al.*⁷ Unfortunately, the very weak 297.1-keV line was not observed in the conversion electron spectrum, but the much more intense transitions to the $1/2^-$ ground and $3/2^-$ excited states suggest that this state is also $1/2^-$, and that the 297.1-keV transition should be $E2$. The weight of evidence thus supports

our assignment for this state, although $3/2$ cannot be absolutely excluded.

492.3-keV level

The spin and parity of this state, proposed on the basis of angular correlation measurements by Bauer *et al.* to be $3/2^-$, seems confirmed. The 333.9- and 492.3-keV conversion electron data rule out pure $E2$ for these transitions, and the $\log ft$ as well as the transfer reaction data⁴ provide weaker arguments to support the assignment.

750.4-keV level

The transfer reaction data here apparently indicate $\ell = 1$, and although the γ -ray data cannot exclude $I^\pi = 1/2^-$, the comparable feedings to the lower-lying spin $3/2$ and $5/2$ states suggest that the $3/2^-$ assignment proposed by Moyer⁴ is correct.

1221.2-keV level

The $\log ft = 6.1$ seems to argue against a first-forbidden unique beta-decay to this state, although one must not attach too much significance to this number since the Q -value is only an estimate from systematics. On the other hand, the γ -ray feeding to both the ground and 455-keV $1/2^-$ states is quite weak, and this could be seen as an argument for the higher-spin $5/2^-$ assignment. If

I^π for this state were indeed $1/2^-$ then one might account for its decay properties if the state were largely a $p_{1/2}$ neutron coupled to the two phonon 0^+ configuration. In fact, such a state is predicted by the theory discussed in section IV. However, in the absence of conversion data for any of the transitions from this state, the spin and parity assignment remains in doubt.

IV. THEORETICAL

Although a considerable quantity of experimental data has accumulated in this region, the low-spin states in odd mass mercury isotopes have not, to date, been well described by simple nuclear models. The even-even nuclei neighboring ^{199}Hg exhibit structure consistent with a spherical vibrator description; hence the lowest lying states in ^{199}Hg are usually assumed to involve the coupling of an odd nucleon to the vibrational states of an even-even core. In this work we shall develop this approach. It should be pointed out, however, that Stephens *et al.*¹⁵ have described the even parity high spin states in terms of the $i_{13/2}$ neutron decoupled from a slightly oblate core, and deformation energy calculations have also predicted small oblate deformations ($\beta \cong -0.1$) in this region.¹⁶ The lighter isotopes of Hg ($A < 190$) are thought to exhibit larger deformations, a conclusion based on the large measured isotope shifts for those isotopes.

The particle-vibration coupling interpretation was first applied by de-Shalit,¹ who invoked a simple core excitation configuration for

the first excited $5/2^-$, $3/2^-$ (158, 208 keV) doublet. The model has since been extended to include coupling to the second 2^+ phonon state by Kalish and Gal^{2,3} when it was found⁸ that the second $3/2^-$, $5/2^-$ (403, 413-keV) doublet is also populated by Coulomb excitation. Crucial to this core excitation description is the assumption of small single particle admixture into the core excited states, yet spectroscopic factors obtained by Moyer⁴ suggest substantial $p_{3/2}$ and $f_{5/2}$ character in these states in ^{199}Hg . Thus, one is compelled to introduce a more sophisticated description for the low-lying level structure in ^{199}Hg .

Microscopic calculations employing a pairing plus quadrupole interaction have not been entirely successful in this region. The quasiparticle plus one phonon treatment of Kisslinger and Sorensen⁵ fails to reproduce the observed level ordering, and a more sophisticated particle number conserving formalism with two-phonon core states introduces only slight improvement.⁶ The result of the latter microscopic calculations led Lo Iudice⁶ to suggest the need for inclusion of higher phonon excited states. An improvement in the results obtained by variation of the $d_{3/2}$ proton energy also indicates the sensitivity of the level structure to the coupling strength and the choice of single particle energies.

For the remainder of this section we discuss the application to ^{199}Hg of the particle-vibration or intermediate coupling model.¹⁷⁻²⁰ In this model, the coupling of as many of three phonons to the odd nucleon is easily achieved. It is found that most of the level

structure, transition rates, and electromagnetic moments are satisfactorily reproduced by a variation of the coupling strength and the $f_{5/2}$ single particle energy.

Intermediate coupling in the unified model begins with a separation of the Hamiltonian into contributions from the core and odd particle.

$$H = H_c + H_{s.p.} + H_{int.}$$

H_c describes macroscopic core vibration. Although some evidence exists for small ground-state deformation in ^{199}Hg ,¹⁵ it was found that inclusion of second and third order anharmonic terms in the core Hamiltonian does not, for the most part, appreciably affect the results presented. In the absence of sufficient experimental data to determine the magnitude of such terms, we have chosen not to introduce the additional free parameters associated with core anharmonicity. Including only quadrupole phonons, we then write

$$H_c = \hbar\omega \sum_{\mu} (b_{2\mu}^{\dagger} b_{2\mu})$$

where $\hbar\omega$ is the vibrational energy of the phonon and $b_{2\mu}^{\dagger}$, $b_{2\mu}$ are the quadrupole phonon creation and destruction operators respectively.

Intermediate coupling has most frequently been applied to nuclei not more than 3 nucleons removed from a closed shell, where the effects

of the pairing condensation can be neglected. However, as one moves farther away from closed shell nuclei such neglect is inappropriate.²¹ Thus we introduce the quasiparticle approximation²² and write:

$$H_{sp} = \sum_j E_j \alpha_j^\dagger \alpha_j$$

where the E_j are quasiparticle energies and α_j^\dagger (α_j) are quasiparticle creation (destruction) operators.

The interaction of the odd particle with the core is determined by the overlap of the particle wave function with the nuclear surface and is given by:¹⁷

$$H_{int} = -r \frac{dV}{dr} \sum_\mu Y_2^\mu(\theta, \phi) \left(\frac{\hbar\omega}{2C_2} \right)^{1/2} [b_{2\mu} + (-1)^\mu b_{2\mu}^\dagger] \quad (1)$$

where C_2 is the stiffness constant for quadrupole deformation and (r, θ, ϕ) refer to the coordinates of the odd nucleon. We choose a representation

$$|j, NR; IM\rangle = \sum_{m_j M_R} (j m_j R M_R | IM) \alpha_{j m_j}^\dagger \left\{ (b_2^\dagger)^N \right\}_{R M_R} |\tilde{0}\rangle, \quad (2)$$

where N denotes the number of phonons built on the core state $|\tilde{0}\rangle$, and R is the spin of the N coupled phonons. The only off-diagonal elements are due to H_{int} , and the matrix elements of H_{int} are

