ON THE SENSITIVITY OF MASSIVE STAR NUCLEOSYNTHESIS AND EVOLUTION TO SOLAR ABUNDANCES AND TO UNCERTAINTIES IN HELIUM-BURNING REACTION RATES

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Received 2007 May 29; accepted 2007 September 5

ABSTRACT

We explore the dependence of presupernova evolution and supernova nucleosynthesis yields on the uncertainties in helium-burning reaction rates. Using the revised solar abundances of Lodders for the initial stellar composition, instead of those of Anders and Grevesse, changes the supernova yields and limits the constraints that those yields place on the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction rate. The production factors of medium-weight elements ($A = 16$–40) were found to be in reasonable agreement with observed solar ratios within the current experimental uncertainties in the triple-$\alpha$ reaction rate. Simultaneous variations by the same amount in both reaction rates or in either of them separately, however, can induce significant changes in the central $^{12}\text{C}$ abundance at core carbon ignition and in the mass of the supernova remnant. It therefore remains important to have experimental determinations of the helium-burning rates so that their ratio and absolute values are known with an accuracy of 10% or better.

Subject headings: nuclear reactions, nucleosynthesis, abundances — Sun: abundances — supernovae: general

1. INTRODUCTION

Throughout the past three decades much experimental and theoretical effort has been dedicated to determining the rate of the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction. It, and the triple-$\alpha$ ($3\alpha$) reaction, are responsible for both energy generation and the nucleosynthesis of $\text{C}$ and $\text{O}$ during stellar helium burning; the ratio of their rates determines the ratio of carbon to oxygen at the completion of core helium burning. This ratio, in turn, strongly influences the subsequent evolution of Type II supernova (SNII) progenitors ($\gtrsim 9 M_\odot$ stars), affecting both the presupernova stellar structure and the post-explosive nucleosynthesis.

Although progress has been achieved in the laboratory evaluation of the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction rate, $R_{n,12}$, there are significant uncertainties in its extrapolation to the low energies relevant to hydrostatic helium burning in stars ($\approx 300$ keV). In a recent review, Buchmann & Barnes (2006) recommend $S(300 \text{ keV}) = 145$ keV barn with errors in the range of 25%–35% for the measured astrophysical $S$-factor of $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$. Stellar evolution calculations have shown (Weaver & Woosley 1993; M. Boyes et al. 2002, unpublished [reported in Woosley et al. 2003; Woosley & Heger 2007]) that such an uncertainty has major effects on SNII nucleosynthesis and on the mass of the precollapse core.

These calculations examined the changes in the production factors (defined as the ratio of the average isotopic mass fraction of nuclides in the ejecta to their solar mass fraction) induced by varying $R_{n,12}$. For these studies, however, the presupernova isotopic mass fractions were used in determining the production factors. Under the crude assumption that SNII progenitors of close-tosolar metallicity are the main contributors to the observed solar abundances for the medium-weight isotopes ($A = 16$–40), very similar production factors are desirable for those nuclides. The reaction rate that produces the smallest spread in production factors was found; M. Boyes et al. (2002, unpublished) find the smallest spread for a narrow range in $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction rates that is only 10% wide.

The calculations assumed a fixed value of the $3\alpha$ reaction rate, $R_{3\alpha}$. This is a reasonable assumption, since $R_{3\alpha}$ has significantly smaller experimental uncertainties, about 10%–12% (Tur et al. 2006; Austin 2005). However, if an accuracy of 10% in the ratio $R_{3\alpha}/R_{n,12}$ is required, the present accuracy of the $3\alpha$ rate is insufficient.

Besides uncertainties in nuclear reaction rates, uncertainties in the initial isotopic composition of stars affect their evolution and nucleosynthesis. Most recent studies of SNII evolution used the abundances of Anders & Grevesse (1989); to our knowledge there are no systematic studies using the more recent abundance set of Lodders (2003).

