NUCLEAR STRUCTURE ASPECTS OF SCHIFF MOMENT AND SEARCH FOR COLLECTIVE ENHANCEMENTS

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Abstract

Nuclear Schiff moment may have a non-zero expectation value if both parity and time-reversal invariance are violated. The Schiff moment induces the atomic electric dipole moment currently searched by a number of experimental groups. The magnitude of the Schiff moment turns out to be sensitive to many features of complex nuclear structure; especially favorable is the combination of quadrupole and octupole deformation. We discuss these aspects, along with the new ideas for the possibility of nuclear enhancements.

1 Introduction to the Schiff moment

The measurement of the electric dipole moment (EDM) of atoms is the goal pursued by several experimental groups. The best limits obtained for the isotopes ^{129}Xe [1] and ^{199}Hg [2] provide an important advance on the way to information about fundamental forces violating both parity ($\mathcal{P}$) and time reversal ($\mathcal{T}$) invariance.

Indeed, as it was argued long ago by Purcell and Ramsey [3], a non-zero expectation value $\langle \mathbf{d} \rangle$ of the atomic EDM in a stationary state of an atom with a certain value $\mathbf{J}$ of angular momentum is possible only under *simultaneous* violation of $\mathcal{P}$- and $\mathcal{T}$-invariance. According to the quantum-mechanical vector model, such an expectation value is determined by the effective operator $\hat{\mathbf{d}}$ acting within the rotational multiplet $|JM\rangle$,

$$\hat{\mathbf{d}} = \langle (\mathbf{d} \cdot \mathbf{J}) \rangle \, \frac{\mathbf{J}}{J(J+1)}. \tag{1}$$

The pseudoscalar $\langle (\mathbf{d} \cdot \mathbf{J}) \rangle$ requires parity non-conservation; on the other hand, due to rotational invariance, its value cannot depend on the angular momentum projection M and therefore should not change under time reversal ($M \to -M$), whereas under this transformation such a time-odd quantity changes sign. Note that these arguments forbid a non-zero expectation value of any time-even polar vector.

The EDM in an atom with closed electron subshells is induced by the corresponding, $\mathcal{P}, \mathcal{T}$-violating, electrostatic potential of the nucleus where it may exist because of the presence of $\mathcal{P},\mathcal{T}$-odd forces acting between nucleons and/or quarks. The simplest consequence of such forces would be the non-vanishing dipole moment of the nucleus. However, this dipole moment is practically completely screened by atomic electrons as follows from the Schiff theorem [4, 5]. As pointed out first in Ref. [6], the operator actually inducing the atomic EDM is the so-called *Schiff moment*, the next vector term of the expansion of the nuclear charge distribution,

$$\mathbf{S} = \frac{1}{10} \sum_a e_a \mathbf{r}_a \left[r_a^2 - \frac{5}{3} \langle r_{\rm ch}^2 \rangle \right] \tag{2}$$

(a similar operator is responsible for the isoscalar giant dipole resonance in nuclei [7]). It was recently suggested [8] that the expression (2) for the Schiff moment has to be supplemented by new terms which bring in a contribution of nucleon-nucleon correlations. An accurate rederivation of the Schiff moment [9] confirms the old result (2). Similarly to eq. (1), the expectation value of the Schiff moment in the state with a certain value of nuclear spin $\mathbf{I}$ is given by

$$\hat{\mathbf{S}} = \langle (\mathbf{S} \cdot \mathbf{I}) \rangle \, \frac{\mathbf{I}}{I(I+1)} . \tag{3}$$

This means that we have to look at the structure of the ground state in the odd-A nucleus ($I \neq 0$). Various aspects of nuclear structure, single-particle and collective, and, in particular, quantum-mechanical symmetry properties emerge as decisive tools we can try to use in order to come to the best experimental candidates.

2 Microscopic calculation of the Schiff moment

The microscopic calculation of the Schiff moment requires assumptions concerning the effective $\mathcal{P},\mathcal{T}$-violating forces; their coupling strengths are to be extracted from the experiments on the EDM (we do not discuss here the important part of the whole approach, namely the high-precision atomic calculations which are necessary for translating the observed EDM value into the unknown force parameters, see, for example, [10]).