$$\begin{aligned} & \langle j', N'R'; IM | H_{int} | j, NR, IM \rangle \\ &= \frac{\alpha}{2} \langle k(r) \rangle \left\{ \frac{\hbar\omega}{2\pi C_2} \right\}^{\frac{1}{2}} (-1)^{I-\frac{1}{2}} \left\{ (2j+1)(2j'+1) \right\}^{\frac{1}{2}} \left\{ \begin{matrix} j & R & I \\ R' & j & 2 \end{matrix} \right\} \\ & (j' \frac{1}{2} j - \frac{1}{2} | 20) \times \left\{ (-1)^{R'} \langle N'R' || b || NR \rangle + (-1)^R \langle NR || b || N'R' \rangle \right\} \\ & (U_j U_{j'} - V_j V_{j'}) \end{aligned}$$

where the coupling strength

$$\langle k(r) \rangle = \langle n'l' | r \frac{dV}{dr} | n\ell \rangle ,$$

is calculated from a Woods-Saxon potential and the U_j and V_j are the appropriate pairing occupation amplitudes which result from the quasiparticle transformation. An assumption frequently employed is to ignore the j dependence of $\langle k(r) \rangle$. We find, however, that this term varies by about 40% over the j values of interest in this work, so we calculate $\langle k(r) \rangle$ explicitly.

The reduced matrix elements of the phonon annihilation operator are consistent with Racah's definition. Conversion to other notations is discussed in ref. 18.

In the literature^{18,20} the factor

$$\eta = \langle k(r) \rangle \left[\frac{\hbar\omega}{2\pi C_2} \right]^{\frac{1}{2}}$$

is usually varied to obtain a best fit to experiment. If C_2 is estimated from the $B(E2, 0_1^+ \rightarrow 2_1^+)$ for the neighboring even-even nuclei, then the average value $\langle k(r) \rangle$ obtained from the fit is generally in the range 20 - 40 MeV, and less than the calculated value ($\langle k(r) \rangle_{\text{calc}} \geq 40$ MeV). To correct for the tendency of the model to over-estimate the coupling strength, we define a new phenomenological coupling parameter, α (cf. eqtn. 3). In this way the physical meaning of C_2 and the j dependence of $\langle k(r) \rangle$ is maintained. This reduction factor, α , is normally in the range 0.4-1.0 and is reminiscent of a similar factor in Coriolis coupling fits to experimental data for rotational nuclei. Recently, the latter phenomenon has been associated by Ring²³ with a renormalization of the inertial mass by the odd particle. We can speculate that such a renormalization may also account for the diminished particle-vibration coupling.

The magnetic dipole operator we write as:

$$M(M1, \mu) = \left(\frac{3}{4\pi}\right)^{\frac{1}{2}} \left[g_{\ell} \ell_{\mu} + g_S s_{\mu} + g_R R_{\mu} \right] \text{ (n.m.)}$$

where g_{ℓ} and g_S are the orbital and spin g factors for the odd particle and $g_R = \frac{Z}{A}$ is the g factor for the core.

The magnetic moment of a state i is then given by:

$$\mu = \left[\frac{4\pi I}{3(I+1)(2I+1)} \right]^{\frac{1}{2}} \langle iI \| M(M1) \| iI \rangle$$

and the magnetic dipole reduced transition probability is:

$$B(M1; iI \rightarrow i'I') = \frac{1}{2I+1} \left| \langle i'I' \| M(M1) \| iI \rangle \right|^2$$

where

$$\langle i'I' \| M(M1) \| iI \rangle = - \left[\frac{3(2I+1)(2I'+1)}{4\pi} \right]^{\frac{1}{2}} \sum_{\ell_j, NN'R} C_{i', (\ell_j, NR; I')} C_i (\ell_j, NR; I)$$

$$\left\{ (-1)^{R+I+j'+\ell} \begin{Bmatrix} j' j 1 \\ I I' R \end{Bmatrix} \right\} \left\{ (2j+1)(2j'+1) \right\}^{\frac{1}{2}} \left[(-1)^{j'-\frac{1}{2}} \begin{Bmatrix} \ell \ell 1 \\ j j' \frac{1}{2} \end{Bmatrix} \right]$$

$$\times g_{\ell} \left\{ \ell(\ell+1)(2\ell+1) \right\}^{\frac{1}{2}} + (-1)^{j-\frac{1}{2}} \begin{Bmatrix} \frac{1}{2} \frac{1}{2} 1 \\ j j' \ell \end{Bmatrix} g_s (3/2)^{\frac{1}{2}} \Big]$$

$$+ (-1)^{R+I'+j} \begin{Bmatrix} R R 1 \\ I I' j \end{Bmatrix} g_R \left\{ R(R+1)(2R+1) \right\}^{\frac{1}{2}} \delta_{j'j} \left\{ \left(U_j U_j + V_{j'j} V_{j'j} \right) \right\}$$

For the electric quadrupole operator we write:

$$M(E2, \mu) = \frac{3}{4\pi} ZeR_o^2 \left(\frac{\hbar\omega}{2C_2} \right)^{\frac{1}{2}} \left[(-1)^\mu b_{2\mu} + b_{2\mu}^\dagger \right]$$

$$+ \sum_n \left[\left(e_n - \frac{Ze}{A^2} \right) r_n^2 Y_2^\mu(\theta, \phi) \right]$$

Since the calculation should determine the degree of core polarization, we choose not to introduce the additional degree of freedom of an effective neutron charge, i.e. we take $e_n = 0$. Then dropping the recoil

term, $\left(\frac{Z}{A^2} \ll 1\right)$, the expression for the electric quadrupole moment simplifies to:

$$Q = \left[\frac{16\pi(2I-1)I}{(2I+1)(2I+3)(I+1)} \right]^{\frac{1}{2}} \langle iI \parallel M(E2) \parallel iI \rangle$$

The electric quadrupole reduced transition probability is then:

$$B(E2; iI \rightarrow i'I') = \frac{1}{2I+1} |\langle i'I' \parallel M(E2) \parallel iI \rangle|^2$$

where

$$\langle i'I' \parallel M(E2) \parallel iI \rangle = \left[(2I+1)(2I'+1) \right]^{\frac{1}{2}} \times$$

$$\sum_{\ell j N R N' R'} C_{i'}(\ell j, N' R'; I') C_i(\ell j, N R; I) \frac{3}{4\pi} Z e R_o^2 \left(\frac{\hbar\omega}{2C_2} \right)^{\frac{1}{2}} (-1)^{j+I'} \times$$

$$\left\{ \begin{matrix} R & R' & 2 \\ I' & I & j \end{matrix} \right\} \left[(-1)^{R'} \langle N R \parallel b \parallel N' R' \rangle + (-1)^R \langle N' R' \parallel b \parallel N R \rangle \right]$$

The nucleus ^{199}Hg is sufficiently removed from ^{207}Pb that one would not expect the arrangement of single particle energies to be entirely the same. Indeed, the changes in level orderings in this region suggest that the $f_{5/2}$ neutron level is varying rapidly as one departs from the closed shell configuration. Unfortunately, however, the transfer reaction results are insufficient to indicate the correct level spacings, due to the rather large error bars on the data. In view of this difficulty and the sensitivity of the results to the choice of single particle energies, the $f_{5/2}$ single particle energy as well as the coupling coefficient α were varied in order to optimize the fit of the calculated eigenvalues to the experimental spectrum. The other level energies were taken from ^{207}Pb .

