

# A high-current EBIT as a charge breeder for the reacceleration of rare isotopes at the NSCL \*

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Reacceleration of low-energy rare isotope beams available from gas stopping of fast-fragment beams or from an ISOL target station to energies in the range of 0.3-12 MeV/nucleon is needed for experiments such as low-energy Coulomb excitation and transfer reaction studies and for the precise study of astrophysical reactions. The implementation of charge breeding as a first step in a reaccelerator is a key to obtaining a compact and cost-efficient reacceleration scheme. For highest efficiency it is essential that single charge states are obtained in a short breeding time. A low-emittance beam must be delivered. An Electron Beam Ion Trap (EBIT) has the potential to meet these requirements. An EBIT-based charge breeder is presently under design and construction at the NSCL as part of the construction of a reaccelerator for stopped beams from projectile fragmentation. This new facility will have the potential to provide low-energy rare isotope beams not yet available elsewhere.

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## I. INTRODUCTION

The increasing availability of stopped beams at fragmentation facilities [1, 2] and the success of the experiments taking advantage of these unique beams [3–5] have led to the demand for reaccelerated beams in the energy range of 0.3-12 MeV/nucleon. Beams from fast fragmentation at these energies will allow for the study of safe Coulomb excitation, transfer reactions and astrophysical reactions of nuclides not available at ISOL facilities. For this reason reacceleration of stopped fragmentation beams is an important component of new and proposed rare-isotope research facilities [6, 7]. The coupled cyclotron facility at the NSCL was the first facility to provide thermalized beams [8] from projectile fragmentation for experimental studies [3, 5, 9]. An expansion of the experimental opportunities is ongoing with the design and construction of a re-accelerator.

For optimum conditions for experiments with reaccelerated beams, the reaccelerator scheme must offer a) high efficiency for ions of all elements available at the facility, b) beam rate capacity that matches the maximum secondary beam rates, c) high beam purity to minimize

background and d) variable time structure of the reaccelerated beam from microsecond pulses to continuous beams.

The implementation of charge-state boosting as the first step in a reaccelerator is recognized as the best way to obtain a very compact and cost-efficient reacceleration scheme while meeting the above requirements. This was first demonstrated with REX-ISOLDE [10] at CERN, which uses an electron beam ion source (EBIS), REX-EBIS [11] to charge-breed rare isotopes delivered from the ISOLDE mass separator. For the NSCL, a similar reacceleration scheme has been chosen, as illustrated in Fig. 1. The high-energy fragment beams will first

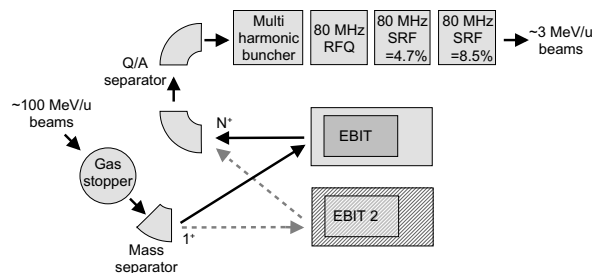


FIG. 1: Concept of EBIT-based reacceleration at the NSCL

be converted into a keV-energy beam in a gas stopper [1, 12, 13]. After mass separation, the singly-charged low-energy beams will be injected into an Electron Beam Ion Trap (EBIT) charge breeder. The EBIT will be lo-

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cated on a high voltage platform, which can be raised to a potential of a few tens of kV during breeding and before extraction. This is needed to match the velocity of the ejected highly charged ions (“ $n^+$ ”) to the requirements of the following radiofrequency-quadrupole (RFQ) accelerator. The  $n^+$  ions will pass through an achromatic Q/A separator to select the desired charge state and to suppress unwanted background ions before entering the accelerator. The compact linac consists of a multi-harmonic buncher, a room-temperature RFQ and superconducting RF structures [14]. A second EBIT, operated in a “push-pull” mode, is foreseen as an option to maximize efficiency in both continuous beam and pulsed beam operation.