In this paper we describe an extensive set of calculations to determine how SNII nucleosynthesis and other stellar properties vary when $R_{3\alpha}$ and $R_{n,12}$ are varied. These calculations were repeated for the two abundance sets: Anders & Grevesse (1989) and Lodders (2003). We repeated the calculations of M. Boyes et al. (2002, unpublished) to ensure that any small changes in procedures are unimportant. Another improvement is that the results of M. Boyes et al. (2002, unpublished), as well as those of Weaver & Woosley (1993), were based on pre-SN nucleosynthesis, but some of the abundances examined are known (Weaver &
Woosley 1993; Woosley et al. 2002) to be modified in the SN explosion. The simulations presented in this paper include explosive nucleosynthesis.

A description of the stellar models and the range of the calculations is given in § 2. The differences in the stellar structure and nucleosynthesis resulting from differences in solar abundance sets are presented in § 3. In § 4 we compare the stellar evolution implications of the uncertainties in the $3\alpha$ and $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ rates.

2. COMPUTED MODELS

Stars with initial masses from 13 to 27 $M_\odot$ were considered. All models were calculated using the implicit, one-dimensional, hydrodynamical stellar evolution code KEPLER. Since its first implementation in 1978 (Weaver et al. 1978) the code has undergone several major revisions with improvements to the physical modeling of the stellar structure and to the nuclear reaction networks (Woosley & Weaver 1995; Rauscher et al. 2002; Woosley et al. 2002). A small network directly coupled to the stellar model calculation provides the approximate nuclear energy generation rate. A larger “adaptive” network is used to track nucleosynthesis. The large network automatically adjusts its size to accommodate the current nuclear flows and progressively grows from several hundred isotopes during hydrogen burning to more than 2000 isotopes at explosive burning. The treatment of convection, semiconvection, and overshoot mixing is as described in Woosley & Weaver (1988) and Woosley et al. (2002). We do not include the effects of rotation and magnetic fields. Stars are first evolved from the zero-age main sequence to presupernova, i.e., from central hydrogen burning to iron core collapse, and are then exploded. The explosion is parameterized by a piston at a constant Lagrangian mass coordinate and has two important specifications: its location in mass (i.e., the initial mass cut) and the total kinetic energy of the ejecta at infinity (here, 1 yr after the explosion). See Woosley & Heger (2007) for a more complete description of the piston parameters.

The values of these two parameters are chosen to fit reasonably well within the range of observational constraints. The explosion energy was set to 1.2B (B for Bethe, 1B = 10$^{51}$ ergs). Supernova 1987A is thought to be an $18-20$ $M_\odot$ star which exploded with an estimated energy in the range 0.6B–1.5B with an uncertainty of perhaps 50% based on the observed light curve and velocity (Arnett et al. 1989). The initial mass cut was placed at the base of the oxygen-burning shell, a location associated with a large density drop, and hence dynamically important to generating successful explosions (H.-T. Janka 2007, private communication). Specifically, we chose to place the piston at the location in the star where the entropy $S$ reaches a value of $S = 4k_B$/baryon (Woosley & Heger 2007), beyond which a large rise in entropy, and hence a drop in density, is observed. The piston location cannot be below the surface of the iron core or neutron-rich species in the iron group will be overproduced; it cannot be above the base of the oxygen shell or typical neutron star masses will be too large (Woosley & Heger 2007). Our nucleosynthesis studies do take into account all strong and weak reactions during oxygen shell burning, including the slight neutron excess resulting in this burning phase. KEPLER calculations by Woosley & Heger (2007) showed that explosion energies of either 1.2B or 2.4B and mass cuts at the base of the oxygen-burning shell or at the edge of the iron core gave very similar nucleosynthesis, except for the iron peak nuclei. We note, however, that in a recent study by Young & Fryer (2006) both elemental and isotopic yields beyond silicon were found to be very sensitive to the explosion energy.

Three separate studies were done for stars of 15, 20, and 25 $M_\odot$ for both solar abundance sets, Anders & Grevesse (1989) and Lodders (2003): (A) $R_{3\alpha}$ was kept constant (at its value from Caughlan & Fowler 1988) and $R_{\alpha,12}$ was varied; (B) both rates were varied by the same factor, so their ratio remained constant; and (C) $R_{\alpha,12}$ was held constant at 1.2 times the rate recommended by Buchmann (1996) and $R_{3\alpha}$ is varied (C).

