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CYCLOTRON LABORATORY

FOCUSING PROPERTIES OF SUPERCONDUCTING CYCLOTRON MAGNETS

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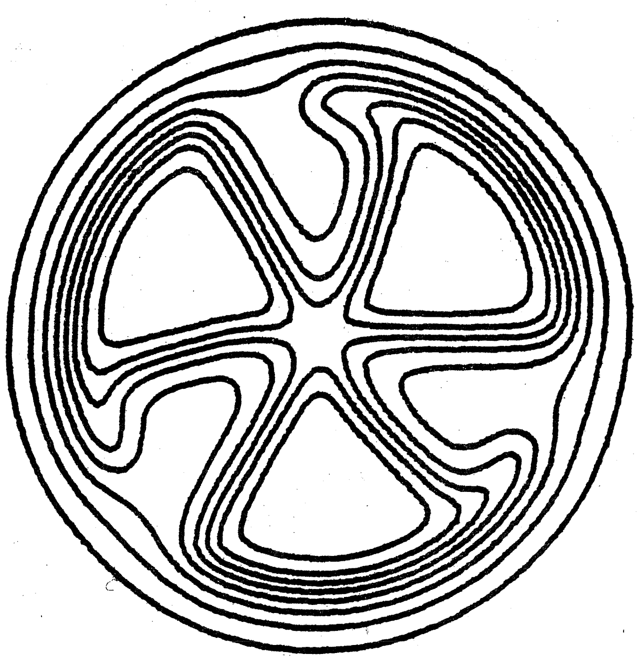

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ABSTRACT

Superconducting main coils combined with conventional sectored iron pole tips constitute a promising design configuration for a new generation of compact, low-cost heavy-ion cyclotrons. This paper gives results of focusing strength calculations for such cyclotrons for a variety of possible pole tips. The focusing is found to be highly sensitive to the average strength of the magnetic field, moderately sensitive to the magnet gap in the hills, and relatively independant of the number of sectors, the relative width of hill and valley and the magnet gap in the valleys. Spiraling of the magnet pole tips increases the focusing but substantially less than expected. Several of these properties are strikingly different from the corresponding properties of conventional low field magnets. The focusing limit of high field magnets leads to a linear dependance of the energy per nucleon on the charge to mass ratio of the ion over much of the operating range which contrasts with the usual quadratic dependance.

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I. INTRODUCTION

Large superconducting coils with fields of 3 tesla and higher are now available in reliable low cost packages.[1] Fraser, et al. have proposed to use such coils in the magnet of a heavy-ion cyclotron with expected advantages in cost and compactness.[2] An important limitation on the performance of such a cyclotron is the focusing, which would still be obtained from sectors of iron as in present cyclotrons.[3] In view of the importance of the focusing we have undertaken a survey of possible pole-tip geometries to ascertain their focusing effectiveness and to determine general design guidelines. This paper presents results and conclusions from this study.

II. UNIFORM M APPROXIMATION

In the limit of high fields, iron "saturates" and it becomes a reasonable approximation to assume that the magnetic moment distribution in the iron is everywhere uniform and equal to its saturation value, i.e. $\tilde{M} = \tilde{k}M_s$ where $\tilde{k}$ is a unit vector in the z direction. Given $\tilde{M}$, one can immediately proceed to compute fields, since a magnetic moment distribution is equivalent to either an array of volume currents $\tilde{J}_M=\text{curl } \tilde{M}$ and surface currents $\tilde{j}_M=\tilde{M}\text{x}\tilde{n}$ or to an array of magnetic poles $\rho_M=-\text{div } \tilde{M}$ and $\sigma_m=\tilde{M}\cdot\tilde{n}$.[4] Calculations of this type have long been used to approximately design high field pole tips[5] and magnetic channels.[6]

In order to quantitatively assess the validity of this "uniform M" approximation we have made a comparison with an accurate relaxation calculation for two dimensional magnets.[7] The results of this comparison are shown in Fig. 1, the crosses showing the relaxation results, the curves the results from the uniform M approximation. The geometries are reasonable two dimensional representations of cyclotron pole tips with three choices of parameters as indicated in the figure. The uniform M calculations are seen to be in good agreement with the relaxation calculations and the latter in turn are essentially equivalent to experiment. The uniform M calculations are quick and, in view of the indicated accuracy, we proceed to use such calculations to compute the fields of a large number of possible three dimensional pole tips in order to evaluate their probable focusing performance.