An EBIS based charge breeder is presently being developed at Brookhaven [15]. This system is optimized to provide very high intensity pulses of heavy ions for the Relativistic Heavy Ion Collider (RHIC). An EBIT type charge breeder [16] has been built by the Max Planck Institute for Nuclear Physics in Heidelberg in collaboration with TRIUMF. This breeder will be used for TITAN [17], an ion trap project for the study of highly charged-ions of rare isotopes produced via the ISOL technique at ISAC. The TITAN-EBIT is the first EBIT system optimized for fast charge breeding of externally injected ions. The system has successfully passed first off-line tests and is now installed and being brought into operation at TRIUMF. Among existing EBIS/EBIT-type charge breeders, the expected performance of the TITAN appears to be closest to that required for an efficient reacceleration scheme. Therefore the TITAN EBIT design has been chosen as the basis for the design of the charge breeder for the NSCL reaccelerator.

The breeding efficiency of the EBIT is determined by the acceptance of the incoming singly charged ion beam, the efficiency of breeding into a particular charge state in a given time, and the extraction efficiency. The timescale of the charge breeding of ions inside an electron beam is set by the ionization factor, i.e. the product of breeding time  $\tau$  and electron current density  $j$ . As this product is solely determined by the cross sections for successive electron impact ionization, a short breeding time implies large current density. In order not to lose rare isotopes in the breeding process to radioactive decay, a breeding time of  $\approx 10$  ms is desirable. As an example, an electron current density of  $10^4$  A/cm<sup>2</sup> will be required for the breeding of ions up to  $Z \approx 35$  into Ne-like or higher charge states within this time, using cross sections based on W. Lotz’s empirical formula [18]. Assuming appropriate compression of the electron beam, this current density can for example be obtained with an EBIT providing a magnetic field strength of 3 T and electron currents exceeding 0.5 A. The TITAN-EBIT, designed for its magnetic field strength of up to 6 T and electron currents of up to 5 A should be able to provide even larger current densities.

## II. ACCEPTANCE CALCULATION

As a consequence of the constant ionization factor, the highest acceptance for a continuously injected beam is obtained, if a long trap region is combined with a large electron beam radius and high electron current density. In order to explore the operating parameters needed to yield large acceptance for the future NSCL EBIT, injection simulations have been performed based on the TITAN-EBIT configuration. The calculation is carried out in three steps: First the radial coordinates of ions injected into the EBIT are calculated. This information is then used to evaluate the fraction of time the ions spend inside the electron beam when they orbit inside the EBIT. In a third step the actual time the ions linger in the trap is used together with breeding cross-sections and the results from the preceding two simulation steps to obtain the probability of ions ending up in the  $2^+$ -state as a function of beam emittance. As injection mode the overbarrier or “accu-mode” [19] is investigated.

### A. Injection of singly-charged ions

Ions are injected into the EBIT and their radial position in the trap center is recorded as a function of the initial radial coordinates where they started outside the magnetic field. The axial starting position is set 0.6 m away from the trap center, slightly upstream the collector, so the ions will experience the potential over the entire relevant path of the electrons. The shape of the magnetic field has been derived from the position of the coils in the TITAN-EBIT design and then scaled to yield either 6 T or 1 T in the trap center. For the actual injection simulation the axial field is used and the radial components are derived in paraxial approximation. The electric field due to the electrodes has been obtained from Simion calculations. The electrode structure is il-

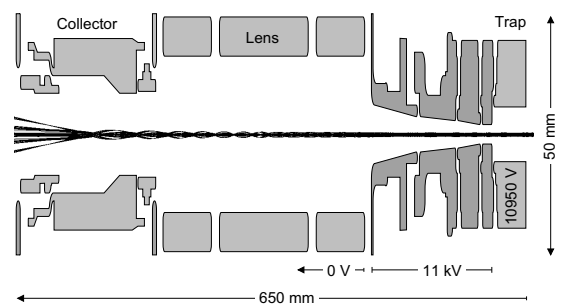


FIG. 2: Electrode structure used for acceptance calculations.

lustrated in Fig. 2 together with typical voltages used in the simulations and sample trajectories of injected ions.