3. SENSITIVITY TO DIFFERENCES IN SOLAR ABUNDANCES AND REACTION RATES

The differences in the two recent solar abundance determinations are shown in Figure 2. For Lodders (2003) compared to Anders & Grevesse (1989) the abundances are the following: for
CNO they are lower by about 30%; for Cl, Kr, Xe, and Hg they are higher by more than 40%; and for most other metals they are higher by about 15%. As a consequence, the overall solar mass fractions change from $X_0 = 0.7057$, $Y_0 = 0.2752$, and $Z_0 = 0.0191$ for the old set to $X_0 = 0.7110$, $Y_0 = 0.2741$, and $Z_0 = 0.0149$ for the new set.

### 3.1. The Effect on the Production Factors

The study with $R_{3\alpha}$ constant and $R_{\alpha,12}$ varied, is an elaboration of two previous studies using the Anders & Grevesse (1989) abundances (Weaver & Woosley 1993; M. Boyes et al. 2002, unpublished). Relative to Weaver & Woosley (1993) our models also include mass loss due to stellar winds, as described in Woosley & Heger (2007). As noted above, explosive nucleosynthesis is also included. The same study done with the Lodders (2003) solar abundances is entirely new and demonstrates the uncertainties in determining $R_{3\alpha}$ and $R_{\alpha,12}$ using astrophysical models.

Based on SNII nucleosynthesis considerations, Weaver & Woosley (1993) predicted an $S$-factor at 300 keV of ~170 keV barn, or more precisely a rate of $1.7 \pm 0.5$ times that of Caughlan & Fowler (1988). This constrained $R_{\alpha,12}$ to a range of about 30%. The same study was repeated later by M. Boyes et al. (2002, unpublished; reported in Woosley et al. 2003; Woosley & Heger 2007) with improved stellar models [newer opacities, added mass loss, finer stellar zoning, and finer grid of $^{12}$C($\alpha$, $\gamma$)$^{16}$O rates] and found a best fit of 175 keV barn or about 1.2 times the Buchmann (1996) rate. This is unfortunate, since it means that one cannot so strongly limit the uncertainty in $R_{\alpha,12}$ using SNII calculations of production factors.

Our results are for postexplosion values of the production factors, whereas the two previous studies stopped at the presupernova stage. We found that nuclides beyond $^{28}$Si, such as $^{40}$Ca, $^{36}$Ar, and $^{32}$S, were significantly modified during explosion, often by a factor of 1.5 or more for the Anders & Grevesse (1989) abundances. Yet, these modifications did not greatly change the earlier results for production factors.

The 20 $M_\odot$ star showed a peculiar behavior: large overproductions were found for $^{31}$P, $^{35}$Cl, and $^{39}$K for some choices of $R_{3\alpha}$ and $R_{\alpha,12}$. This was also observed and explained in Rauscher et al. (2002). The overproductions are attributed to the merging of the convective oxygen-, neon-, and carbon-burning shells about 1 day before the explosion, thereby carrying neutron sources such as $^{22}$Ne and $^{26}$Mg to depths where they burn rapidly and provide neutrons for capture reactions. Because of these peculiarities, we excluded the 20 $M_\odot$ star from our results.

### 3.2. Variations in the Carbon Mass Fraction at Central Carbon Ignition and in the Remnant Masses

We also explored the change of the central carbon mass fraction at core carbon ignition and of the remnant mass after explosion, both as a function of the initial solar abundances of the stars and as a function of variations in the helium-burning reaction rates. We illustrate the results in Figures 4 (carbon mass fraction), and 5 (remnant masses). The remnant masses are the gravitational masses of the resulting neutron stars or black holes. They are based on the baryonic mass below the piston (i.e., the mass enclosed within a radius reaching out to the base of the oxygen shell at the presupernova stage) corrected for the binding energy (Zhang et al. 2007) according to the approximation given by Lattimer & Prakash (2001). In our study, none of the stars had any significant fallback after explosion.

The variations in the central carbon mass fractions are smooth, but we see a very sensitive dependence of the remnant masses on the solar abundance set used for the initial stellar composition. To disentangle and assess the magnitude of the effects compared...
to observational data, however, would require a detailed population synthesis study of remnant masses as a function of metallicity which is beyond the scope of this paper. The predicted remnant masses are also strongly dependent on the precise reaction rates used in the presupernova evolution, often varying by $0.2 \, M_{\odot}$ or more, over ranges of ±2σ experimental errors of the reaction rates.