III. FIELD FLUTTER FOR VARIOUS POLE TIP SHAPES

Our survey is based on a standard case geometry show in Figure 2 with parameter values $g_H=3''$, $g_V=36''$, $\theta_H=\theta_V=45°$ (4 sectors) $r_{max}=24''$ and $\langle B\rangle_0 = 3.5$ tesla. We have then varied each of these parameters over a plus-minus range and computed fields. (In Section IV we consider cases where the pole tips are spiraled). A quick estimate of the focusing power of a given field is obtained by computing the "flutter",

$$F(r) = \langle (B-\langle B\rangle)^2\rangle/\langle B\rangle^2 \qquad (1)$$

where

$$\langle \ldots \rangle = (2\pi)^{-1} \int_0^{2\pi} \ldots \, d\theta$$

The focusing energy limit of an isochronous field with given F is then approximately

$$(\gamma-1) = F/2 \qquad (2)$$

where γ is the usual ratio of total energy to rest energy. (The accuracy of this simple formula for the energy limit is compared with exact calculations in Section IV.)

The flutter calculated for the 4-sector standard case magnet is shown in Figure 3 plotted vs. radius. In the calculation it is assumed that coils provide an axially symmetric field which adds to the average field of the iron such that the total average field is given by

$$\langle B\rangle = [1-(r/138'')^2]^{-1/2} \times 3.5 \text{ tesla} \qquad (3)$$

for $0\leq r\leq 24''$ and $\langle B\rangle(25'') = 3.471$ tesla and $\langle B\rangle(26'') = 3.347$ tesla. This is then an average field which would be exactly isochronous for circular orbits going to 15 MeV/nucleon at r=24" and with a turning over of the edge as appropriate for a realistic field. (In later figures this average field is designated $\langle B\rangle_{circ}$--in Section IV this circular orbit isochronous field is compared with fields which are isochronous for actual orbits.) The maximum value of the flutter in Figure 3 is 0.032 and hence the iosochronous energy limit of the field from (2) is about 16 MeV/ nucleon.

Figure 4 shows the effect of raising and lowering the sector number to 6 and 3, respectively, keeping equal width hills and valleys and with other parameters fixed as for Figure 3. At large radii the flutter is seen to be rather insensitive to the sector number in much the same fashion as one would find in low field magnets.

Figure 5 shows the effect of varying the hill width by ±10° keeping the sector number and other parameters at their standard case values. The flutter is seen to be relatively insensitive to the hill width and the ±10° changes both lower the flutter by an exactly equal amount. This is in sharp contrast to the normal low field behavior where flutter varies approximately as the inverse square of hill width. The origin of this difference is that the high field calculation assumes coils which bring <B> to its stipulated value (3) independent of sector width whereas in the usual low field situation the <B> is due to the iron and therefore varies with hill width.

Figure 6 and 7 show the effect on F of increasing and decreasing the hill gap and the valley gap respectively. The dependence on hill gap is strong (approximately linear) although not so strong as would be expected in a low field magnet. The effect of changing the valley gap is small.

Figure 8 shows the effect on F of changing the average field by ±1.5 tesla. The flutter varies inversely as the square of <B>--the variation is strong and in marked contrast with the normal low field behavior where F is largely independent of magnet excitation.

The high field relation between F and <B> leads to a revised energy limitation for superconducting cyclotrons as sketched in Figure 9. For a given magnet excitation there is a bending

limit characterized by the usual

$$E/A = K(Z/A)^2 \tag{4}$$

where $K=(eB\rho)^2/2m_o$ and the charge and mass of the ion are respectively $q=Ze$ and $m=Am_o$, and a focusing limit characterized by E/A=constant--the dashed line of Figure 9. The useful operating operating range is the crosshatched region below both limits. On the low Z/A side of the point where the two limits cross the energy per nucleon has its traditional $(Z/A)^2$ dependence. On the high Z/A side of this point it is not possible to operate at full field since focusing would be inadequate. The field must be reduced but as the field is lowered the bending limit falls like B^2 while the focusing strength increases like $1/B^2$ as indicated in Figure 8. The field strength at which focusing and Bρ limits coincide is the real operating limit and equating the expressions for focusing and Bρ leads to the result that the energy in this region varies linearly with Z/A. We therefore introduce a new constant K_f such that the maximum E/A which can be focused is given by

$$E/A = K_f(Z/A) \tag{5}$$

We call K_f the focusing energy limit.