The electron beam is assumed to be box-shaped. The electric potential  $\phi$  for this approximation as a function

of radius  $r$  is given by the following formula [20]:

$$\phi = \frac{Q_e}{2\pi\epsilon_0} \begin{cases} \left[ \frac{1}{2}(1 - r^2/b^2) + \ln(a/b) \right] : 0 \leq r \leq b \\ \ln(a/r) : b < r \leq a \end{cases}, \quad (1)$$

with the linear charge density  $Q_e = -I_e/\beta c < 0$ , the radius of the electron beam  $b$  and the radius of the beam tube  $a$  and the dielectric constant  $\epsilon_0$ .  $I_e$  denotes the electron current,  $\beta \cdot c$  is the velocity of the electron beam. For the radius of the electron beam the Herrmann radius [21] is used:

$$r_H = r_b \sqrt{\frac{1}{2} + \frac{1}{2} \sqrt{1 + 4 \left( \frac{8m_e k T r_c^2}{e^2 r_b^4 B^2} + \frac{B_c^2 r_c^4}{B^2 r_b^4} \right)}} \quad (2)$$

with the cathode radius  $r_c$ , the cathode temperature  $T$ , Boltzmann's constant  $k$ , the magnetic field strength  $B$  and the mass and charge of the electron,  $m_e$  and  $e$ .  $r_b$  denotes the Brillouin radius [22]

$$r_b = \frac{1}{B} \sqrt{\frac{I_e}{\pi\epsilon_0}} \left( \frac{2m_e^3}{E_e e^2} \right)^{1/4} \approx \frac{150 \mu\text{m}}{B[\text{T}]} \frac{\sqrt{I_e[\text{A}]}}{(E_e[\text{keV}])^{1/4}}, \quad (3)$$

which depends on the energy of the electron beam  $E_e$ .

## B. Fraction of time spent inside electron beam

In the simple model of a box-shaped electron beam an ion can obviously only be charge bred if it spends some of its time inside the electron beam ( $r \leq b$ ). Depending on the way it is injected into the EBIT, it can either spend some, all or even none of its time with the electrons. This is illustrated in figure 3.

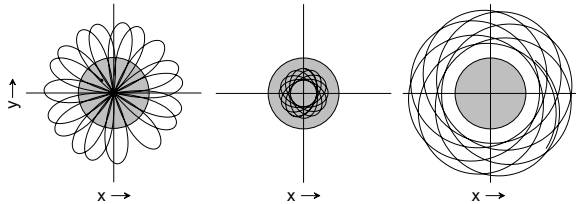


FIG. 3: The three possible cases of overlap of an ion with the electron beam. Left: ion spends time both inside and outside the electron beam. Middle: ion spends all of its time in the e-beam. Right: ion orbits around the electron beam.

For a given set of radial coordinates one can predict the fraction of the time an ion will spend inside the e-beam by integrating the equations of motion. According to eqn. (1) the electric potential changes at the boundary of the electron beam, thus the equations of motion in the transverse coordinates  $x$  and  $y$  for  $r \leq b$  separate into the simple set

$$\begin{aligned} \ddot{x} &= \frac{q}{m} \frac{Q_e}{2\pi\epsilon_0} \frac{1}{b^2} x + \frac{q}{m} B \dot{y} \\ \ddot{y} &= \frac{q}{m} \frac{Q_e}{2\pi\epsilon_0} \frac{1}{b^2} y - \frac{q}{m} B \dot{x} \end{aligned} \quad (4)$$

and for ( $b < r \leq a$ ) into the more complicated versions

$$\begin{aligned} \ddot{x} &= \frac{q}{m} \frac{Q_e}{2\pi\epsilon_0} \frac{1}{x^2+y^2} x + \frac{q}{m} B \dot{y} \\ \ddot{y} &= \frac{q}{m} \frac{Q_e}{2\pi\epsilon_0} \frac{1}{x^2+y^2} y - \frac{q}{m} B \dot{x}, \end{aligned} \quad (5)$$