The remnant mass may determine the relative population of neutron stars and black holes resulting from SNII explosions. Lattimer & Prakash (2007) have surveyed the available data on neutron star masses. Their conclusions are the following: (1) While some masses in excess of $2 \, M_{\odot}$ have been reported, “it is furthermore the case that the 2σ errors for all but two systems extend into the range below 1.45 $M_{\odot}$, so caution should be exercised before concluding that firm evidence of large neutron star masses exists.” And (2) the smallest “reliably estimated neutron star mass is $1.18 \, M_{\odot}$.” While these uncertainties make it difficult or impossible to use neutron star masses to place limits on reaction rates or abundances, for orientation we have placed lines in Figure 5, at values of 1.7 and 2.0 $M_{\odot}$ as possible maximum masses for neutron stars.

4. COMPARING CHANGES IN TRIPLE-α AND 12C(α, γ)O RATES

In this section, we discuss the relative importance of the uncertainties in $R_{\alpha,12}$ compared to the 2 times larger uncertainties in $R_{3,\alpha}$. Figures 6a and 6b show the production factors of some medium-weight isotopes (the same set as M. Boyes et al. 2002, unpublished) as a function of the triple-α reaction rate in two of our studies: $R_{3,\alpha}$ varied and $R_{12,\alpha}$ constant, and both reaction rates varied by the same factor. The variations in the production factors (Fig. 6a) over a range of one standard deviation (3σ multiplier from 0.88 to 1.12) are small, although there are larger deviations for 2σ differences.

We find a very sensitive dependence of the remnant masses on the helium-burning reaction rates, and on the initial solar abundance set used. The smooth decrease in the carbon mass fraction as a function of increasing $R_{12,\alpha}$, or decreasing $R_{3,\alpha}$, is expected. The following argument is commonly given to explain the general increasing trend of the remnant masses when the $R_{12,\alpha}$ is increased (seen in Fig. 5a, for instance): a smaller rate gives a larger carbon abundance after helium burning. During carbon shell burning, this larger abundance supports longer and more energetic burning which allows the central regions to cool to lower entropy. The lower entropy, in general, gives smaller iron cores (hence remnants) for stars of a given main-sequence mass (Woosley et al. 2003). Figure 4 also shows that smaller stars make more carbon than larger ones, reflecting their higher density, and tend to have smaller remnants following explosion (as seen in Fig. 5), which supports the previous argument. When looking at the remnant masses for the 25 $M_{\odot}$ star (Fig. 5c) the same argument seems to break down at $R_{\alpha,12}$.
least partly. One expects a general decreasing trend in the remnant masses for higher triple-\(\alpha\) rates, whereas one sees an increase for a multiplier larger than one. The nonmonotonic behavior of remnant masses can be understood as a result of the interaction of subsequent burning shells. This causes the behavior of the remnants for the 25 \(M_\odot\) star of Figure 5c.

In Figure 5, variations within the current experimental range of uncertainties (2 \(\sigma\)) of both \(R_{3\alpha}\) and \(R_{\alpha,12}\) cause significant changes in the remnant mass. The remnant mass curves look smooth for the 25 \(M_\odot\) star, but they show an oscillatory behavior with rapid variations (over a small rate multiplier range) for our 15 and 20 \(M_\odot\) stars. In particular, Figure 5d shows very strong fluctuations in remnant masses, when the ratio of the helium-burning reactions is kept constant (LB), despite the very smooth change of the carbon mass fraction. These oscillations are likely due to small numerical noise in the models originating from temporal and spatial discretization, combined with a sharp transition in the stellar evolution past helium burning as a function of the carbon mass fraction, where an additional burning shell ignites or does not ignite beyond a certain threshold.

These observations lend support to the idea that variations in both \(R_{3\alpha}\) and \(R_{\alpha,12}\) are important, not just their ratio or their relative variations. An increase of 10\% in \(R_{3\alpha}\) gives the same amount of increase in the central carbon mass fraction as an 8\% decrease in \(R_{\alpha,12}\), in close agreement with the findings of Woosley & Heger (2007) for a simple calculation at given temperatures and densities. A 27\% decrease in both reaction rates is required to produce the same amount of increase in the central carbon mass fraction when the two rates are multiplied by the same factor.