The single particle energies used in the calculations are shown in Table 2 and compared with the levels in ^{207}Pb . Although the $f_{5/2}$ energy has changed considerably, the magnitude of the $f_{5/2} - f_{7/2}$ split is comparable with that calculated from the Woods-Saxon parameters given later in this section. The calculations were found to be sensitive to the position of the $p_{3/2}$ neutron state as well as the $f_{5/2}$. For comparison, results are also presented with the $p_{3/2}$ neutron state depressed by 200 keV. These results are labeled ThII. Results obtained from variation of only the $f_{5/2}$ energy are labeled ThI.

For the coupling renormalization coefficient, α , a value of 0.50 was found to give reasonable agreement with experiment. The phonon energy, $\hbar\omega = 0.390$ MeV, was obtained from the average excitation energy for the first 2^+ state in ^{198}Hg and ^{200}Hg . Likewise, $C_2 = 86$ MeV was obtained

from the $B(E2; 0_1^+ \rightarrow 2_1^+)$ in ^{198}Hg and ^{200}Hg .^{9,10} Quasiparticle energies and occupation probabilities were obtained from the usual Bogoliubov equations.²² The pairing gap parameter, Δ , was determined from the odd-even mass difference to be 0.76 MeV. From the requirement that the average nucleon number be conserved, the Fermi energy, λ , was determined to be -0.10 MeV for ThI and -0.13 MeV for ThII, relative to the $p_{1/2}$ single particle energy.

The matrix element $\langle k(r) \rangle$ is not particularly sensitive to the choice of Woods-Saxon parameters. For the calculations the following parameters were employed:

$$\begin{array}{ll} V_o = 44 \text{ MeV} & R_o = R = 7.15 \text{ fm} \\ V_{SO} = 32 \text{ MeV} & a_o = 0.67 \text{ fm} \end{array}$$

where the notation is that of Blomquist and Wahlborn.²⁴

The eigenvalues of the intermediate coupling Hamiltonian are shown in Figure 4. Lines are drawn between states with similar experimental and theoretical properties. For the most part, only the most experimentally accessible levels are included in the figure (i.e. levels with single quasiparticle or one phonon admixture greater than or equal to about 1%). The agreement with experiment up to about 0.6 MeV is seen to be quite good. The uncertainty in the spin and parity assignments for the higher-lying states precludes unambiguous comparison between theory and experiment, with the possible exception of the $\ell = 3$ strength at 1.454 MeV which compares quite favorably with the $f_{7/2}$ single quasiparticle strength predicted by the theory at this energy.

Spectroscopic factors and energies for the levels indicated on Fig. 4 are summarized in Table 3. The experimental spectroscopic factors were unnormalized and have been normalized to the average of the spectroscopic strength predicted for the $1/2^-$ ground state in ThI and ThII.

Magnetic dipole and electric quadrupole moments are given in Table 4, while the transition probabilities for a few low-lying transitions are presented in Table 5. Experimental energies have been used to derive the magnitude of the E2/M1 mixing ratios and the total branching ratios which are given in Table 6.

Comparison between experiment and theory leads us to the following statements concerning the configuration of the lowest lying states in ^{199}Hg :

$1/2^-$ ground state

The measured spectroscopic strength and magnetic moment are in good agreement with the calculations. Not surprising is the implication that the state is nearly pure single quasiparticle in nature, about 80% $|p_{1/2}, 00; 1/2\rangle$.

$5/2^-$, $3/2^-$ (158, 208 keV) and $3/2^-$, $5/2^-$ (403, 413 keV) doublets

The total collective and single quasiparticle strength distributed between these states is correctly predicted by the theory. This is an improvement over the core excitation treatment. Although the

precise division of single quasiparticle and collective components is not well reproduced in the present work, we nevertheless conclude that these states result from considerable mixing of the $|p_{3/2}, 00; 3/2^- \rangle$ and the $|p_{1/2}, 12; 3/2^- \rangle$ configuration and similarly the $|f_{5/2}, 00; 5/2^- \rangle$ and the $|p_{1/2}, 12; 5/2^- \rangle$ configurations. The $p_{3/2}$ admixture into the first $3/2^-$ state is over-estimated, as is indicated by the spectroscopic factor in Table 3. Similarly, the $B(E2)$'s in Table 5 suggest that the configuration of the first phonon coupled to the $p_{1/2}$ quasiparticle is weighted too heavily into the second doublet. This discrepancy between theory and experiment is due at least somewhat to the particular choice of single particle energies, as shown by the improvement obtained in the ThII results.

The calculated magnetic moment for the first excited state is in acceptable agreement with experiment. The discrepancy between the experimental and predicted magnetic moment for the $3/2^-$ (208-keV) state, although striking, is not entirely unreasonable. The dipole moment for this state is quite sensitive to the $p_{3/2}$ single quasiparticle admixture which the theory has overestimated somewhat. If the $|p_{3/2}, 00; 3/2^- \rangle$ amplitude is constrained to agree with transfer reaction results for the spectroscopic factor of this state, then within the limits of error, one can obtain a positive magnetic moment for this state comparable in magnitude to the experimental value. Still, the discrepancy is significant, and similar $3/2^-$ states in ^{193}Hg and ^{201}Hg as well as ^{195}Pt are well known²⁸ to have

a magnetic moment of the same magnitude but negative sign. These considerations seem to indicate further investigation of the magnetic moment of this state would be worthwhile.

Other interesting consequences of the theory are the occurrence of appropriately small transition probabilities for the 205-keV and 245-keV transitions, in agreement with the virtual absence of these lines from the experimental spectrum. The hindrance of the 255.5-keV transition predicted in this work is a substantial improvement over the core excitation treatment.² The branching ratio from the $3/2^-$ (208 keV) level is too large by several orders of magnitude. This is again due to the over-estimation of the single quasiparticle character of the first $3/2^-$, $5/2^-$ doublet since the $B(M1)$ matrix element between these states is quite sensitive to the inclusion of configurations coupling $p_{1/2}$ to the first phonon. Finally of interest is the lack of $|p_{1/2}, 22; 5/2^- \rangle$ or $|p_{1/2}, 22; 3/2^- \rangle$ admixture into these doublets which has been assumed in the core excitation treatment,^{2,3} but is not reproduced by the intermediate coupling calculations.

$1/2^-$ (455 keV) state

The theoretical results for this state are quite dependent on the choice of single particle energies. The level indicated in the ThI results of Figure 4 is included because it is the only state

with decay properties similar to those of the experimental level, even though the $p_{1/2}$ single quasiparticle amplitude of this state is less than 1%. If one accepts the correspondence between theory and experiment, then this state is due in large part to an $|f_{5/2}, 12; 1/2^- \rangle$ configuration.