The focusing limit K_f is a key design parameter for a high field magnet. Figure 10 for example compares performance of a K=340, K_f=120 magnet versus a K=510, K_f=70 magnet. The stronger focusing of the smaller magnet makes its actual energy higher over much of the operating range. Judgements as to the relative importance of high energy light ions vs the heavier ions must then be made in order to decide whether to emphasize focusing or bending in a particular design.

IV. EFFECT OF SPIRAL POLE TIPS

Spiraling of pole tips introduces alternating gradient components which increase the net focusing of the field as is well known.[8] The effective value of the flutter in such a field is multiplied by the factor

$$S(r) = 1+2\tan^2\alpha(r) \qquad (6)$$

where $\alpha(r)$ is the angle between the radius vector at radius r and the tangent to the pole tip edge at radius r. The actual gain in focusing is less than that expected since the spiral in the field is less than the spiral in the pole tips due to edge effects and also the flutter falls as spiral is introduced since the linear distance between pole tips is reduced by the spiral. Eq. 6 is then not adequate for accurately assessing the effect of spiral. Our procedure therefore is to compute fields from the uniform M assumption and then to use an exact equilibrium orbit program[9] for computing the axial focusing frequency ν_z.

The results of such a study are presented in Figure 11 for five pole tip spirals, the polar coordinate location of the sector edges being given by $\phi(r) = a(r/24") + \phi_o$ with a=0,0.8,1.2,1.6 and 2.0. The focusing increases steadily but

not with the a^2 dependence which would be expected from (6). The actual increase in focusing strength is compared with the expected increase in Table I. The ratio of actual to expected is seen to steadily decrease down to 0.53 for the a=2.0 case. Nevertheless the spiral has a strong effect in **increasing the focusing and hence in raising the focusing limit** K_f.

The equilibrium orbit calculations let us check two of the previous approximations, namely whether (2) is a good representation of the maximum useable energy and whether (3) is a good isochronous field. Figure 12 gives the results of this check for the standard case. In the curve labeled "Smooth Approximation" ν_z is taken to be $(F-(r/\langle B\rangle)d\langle B\rangle/dr)^{1/2}$ which is equivalent to (2) while the two curves labeled "E.O." give the actual ν_z for the average field of equation (3), "$\langle B\rangle_{circ}$", and for an accurately isochronized 15 MeV/nuc average field "$\langle B\rangle_{isoch}$". The smooth approximation differs substantially from the E.O. curve in the large radius region but the E.O. curve for the carefully isochronized field moves in the opposite direction and gives $\nu_{z\ min.} = 0.08$ for 15 MeV/nucleon. A ν_z of less than 0.08 is generally troublesome so that in total the estimate of 16 MeV/nuc from Equation (2) is about right and certainly sufficient for purposes here.

V. CONCLUSIONS

The results presented herein can be summarized in a listing of high field magnet design guidelines as follows:

1. The focusing F(r) is approximately independent of hill width and strongest for equal width hills and valleys.

2. The focusing is weakly dependent on sector number and strongest for low sector number.

3. The focusing is inversely proportional to the hill gap and approximately independent of valley gap.

4. The focusing is inversely proportional to the square of the average field strength. The maximum usable energy therefore varies as $E/A=K_f(Z/A)$ over much of the operating range.

5. Spiraling of the pole tips increases focusing but less than expected depending on the degree of spiral.

These guidelines are based on the uniform M approximation for the magnet iron which appears to be an accurate description. They are valid over a wide range of parameters typical of most proposed superconducting magnets, and provide planning guidelines for the designer in the development of final pole tips optimized for particular design requirements.

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TABLE I.--Comparison of expected increase in focusing strength (second column) with actual increase in focusing strength (third column) and ratio of actual to expected (fourth column) for various pole tip spirals (first column). All quantities are evaluated at 18" radius which is a typical point. S(r) is from Eq. (6), ν_z is from Fig. 11, k is (r/<B>)d<B>/dr, and F is from eq. 1.

$\phi(r)$	S(r)	$(\nu_{z_{E.O.}}^2 + k)/F$	ratio
0	1	1	1
$0.8 \frac{r}{24''}$	1.72	1.427	0.83
$1.2 \frac{r}{24''}$	2.62	1.839	0.70
$1.6 \frac{r}{24''}$	3.88	2.412	0.62
$2.0 \frac{r}{24''}$	5.50	2.917	0.53