with  $q$  and  $m$  being charge and mass of the ion, respectively. The equations for  $r \leq b$ , eqn. (4), are formally identical to the corresponding ones for the Penning trap. The only difference is that the factors in the linear terms ( $\frac{q}{m} \frac{Q_e}{2\pi\epsilon_0} \frac{1}{b^2}$ ) are negative due to the negative charge density. To simplify the notation, the negative term  $\frac{q}{m} \frac{Q_e}{2\pi\epsilon_0}$  will be called  $U$  and  $\omega_E$  will be shorthand for the frequency  $\sqrt{\frac{-2U}{b^2}} \cdot \frac{q}{m}$ . This is the quantity analog to the axial frequency in the case of the Penning trap. Using the frequency  $\omega_E$  and the well-known cyclotron frequency  $\omega_c = q/m \cdot B$ , one can define two more frequencies similar to the 'reduced cyclotron' and 'magnetron' frequencies  $\omega_{\pm} = \frac{\omega_c}{2} \pm \sqrt{\frac{\omega_c^2}{4} + \frac{\omega_E^2}{2}}$ . Just as in the Penning trap case,  $\omega_c = \omega_+ + \omega_-$  holds, but note that  $\omega_-$  is negative here. The 'inside electron beam' equations of motion can be solved in the same way as for the Penning trap and one obtains for the coordinates  $x$  and  $y$ :

$$\begin{aligned} x &= \rho_+ \sin(W_+ t + \xi_+) + \rho_- \sin(W_- t + \xi_-) \\ y &= \rho_+ \cos(W_+ t + \xi_+) - \rho_- \cos(W_- t + \xi_-) \end{aligned} \quad (6)$$

$W_{\pm}$  are the absolute values of the two frequencies  $\omega_{\pm}$ . The radii  $\rho_{\pm}$  and phases  $\xi_{\pm}$  need to be determined from the initial conditions. Inspection of the two equations (6) shows that the time until an ion reaches the radius of the electron beam  $r_e$  can be calculated as

$$t = \frac{\arccos\left(\frac{r_e^2 - \rho_+^2 - \rho_-^2}{-2\rho_+\rho_-}\right) - \xi_+ - \xi_-}{W_+ + W_-} \quad (7)$$

This relation allows one to calculate the time an ion spends inside the electron beam, if the initial conditions permit the ion to be there.

For the time the ion spends on the outside of the electron beam there may not be a closed solution due to the complicating  $1/(x^2 + y^2)$ -terms in eqn. (5). However one can still do better than to resort to numerical integration of the equations of motion. The generalized angular momentum  $p_{\phi}$  and energy  $E$  are conserved quantities:

$$\begin{aligned} p_{\phi} &= mrv_{\phi}(r) + qr^2 \frac{B}{2} \\ E &= \frac{m}{2} \dot{v}^2(r) + q\phi(r). \end{aligned} \quad (8)$$

Solving these two equations for the radial velocity  $v_r$  as a function of the azimuthal velocity  $v_{\phi}$  one obtains

$$|v_r| = \sqrt{\frac{2E}{m} - \frac{2q}{m} \phi(r) - v_{\phi}^2(r)}, \quad v_{\phi} = \frac{p_{\phi}}{mr} - \frac{q}{m} \frac{B}{2} r. \quad (9)$$

Integration of  $1/|v_r|$  over  $r$  from the minimum radius (the electron beam radius, or the limit given by the initial conditions) to the maximum radius yields half the round-trip time outside the electron beam. The maximum radius is obtained by solving eqn. (9) for its larger root.

### C. Breeding into charge state $2^+$

The ions injected into the trap region are singly-charged. In order to confine these ions inside the axial potential well, they need to lose at least one more electron by the electron beam. The cross-sections for this process  $\sigma_{1 \rightarrow 2}$  are calculated with the Lotz formula [18]; they generally drop by an order of magnitude from their maxima at low energy towards their values near 10 keV.