$3/2^-$ (493 keV) state

The theory appears to predict the properties of this state reasonably well. The spectroscopic strength is indicative of small $p_{3/2}$ admixture. The 37-, 78-, and 89-keV deexciting transitions are predicted to be sufficiently small to account for their absence from the experimental spectrum. The theory finds the $3/2^-$ (493 keV) state to be predominantly the result of an $|f_{5/2}, 12; 3/2^- \rangle$ coupling.

$13/2^+$ (532 keV) state

The predicted magnetic moment and spectroscopic strength are slightly overestimated. The general agreement, however, is not unsatisfactory. The theory would predict that this state is about 45% $|i_{13/2}, 00; 13/2^+ \rangle$ with nearly as much $|i_{13/2}, 12; 13/2^+ \rangle$ admixture. An interesting consequence of this result is that the reduction in single particle character combined with the effects of pairing condensation should account for much of the M4 hindrance observed in the decay from this state to the $5/2^-$ (158 keV) level. The implication is then that the estimate of the single particle character in

the $5/2^-$ state which was deduced in ref. 2 on the basis of the M4 hindrance factor is too small, and that the presence of sizable $f_{5/2}$ admixture is not inconsistent with the data presented in ref. 2.

$5/2^-$, $5/2^-$ (567, 698 keV) doublet

Unfortunately only transfer spectroscopic data are available to compare with the theory for these states, although (n,γ) studies establish the $5/2^-$ spin and parity. No state in ThII exhibits enough spectroscopic strength to compare well with the first $5/2^-$ level. The corresponding state from ThI is highly admixed having a largest component ($\sim 35\%$) due to $|p_{3/2}, 12; 5/2^- \rangle$. The second $5/2^-$ state appears to be dominated by an $|f_{5/2}, 12; 5/2^- \rangle$ configuration.

$(5/2^-, 7/2^-)$ (1454 keV) state

If this state has $I^\pi = 7/2^-$ then the $f_{7/2}$ spectroscopic strength compares well with the $|f_{7/2}; 00; 7/2^- \rangle$ amplitude predicted by the theory at this energy.

V. SUMMARY

The electron capture decay of ^{199}Tl to levels in ^{199}Hg has been reinvestigated. In addition to the placement of 20 new γ -rays in the

decay scheme, the results of the present work have confirmed the placement of two new levels at 750.4 keV ($I^\pi = 1/2, 3/2^-$) and 1221.2 keV ($I^\pi = 1/2, 3/2, 5/2^-$).

The lowest lying levels in ^{199}Hg for which spins and parities are well established have been interpreted in terms of an intermediate coupling model. The model introduces several improvements over previous core excitation interpretations which could not account for spectroscopic factors and some decay properties. Even though it was not possible to admix sufficiently the collective and single quasiparticle components of the first two $3/2^-$, $5/2^-$ doublets by a variation of only the $f_{5/2}$ neutron energy and coupling coefficient α , most of the low energy nuclear structure is satisfactorily reproduced for ^{199}Hg in the intermediate coupling model.

VI. ACKNOWLEDGEMENTS

The authors wish to thank W. B. Walters at the University of Maryland and R. A. Warner, R. W. Gales, R. R. Todd, and R. B. Firestone at Michigan State University for helpful discussions and assistance during various phases of this work.

REFERENCES

- * Work supported by the U. S. Atomic Energy Commission under contract AT(11-1)-1779 and by the National Science Foundation.
1. A. de-Shalit, Phys. Rev. 122, 1530 (1961).
 2. R. Kalish and A. Gal, Nucl. Phys. A175, 652 (1971).
 3. R. Kalish, B. Herskind, R. R. Borchers, and G. M. Heestand, Nucl. Phys. A206, 558 (1973).
 4. R. A. Moyer, Phys. Rev. C5, 1678 (1972).
 5. L. S. Kisslinger and R. A. Sørensen, Rev. Mod. Phys. 35, 853 (1963).
 6. N. Lo Iudice, D. Prosperi, and E. Salusti, Nucl. Phys. A127, 221 (1969).
 7. R. W. Bauer, L. Grodzins, and H. H. Wilson, Phys. Rev. 128, 694 (1962).
 8. R. Kalish, R. R. Borchers, and H. W. Kugel, Nucl. Phys. A161, 637 (1970).
 9. R. L. Auble, Nucl. Data B6, 319 (1971).
 10. M. J. Martin, Nucl. Data B6, 387 (1971).
 11. J. T. Routti and S. G. Prussin, Nucl. Instr. and Method. 72, 125 (1969).
 12. B. Jung and J. Svedberg, Nucl. Phys 20, 630 (1960).
 13. R. S. Hager and E. C. Seltzer, Nucl. Data A4, 1 (1968).
 14. M. B. Lewis, Nucl. Data B6, 3555 (1971).
 15. D. Proetel, D. Benson, M. R. Maier, R. M. Diamond, and F. S. Stephens, in *Proceedings of the International Conference on Nuclear Physics*, Munich, Germany, 1973, edited by J. de Boer and H. J. Mang (North-Holland, Amsterdam, 1973), p. 194.
 16. A. Faessler, U. Götz, B. Slavov, and T. Ledergerber, Phys. Lett. 39B, 579 (1972).
 17. A. Bohr and B. R. Mottelson, Mat. Fys. Medd. Dan. Vid. Selsk., 27, No. 16 (1957).

18. K. Heyde and P. J. Brussard, Nucl. Phys. A104, 81 (1967).
19. V. Parr, Nucl. Phys. A164, 576 (1971).
20. A. Covello and G. Sartoris, Nucl. Phys. A93, 481 (1967).
21. K. W. C. Stewart, B. Castel, and B. P. Singh, Phys. Rev. C4, 2131 (1971).
22. N. N. Bogoliubov, Zh. Eksperim, Soviet Phys. JETP 7, 51 (1958).
23. P. Ring and H. J. Mang, University of California, Lawrence Berkeley Laboratory preprint LBL-2968 (to be published).
24. J. Blomquist and S. Wahlborn, Arkiv für Fysik 16, 545 (1959).
25. L. Grodzins, R. W. Bauer and H. H. Wilson, Phys. Rev. 124, 1897 (1961).
26. R. J. Reimann and M. N. McDermott, Phys. Rev. C5, 2065 (1973).
27. J. Bonn, G. Huber, H. J. Kluge, U. Köpf, L. Kluger, E. W. Otten, and J. Rodriguez, *Proceedings of the International Conference on Nuclear Moments and Nuclear Structure, 1972*, J. Phys. Soc. Japan, 34, Suppl. 317 (1973).
28. G. H. Fuller and V. W. Cohen, Nucl. Data. A5, 433 (1969).