In the first order with respect to nucleon velocities, the $\mathcal{P},\mathcal{T}$-odd forces between the nucleons a and b have the structure [6]

$$W_{ab} = \frac{G}{\sqrt{2}\,2m} \Big((\eta_{ab}\vec{\sigma}_a - \eta_{ba}\vec{\sigma}_b)\cdot\nabla_a \delta(\mathbf{r}_a - \mathbf{r}_b) + \eta'_{ab}[\vec{\sigma}_a \times \vec{\sigma}_b]\cdot\{(\mathbf{p}_a - \mathbf{p}_b), \delta(\mathbf{r}_a - \mathbf{r}_b)\}_+ \Big), \tag{4}$$

where η_{ab} and η'_{ab} are to be determined by data, and G is the Fermi weak interaction constant. It is believed that the main contribution to the Schiff moment comes from the coherent mean-field part of the interaction (4) and can be written as a one-body operator,

$$\overline{W}(\mathbf{r}) = \frac{G}{\sqrt{2}} \frac{1}{2m} \frac{\eta}{4\pi} (\vec{\sigma} \cdot \nabla)\rho(\mathbf{r}), \tag{5}$$

where $\rho(\mathbf{r})$ is the nuclear density.

Using standard perturbation theory, we find the Schiff moment $\langle \mathbf{S} \rangle$ of the ground state $|I, M = I\rangle \equiv |0\rangle$ of an odd-A nucleus as a sum over intermediate states $|n\rangle$ of the same spin $I \neq 0$ but opposite parity admixed to the ground state by the interaction W,

$$\langle \mathbf{S} \rangle = 2 \sum_n \frac{\langle 0|\mathbf{S}|n\rangle\langle n|W|0\rangle}{E_0 - E_n}. \tag{6}$$

Here it is assumed that the matrix elements of W and $\mathbf{S}$ are real.

In the simplest approximation, the ground state has one unpaired particle and we admix the *single-particle* orbitals of opposite parity. For a spherical nucleus, the mixed orbitals should have the same angular momentum, $j' = j$, but different orbital momenta, $l' = l \pm 1$. In a deformed nucleus, we need to have Nilsson orbitals of opposite parity originated from such spherical levels. Rare accidental proximity can lead [11] to the small energy denominator in (6). However, in the case of single-particle mixing, this hardly can enhance the outcome since the matrix elements of W, eq. (5), are roughly proportional to those of the single-particle momentum that cancels the energy difference [6].

The single-particle properties in the nucleus are modified by the residual strong interaction. The effects of the core polarization dress the quasiparticle states and renormalize all observables. Realistic calculations performed in various versions of configuration mixing [12, 13, 14, 15] showed that the results for the Schiff moment may differ from those in the single-particle approximation by a factor of about 2.

3 Coherent mechanism: octupole deformation

For a long time it was known that, in a system of interacting particles, there exist powerful *many-body* mechanisms which, under certain conditions, can substantially increase effects of weak perturbations. For example, parity nonconservation is strongly enhanced in scattering of slow polarized neutrons and fission in the region of narrow neutron resonances [16, 17]. This enhancement is essentially of *statistical* character as can be seen from the estimates of the level spacing, $D \propto N^{-1}$, and scaling, $\propto N^{-1/2}$, of mixing matrix elements of the weak interaction between the compound states of s- and p-wave resonances; here $N \sim 10^6$ is the degree of complexity of neutron resonance states (a typical number of simple shell-model components in the chaotic wave functions), see reviews [18, 19] and references therein. The corresponding enhancement factor

$\sim \sqrt{N} \sim 10^3$ is multiplied, in the case of neutron scattering, by the *kinematic* coefficient related to the ratio of s- and p-wave neutron widths, $\sim (\Gamma_s/\Gamma_p)^{1/2}$, that can bring the observed effect of the longitudinal asymmetry (the difference of the total cross sections for neutrons with opposite helicity) from the typical estimate of $10^{-(7\div 8)}$ up to 10%.

The statistical mechanisms are presumably absent in the ground state structure of the nucleus. Therefore we have to search for the specific structural features which can bring closely levels of opposite parity that can have a large probability of being mixed by the interaction W. These features are related to the possible *coherent* mixing. The main efforts in this direction tried to use *deformed* nuclei as the appropriate arena for the combined action of intrinsic symmetry and weak interactions.

Let us consider an axially symmetric deformed odd-A nucleus. In the usual adiabatic approximation, the nuclear rotation (which restores the proper quantum numbers of angular momentum) is adiabatic with respect to intrinsic excitation. The full wave function can be presented as the product of the rotational Wigner function D^I_{MK} depending on the orientational angles and the intrinsic function χ_K. Here M is the angular momentum projection in the lab-fixed frame while K is the quantum number of the projection onto the intrinsic symmetry axis, $K = (\mathbf{I} \cdot \mathbf{n})$, where $\mathbf{n}$ is the unit vector along this axis; K is the intrinsic pseudoscalar.