The average time it takes for a singly-charged ion to lose one electron in an electron beam of current density  $j_e$  is  $t_{1 \rightarrow 2} = e/(\sigma_{1 \rightarrow 2} \cdot j_e)$ . The probability of this process to happen as a function of the time of the ion in the e-beam  $t_{iEB}$  is calculated with the exponential distribution  $p = 1 - \exp(-t_{iEB}/t_{1 \rightarrow 2})$ .

### D. Results

Three combinations of magnetic field strengths at the cathode  $B_c$  and at the trap center  $B$  have been investigated to explore the possible variations in acceptance:  $(B_c = 0, B = 6 T)$ ,  $(B_c = 0, B = 1 T)$ ,  $(B_c = 800 G, B = 6 T)$ . Each of these cases was checked for three cathode configurations:  $(r_c = 1.7 mm, I_e = 0.5 A)$ ,  $(r_c = 3.125 mm, I_e = 1.5 A)$  and  $(r_c = 5 mm, I_e = 5 A)$ .

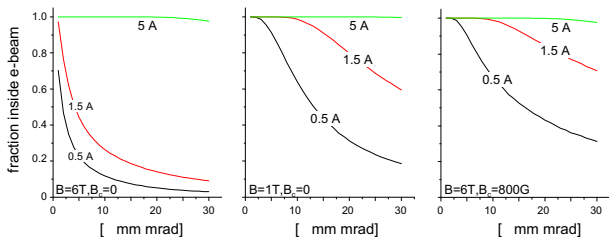


FIG. 4: The fraction of injected beam that ends up inside the electron beam as a function of beam emittance for three combinations of magnetic field strength at the cathode  $B_c$  and in the trap  $B$ . The three curves in each panel are for three different cathode options; see text.

Figure 4 shows the average fraction of time the injected ions spend inside the electron beam as a function of beam emittance (initial ion energy:  $\approx 11050 eV$ ). The values shown here are not to be confused with the probability to end up in the second charge-state, they represent the geometrical acceptance and result from the ions' paths through or around the electron beam as dictated by the radial coordinates of the injected beam. For the  $(B_c = 0, B = 6 T)$  case, i.e. maximum compression of the e-beam, the geometrical acceptance for the low-current cathodes is relatively small compared to all other cases. The evaluation of the  $1 \rightarrow 2$  breeding times will decide if a lower geometrical acceptance can be compensated for by larger electron current densities or longer times inside the trap.

As mentioned above, the probability of the  $1 \rightarrow 2$  breeding process to happen as a function of the time of the ion in the e-beam  $t$  is calculated with the exponential distribution  $p = 1 - \exp(-t_{iEB}/t_{1 \rightarrow 2})$ . Considering the fraction of time  $f$  an ion spends inside the electron beam this distribution changes to  $p = 1 - \exp(-f \cdot t_{trap}/t_{1 \rightarrow 2})$ , when expressed as a function of the time spent in the trap  $t_{trap}$ . As a consequence of this simple relationship a poor overlap with the electron beam can be compensated if the time  $t_{trap}$  is long enough.

The final set of calculations combines injection properties, overlap factors, breeding cross sections and trapping times into the probabilities of ions to end up in the  $2^+$ -state. As a reference case, it has been assumed that the ion beam is decelerated to an axial energy of 100 eV as it makes its way through the trap section of length  $l_{trap}$ . All results shown in Fig. 4 have been scaled according to the breeding probability as discussed above to yield the capture probability as a function of emittance. The results are shown in Fig. 5 for the same cathode configurations as before (0.5A, 1.5A and 5A, all 8 keV energy) and four different trap lengths:  $l_{trap} = 80 mm, 250 mm, 400 mm$  and  $800 mm$  (bottom-to-top, set of lines for each cathode). The breeding times  $t_{1 \rightarrow 2}$  range from  $0.54 \mu s$  to  $1.53 \mu s$  ( $B_c = 0, B = 6 T$ ),  $8.35 \mu s$  to  $13.1 \mu s$  ( $B_c = 0, B = 1 T$ ) and  $63.8 \mu s$  to  $73.7 \mu s$  ( $B_c = 800 G, B = 6 T$ ). Only the high and maximum-compression cases ( $B_c = 0, B = 1$  or  $6 T$ ) yield short breeding times and approach a capture probability of unity for high-current and/or low-emittance conditions. In the rather low compression case ( $B_c = 800 G, B = 6 T$ ) even the  $I_e = 5 A, l_{trap} = 0.8 m$  condition cannot exceed 46% due to the long required breeding time.