TABLE 1. Gamma-ray Transitions from Decay of ^{199}Tl to ^{199}Hg

Energy (keV)	Relative Intensity	Conversion Coefficients ^a						Multipolarity
		α_K	α_{L_I}	$\alpha_{L_{II}}$	$\alpha_{L_{III}}$	α_{M_I}	$\alpha_{M_{II}}$	
10.4 ^b 1	(0.08) ^c							
36.83 ^d 3	(0.11)		17 4			6 2		M1
49.83 ^e 4	3.6 ^e		11 3	1.0 3	0.19 7	2.4 7		M1
51.93 ^d 6	(0.21)		7 3			3 1		M1
158.36 3	40 2	0.30 ^a		0.31 7	0.18 4		0.14 3	E2
195.30 5	2.1 2	1.0 2	0.19 5					M1 + E2
205.(6) ^b	0.08 3							
208.20 3	99 5	0.8 1	0.15 3		0.009 4	0.04 1		M1 + E2
245.(1) ^b	≤ 0.3							
247.26 3	75 4	0.5 1	0.10 2					M1
255.5 1	0.10 3							
258.14 11	0.58 6							
284.09 3	17.8 9	0.39 8	0.07 2					M1
294.94 10	0.42 4							

Footnotes to Table 1:

- a Conversion coefficients calculated from electron intensities of Jung and Svedberg,¹² assuming α_K for the 158.36-keV transition is that for a pure E2.¹³
- b Line assigned from coincidence data only.
- c Unlike the other intensities in this table, this number represents *total transition intensity* (γ -ray and conversion electron), deduced from 337-keV coincidence data.
- d Line not seen in this work. The γ -ray intensity is estimated from total conversion electron intensity¹² assuming a pure M1 transition.¹³
- e Energy and intensity (relative to 208.2-keV γ -ray) taken from Ref. 14 based on ¹⁹⁹Au decay data, since those data appear to be more reliable than those of Refs. 7 and 12 for ¹⁹⁹Tl decay.

TABLE 2

Neutron Single Particle Energies (in MeV) Employed in the Calculations

	$3p_{1/2}$	$2f_{5/2}$	$3p_{3/2}$	$1i_{13/2}$	$2f_{7/2}$	$1h_{9/2}$
$\epsilon_j(^{207}\text{Pb})$	0.0	-0.570	-0.898	-1.633	-2.340	-3.409
$\epsilon_j(\text{ThI})$	0.0	0.4	-0.898	-1.633	-2.340	-3.409
$\epsilon_j(\text{ThII})$	0.0	0.4	-1.1	-1.633	-2.340	-3.409

TABLE 3. Energies and Spectroscopic Factors for Levels in ^{199}Hg

E(level) (MeV)	J^π	E_I	E_{II}	$S_I(d,t)$	$S_{II}(d,t)$	$S_{\text{exp}}(d,t)^a$
0.0	$1/2^-$	0.0	0.0	0.72	0.68	0.70 (35)
0.158	$5/2^-$	0.106	0.112	0.97	0.88	1.1 (6)
0.208	$3/2^-$	0.157	0.225	2.29	1.63	0.34 (17)
0.403	$3/2^-$	0.398	0.421	0.20	0.54	0.52 (26)
0.413	$5/2^-$	0.396	0.404	0.07	0.08	1.2 (6)
0.455	$1/2^-$	0.512	0.511	0.001	0.08	0.02 (1)
0.493	$3/2^-$	0.561	0.578	0.11	0.13	0.02 (1)
0.532	$13/2^+$	0.643	0.624	5.6	5.7	3.5 (1.7)
0.667	$5/2^-$	0.538		0.15		0.15 (7)
0.698	$5/2^-$	0.561	0.559	0.13	0.28	0.35 (1.7)
1.459	$(7/2^-)$	1.591	1.520	1.7	1.8	1.3 (6)

^aThe experimental spectroscopic factors given in Ref. 4 were unnormalized and have been derived by normalizing the spectroscopic factor for the $1/2^-$ ground state to the average of the theoretical spectroscopic factors predicted in I and II.

TABLE 4

Magnetic dipole and electric quadrupole moments for levels in ^{199}Hg

E(level) (MeV)	I^π	μ_I (n.m.)	μ_{II} (n.m.)	μ_{exp} (n.m.)	Q_I (eb)	Q_{II} (eb)	Q_{exp} (eb)
0.0	$1/2^-$	0.63	0.62	0.5027(3) ^a	-	-	-
0.158	$5/2^-$	1.32	1.32	1.03(8) ^b	-0.73	-0.74	
0.208	$3/2^-$	-1.62	-1.38	0.52(16) ^c	0.54	0.46	
0.404	$3/2^-$	0.82	0.60		-0.15	-0.08	
0.413	$5/2^-$	1.79	1.67		0.22	0.20	
0.455	$1/2^-$	0.78	0.45		-	-	
0.493	$3/2^-$	0.32	0.40		0.19	0.22	
0.532	$13/2^+$	-1.70	-1.70	-1.0147(8) ^d	2.0	2.0	2.0(1.3) ^e
0.667	$5/2^-$	0.056	-		-0.33	-	
0.698	$5/2^-$	0.713	1.13		-0.10	-0.26	
1.459	$(7/2^-)$	-0.65	-0.22		0.08	0.52	

^a ref. 14^b ref. 25^c ref. 3 See text for comment on the sign of this moment.^d ref. 26^e ref. 27

TABLE 5. Electric quadrupole and magnetic dipole reduced transition probabilities for some low lying transitions in ^{199}Hg

Transition	E_γ (MeV)	$B(E2)_I$ (eb) ²	$B(E2)_{II}$ (eb) ²	$B(E2)_{\text{exp}}$ (eb) ²	$B(M1)_I$ (n.m.) ²	$B(M1)_{II}$ (n.m.) ²	$B(M1)_{\text{exp}}$ (n.m.) ²
$(5/2^-)_1 \rightarrow \text{g.s.}(1/2^-)$	0.1536	0.0640	0.0751	0.125(6) ^a	-	-	-
$(3/2^-)_1 \rightarrow \text{g.s.}$	0.2082	0.0772	0.123	0.124(8) ^a	0.642	0.473	0.03
$(3/2^-)_1 \rightarrow (5/2^-)_1$	0.04983	1.34×10^{-3}	3.61×10^{-3}	0.009(4) ^b	3.80×10^{-4}	2.13×10^{-4}	0.05
$(3/2^-)_2 \rightarrow \text{g.s.}$	0.40350	0.168	0.125	0.085(10) ^a	0.0336	0.0989	0.09
$(3/2^-)_2 \rightarrow (5/2^-)_1$	0.24514	0.0212	0.0223	-	0.0318	0.0362	≤ 0.01
$(3/2^-)_2 \rightarrow (3/2^-)_1$	0.19530	0.0381	0.0778	0.27 ^b	0.0297	0.183	0.12
$(5/2^-)_2 \rightarrow \text{g.s.}$	0.41385	0.182	0.176	0.033(5) ^a	-	-	-
$(5/2^-)_2 \rightarrow (5/2^-)_1$	0.25549	0.0192	0.0243	-	4.07×10^{-3}	2.61×10^{-3}	$\leq 9 \times 10^{-4}$
$(5/2^-)_2 \rightarrow (3/2^-)_1$	0.20565	0.0255	0.0183	-	4.70×10^{-4}	2.34×10^{-3}	≤ 0.001
$(5/2^-)_2 \rightarrow (3/2^-)_2$	0.1035	1.88×10^{-3}	2.85×10^{-3}	-	0.0112	0.0190	≤ 0.002