5. CONCLUSION

Our simulations show that multiple uncertainties significantly influence the evolution and nucleosynthesis of SNII in current one-dimensional massive star and supernova models. The notable effect of differences in solar abundance sets is one example. Using the Lodders (2003) abundances rather than the previous standard set by Anders & Grevesse (1989) appears to reduce the precision with which SNII simulations of production factors can be used to constrain \(R_{\alpha,12}\) to ±25\%. The production factors of medium-weight elements (\(A = 16–40\)) were found to be about constant within the current 1 \(\sigma\) experimental uncertainties in the triple-\(\alpha\) reaction rate. However, variations within the 2 \(\sigma\) experimental errors in either helium-burning reaction rate do induce strong rms deviations for the production factors far from the central values of those rates.

We want to issue a caution, however, about our very approximate treatment of galactochemical evolution. Stars from different initial metallicities contribute to the solar abundance pattern. Here we took the approximation that the stars which contributed most are those of about solar initial abundance, within roughly a factor of 2. Although we did not try to obtain a precise quantification of the uncertainties due to the form of the initial mass function (IMF), the results of our study were not changed in any significant way by substituting a Salpeter IMF for the Scalo IMF used.
throughout this study. Another physics uncertainty which could affect the presupernova structure and supernova nucleosynthesis yields is the treatment of hydrodynamics, including convection and boundary layer mixing such as overshoot and semiconvection. These uncertainties have been shown (Woosley & Weaver 1988; Young et al. 2005) to have effects comparable to uncertainties in nuclear reaction rates, for instance, regarding predictions of both carbon mass fraction and remnant mass. One more issue concerns the poorly understood interactions of burning shells. These effects were discussed in Rauscher et al. (2002), and we have pointed out

![Fig. 5.—Gravitational mass of the remnant (neutron star or black hole) after explosion for 15, 20, and 25 $M_e$ stars. The dotted lines at 1.7 and 2.0 $M_e$ mark possible maximum masses for neutron stars (Lattimer & Prakash 2007); for heavier masses, black holes may be formed. (a) AA series. (b) LA series. (c) LC series. (d) LB series.](image)

![Fig. 6.—(a) Production factors and their rms deviations from the mean for some medium-weight isotopes (the same set as M. Boyes et al. 2002, unpublished) for LC2. (b) Same as (a), but for LB2.](image)
above how they can affect nucleosynthesis for a 20 $M_\odot$ star. Such effects have also been confirmed in multidimensional calculations of presupernova stars (Meakin & Arnett 2006). The effects of uncertainties in the calculation of mass loss and the possible effects of a binary companion could also be important. It would be useful to have a numerical estimate of the implications of all these uncertainties. However, to make something better than a guess would involve a suite of calculations much larger than the already extensive set we have performed. Eventually, perhaps, these effects will be sufficiently well known to permit a reliable estimate of overall uncertainties. However, even then the large effects of uncertainties in the nuclear reaction rates will likely remain.

Within the scope of our study, uncertainties within the current errors in the rates of the helium-burning reactions, both triple-$\alpha$ and $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ have been found to induce strong changes in the remnant mass of massive stars, highlighting the fact that those rates are independently important. The changes in remnant mass can have consequences for the typical neutron star masses. Hence, determining the helium-burning reaction rates is an essential ingredient to the theoretical understanding of the populations of neutron stars and black holes.

Taken together, our results for SNII evolution support the need for improved measurements of both the helium-burning reaction rates, with the goal that their ratio is known to within 10%. This is particularly important if predictions of average remnant masses are to be reliable.

We thank Robert Hoffman for providing the solar abundance sets used in this study and Stan Woosley for helpful discussions, including studies on the relative influence of the two reaction rates. This research was supported in part by the US National Science Foundation grants PHY06-06007 and PHY02-16783, the latter funding the Joint Institute for Nuclear Astrophysics (JINA), a National Science Foundation Physics Frontier Center. A. H. performed his contribution under the auspices of the National Nuclear Security Administration of the US Department of Energy at Los Alamos National Laboratory under contract DE-AC52-06NA25396, and has been supported by the DOE Program for Scientific Discovery through Advanced Computing (SciDAC; DE-FC02-01ER41176).

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