In none of the cases a short-trap/low-current combination can achieve an appreciable capture probability for a realistic beam emittance of  $\approx 15 \pi mm mrad$  or higher. For the medium-current cathode, trap lengths exceeding 400 mm would be required to pass the  $\approx 70\%$  efficiency mark at realistic emittances.

In summary, a not-too-high compression of the electron beam, an electron current of  $\approx 2 A$  and trap lengths of  $>0.3 m$  appears to be a useful combination of parameters to obtain good acceptance. The electron current density in this case is not sufficient to allow for the breeding into higher charge states on a millisecond timescale. The two most attractive options to combine high acceptance and fast subsequent breeding are : a) Use two trap regions with different magnetic field strength; a low-field trap optimized for high acceptance, and a high-field trap for fastest breeding to high charge states. b) Use one trap, but dynamically change the compression of the electron beam. For accumulation one could apply a lower compression of the electron beam than for the breeding process. The compression can for example be adjusted by changing the magnetic field at the cathode (cf. eqn. (2)). These options are currently being investigated.

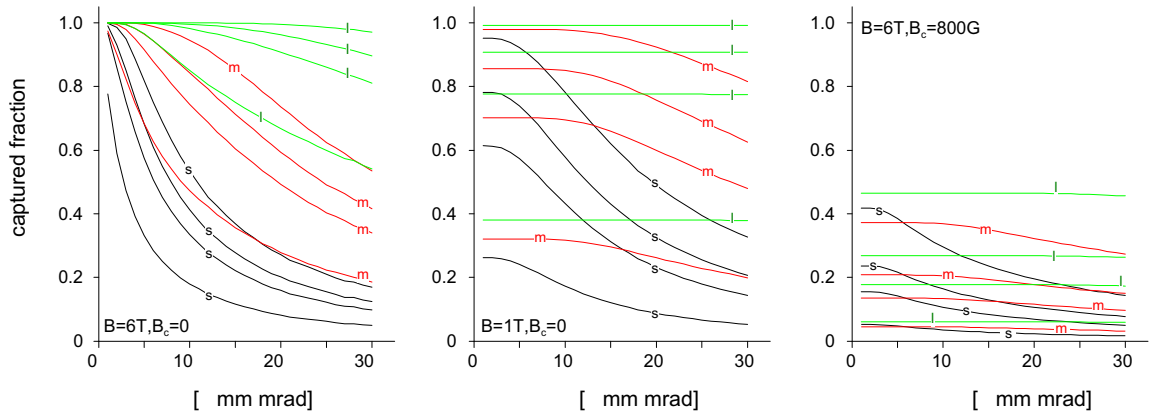


FIG. 5: Capture probability of injected ions as a function of beam emittance for three combinations of magnetic field strength at the cathode  $B_c$  and in the trap  $B$ . The twelve curves in each panel show the effect of three different cathode options (s: 0.5 A, m: 1.5 A, l: 5 A) and four different trap lengths; see text.

### III. SUMMARY

A high-current EBIT is being designed to become part of a compact reaccelerator for stopped fragmentation beams at the NSCL. The design of the NSCL-EBIT will be based on the TITAN-EBIT, as the specifications of

this charge breeder are close to what will be required for an efficient reacceleration scheme. Current simulation efforts aim at determining design and operational parameters (e.g. trap dimensions, electron beam properties) that will ensure both high acceptance and fast charge breeding.

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