^a Derived from Coulomb excitation data.⁸

^b Derived from lifetime and mixing ratio data.¹⁴ The maximum error for these derived experimental values may be taken as 50%. In cases where the E2/M1 mixing ratio has not been measured, these numbers represent an upper limit assuming a pure M1 transition.

TABLE 6. Gamma-ray transition mixing ratios $\left(\left| \delta \right| = \left[\frac{T_Y(E2)}{T_Y(M1)} \right]^{\frac{1}{2}} \right)$ and branching ratios

$$R = \frac{[T_Y(E2) + T_Y(M1)] \text{ ground state transition}}{[T_Y(E2) + T_Y(M1)] \text{ branching transition}}$$

Branching Transition	E_γ (MeV)	$ \delta_I $	$ \delta_{II} $	δ_{exp}^a	R_I	R_{II}	R_{exp}^a
$(3/2^-)_1 \rightarrow \text{g.s.}$	0.20820	0.060	0.089	+0.27 ² _b -0.65 ² _b			
$(3/2^-)_1 \rightarrow (5/2^-)_1$	0.04983	0.078	0.17	+0.017 ⁶	1.3×10^5	1.6×10^5	33 ⁸
$(3/2^-)_2 \rightarrow \text{g.s.}$	0.40350	0.75	0.38	+0.32 ²			
$(3/2^-)_1 \rightarrow (5/2^-)_1$	0.2451	0.17	0.16		7.2	14	≥ 46
$(3/2^-)_2 \rightarrow (3/2^-)_1$	0.19530	0.18	0.10	± 0.24 ⁸	15	5.4	7 ¹
$(5/2^-)_2 \rightarrow (5/2^-)_1$	0.2555	0.47	0.65		19	24	16 ⁶
$(5/2^-)_2 \rightarrow (3/2^-)_1$	0.2056	1.3	0.48		150	60	20 ¹⁰
$(5/2^-)_2 \rightarrow (3/2^-)_2$	0.0104	4.6×10^{-6}	2.7×10^{-6}		1.2×10^5	7.1×10^4	$(1.0 \text{ } 5) \times 10^{4c}$
$(1/2^-)_2 \rightarrow (5/2^-)_1$	0.29707				7.7	1.8	36 ⁵

Continued ..

Table 6 (Contd.)

$(1/2^-)_{2^+}(3/2)_1$	0.24726	0.10	3.4×10^{-3}		0.12	0.16	1.3	1
$(1/2^-)_{2^+}(3/2)_2$	0.05193	0.091	0.083		4.0×10^4	1.5×10^4	500	300^c
$(1/2^-)_{2^+}(5/2)_2$	0.04161				2.3×10^3	8.0×10^7		
$(3/2^-)_{3^+}g.s.$	0.49230	1.0	0.75					
$(3/2^-)_{3^+}(5/2)_1$	0.33393	2.1	3.0	-0.24 2	0.81	1.2	0.86	9
$(3/2^-)_{3^+}(3/2)_1$	0.28409	0.36	0.40		1.5	2.1	0.69	7
$(3/2^-)_{3^+}(3/2)_2$	0.0888	0.089	0.11		14.9	30.3		
$(3/2^-)_{3^+}(5/2)_2$	0.0784	0.042	0.027		190	140		
$(3/2^-)_{3^+}(1/2^-)_2$	0.03683	0.013	0.069		93	2.4×10^3	100	50^c

^a Mixing ratios are from Ref. 14. Branching ratios are from the present work.

^b Experimental measurements inconsistent (cf. Ref. 14).

^c Estimated from conversion electron data.

FIGURE CAPTIONS

Figure 1. Singles γ -ray spectrum from decay of ^{199}Tl to ^{199}Hg .
Contaminant lines from ^{198}Tl and ^{200}Tl are indicated.

Figure 2. Representative γ - γ coincidence spectra from ^{199}Tl decay (90°
geometry). The lines gated are (top to bottom):

- | | |
|-------------|-------------|
| (a) 158 keV | (c) 404 keV |
| (b) 337 keV | (d) 492 keV |

Figure 3. The decay of ^{199}Tl to levels in ^{199}Hg .

Figure 4. Comparison of calculated and experimental level energies
of ^{199}Hg . The experimental levels not assigned from this
decay study are taken from Refs. 2 and 4.

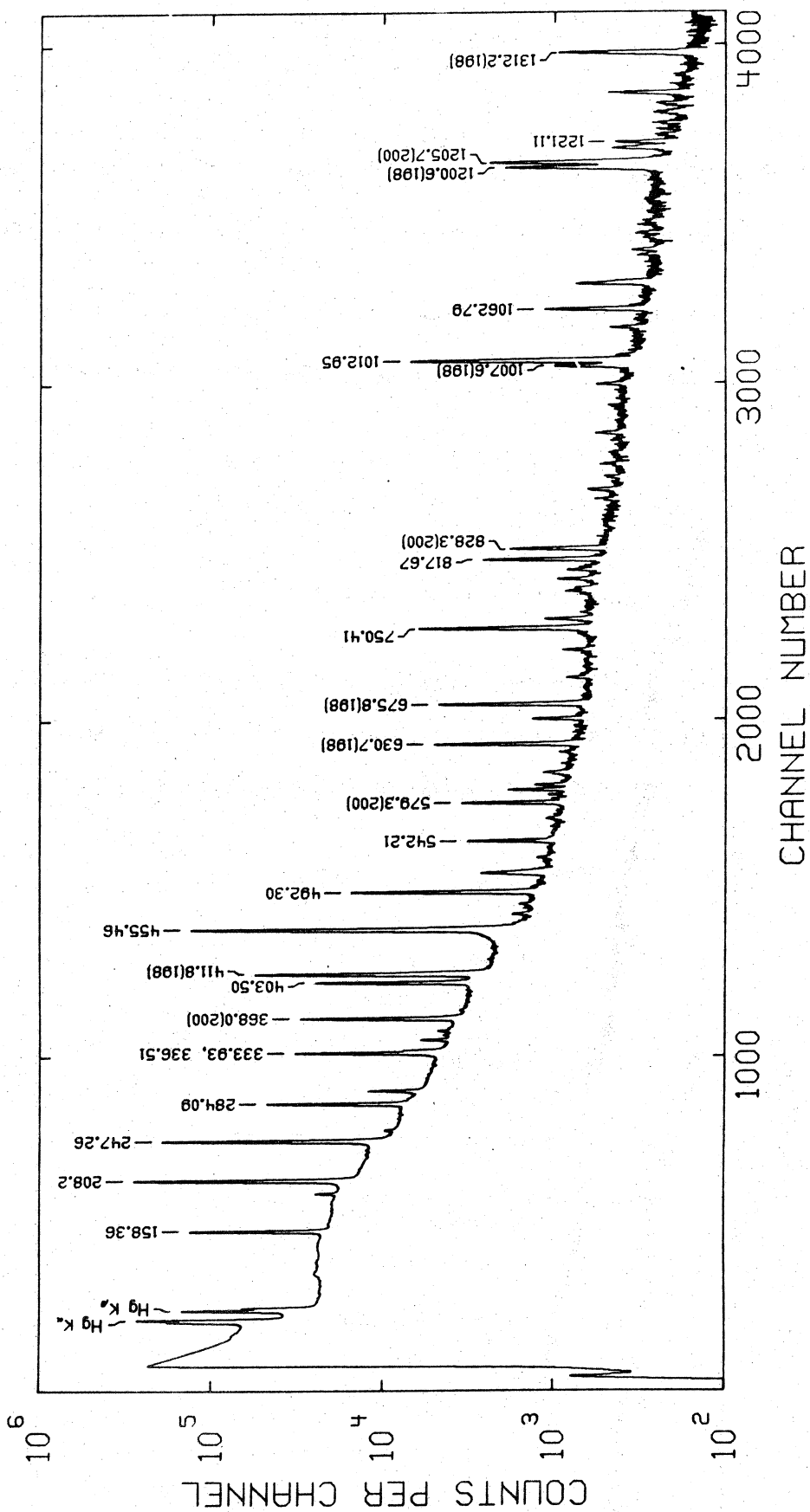


Figure 1

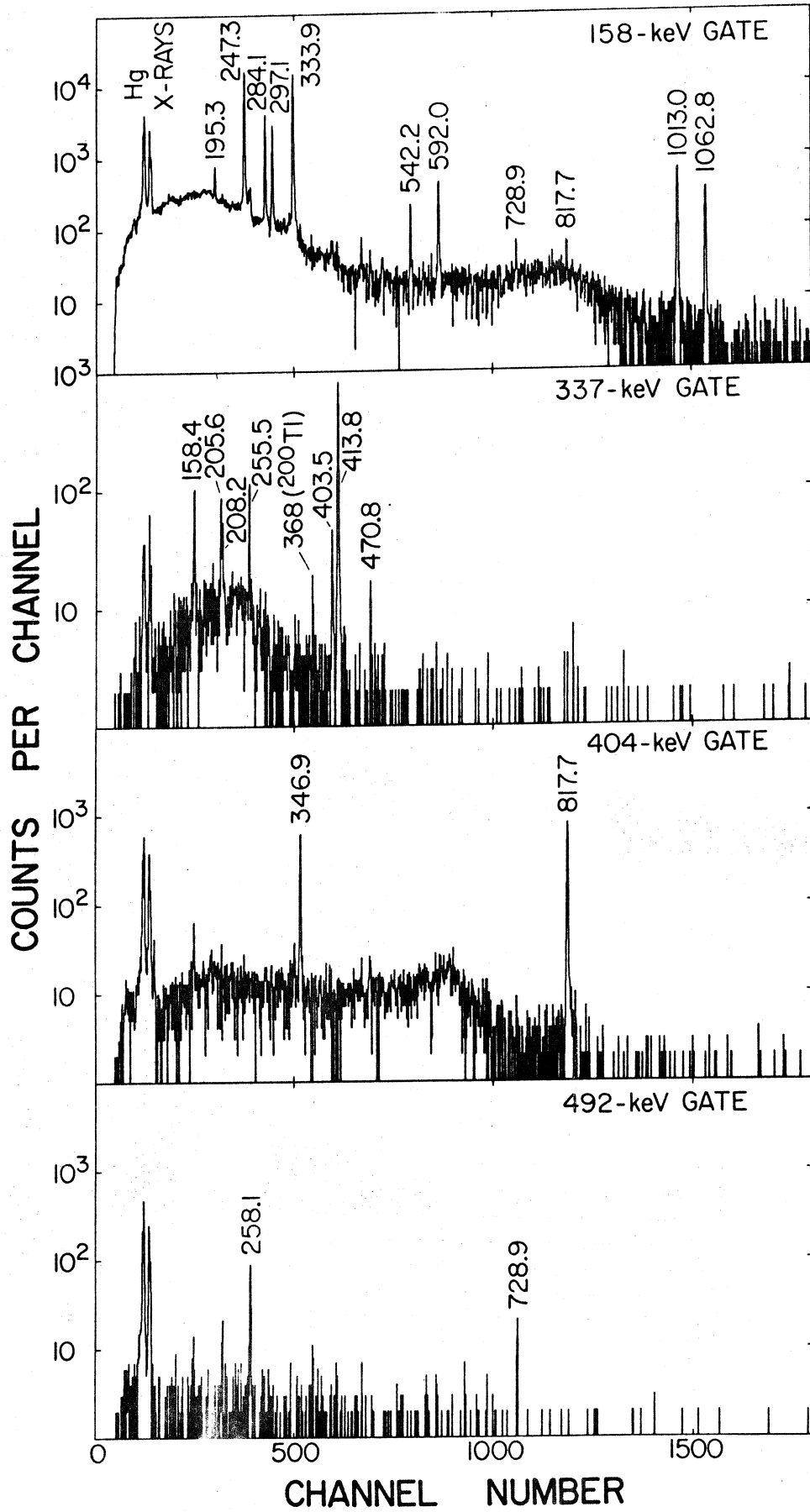


Figure 2

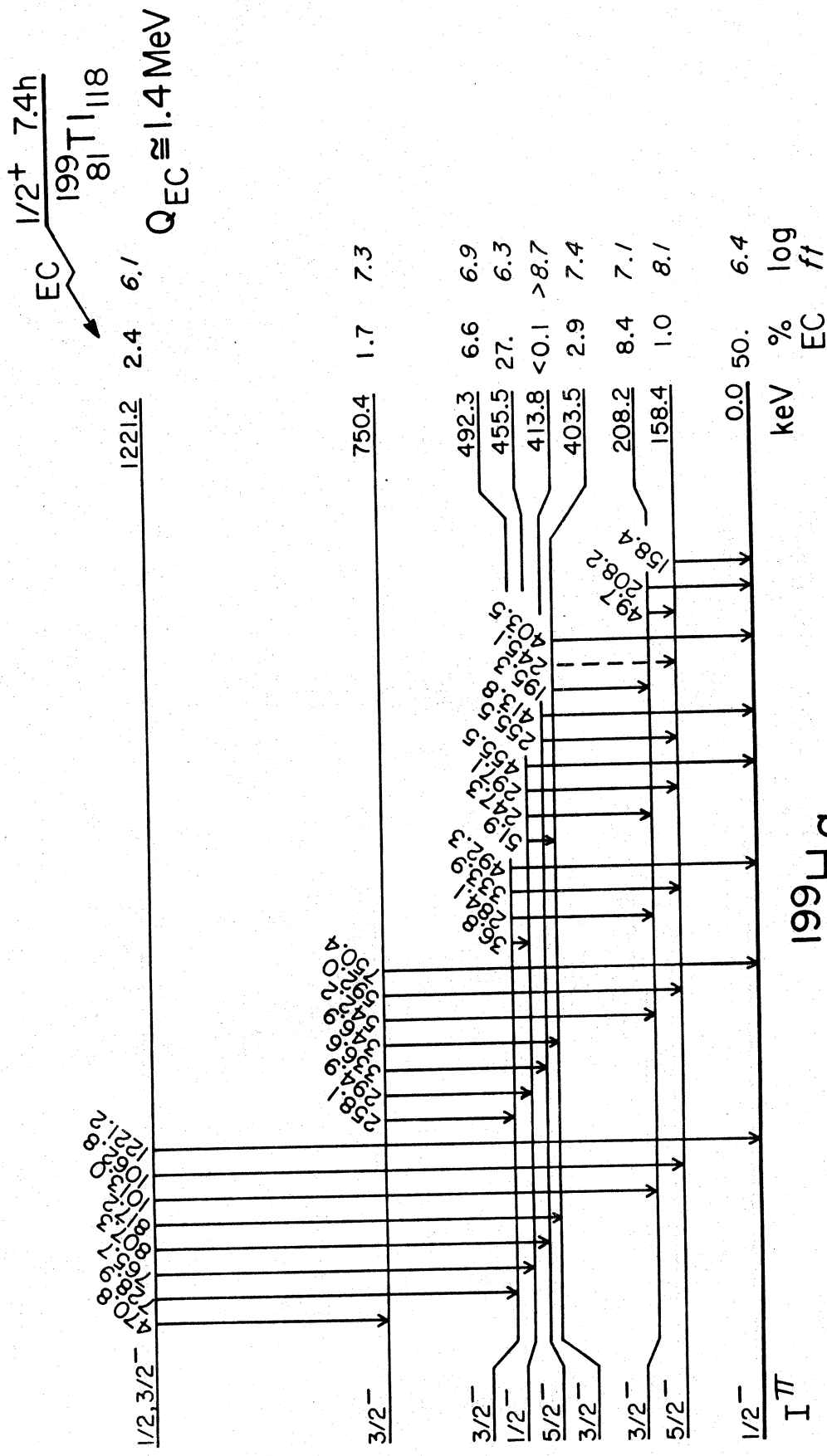


Figure 3

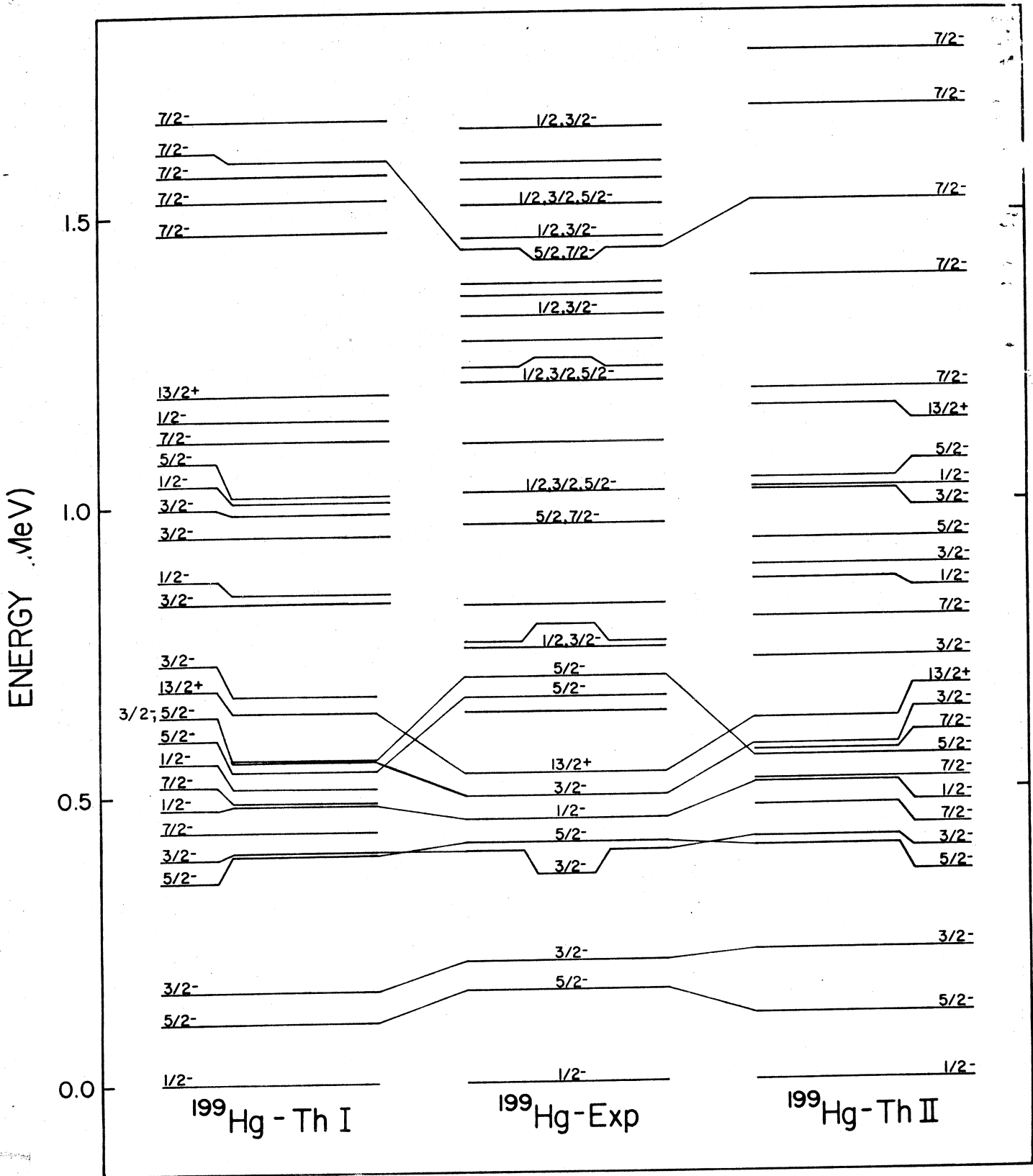


Figure 4