In the frozen body-fixed frame, any polar vector, such as the Schiff moment $\mathbf{S}$, can have a non-zero expectation value $\mathbf{S}_{\rm intr}$ without any $\mathcal{P}$- or $\mathcal{T}$-violation. The symmetry dictates the direction of this vector along the symmetry axis,

$$\mathbf{S}_{\rm intr} = S_{\rm intr}\mathbf{n}. \tag{7}$$

However, this intrinsic vector is averaged out by rotation because the only possible combination in the space-fixed frame is again similar to the one we have seen in eq. (3), namely proportional to the product $\langle(\mathbf{n} \cdot \mathbf{I})\rangle$ that violates $\mathcal{P}$- and $\mathcal{T}$-invariance. If the $\mathcal{P}, \mathcal{T}$-violating forces create an admixture α of states of the same spin and opposite parity, the average orientation of the nuclear axis arises. In the linear approximation with respect to α,

$$\langle(\mathbf{n} \cdot \mathbf{I})\rangle = 2\alpha K, \tag{8}$$

and, therefore, we acquire the space-fixed Schiff moment (3) along the laboratory quantization axis,

$$\langle IM|\hat{\mathbf{S}}|IM\rangle = S_{\rm intr}\,\frac{2\alpha KM}{I(I+1)}. \tag{9}$$

Now the idea is to obtain a large intrinsic Schiff moment and not to lose much in translating the result to the space-fixed frame.

In order to have a significant value of the intrinsic Schiff moment, it is not sufficient to have a standard quadrupole deformation: we need a type of deformation that distinguishes two directions of the axis violating the symmetry with respect to the reflection in the equatorial plane perpendicular to the symmetry axis. The collective effect sought for may be related to the *simultaneous presence of quadrupole and octupole deformation*, the latter creating a pear-shape intrinsic mean field. The importance of octupole deformation for the transmission of statistical parity violation through intermediate stages of the fission process was understood long ago [20]. Now we need the octupole deformation in the ground state.

In the phenomenological collective description of nuclear deformation in terms of the equipotential surfaces,

$$R(\theta) = R\left[1 + \sum_{l=1} \beta_l Y_{l0}(\theta)\right], \tag{10}$$

the vector terms, $l = 1$, emerge, after excluding the center-of-mass displacement, through bilinear combinations of even and odd multipoles,

$$\beta_1 = -\sqrt{\frac{27}{4\pi}} \sum_{l=2} \frac{l+1}{\sqrt{(2l+1)(2l+3)}} \beta_l \beta_{l+1}. \tag{11}$$

The main contribution that comes from the product of the lowest static multipoles, quadrupole and octupole, determines the collective intrinsic Schiff moment [21, 22],

$$S_{\rm intr} \approx \frac{9}{20\sqrt{35}\pi} eZR^3\beta_2\beta_3. \tag{12}$$

The collective character of the octupole moment leads to the strong enhancement of the intrinsic Schiff moment compared to the single-particle estimates. Of course, the results are sensitive to the details of the nuclear models, mean field and effective interactions, but, within a factor of about 2, the Schiff moment may be enhanced up to two to three orders of magnitude [21, 22, 24].

Such results were obtained under an assumption of close levels of opposite parity mixed by the interaction W, with the splitting $\Delta = |E_+ - E_-| \approx 50$ keV. This is a real situation in ^{225}Ra ($\Delta = 55$ keV, $I = 1/2$) and in ^{223}Ra ($\Delta = 50$ keV, $I = 3/2$). The radium and radon isotopes seem to be promising because of clear manifestations of octupole collectivity. In addition, the large nuclear charge is favorable for the enhancement of the atomic EDM [23]. We need to note that the resulting space-fixed expectation value of the Schiff moment, according to eqs. (12) and (9), is proportional to the product $\alpha S_{\rm intr}$ and therefore to β_3^2.

The mixing can be particularly enhanced if the admixed states are *parity doublets* [21, 22, 25, 26, 27]. In the presence of the octupole deformation (or for any axially symmetric shape with no reflection symmetry in the equatorial plane), the states of certain parity Π are even and odd combinations of intrinsic states $\chi_{\pm K}$ with the quantum numbers $\pm K \neq 0$ and the intrinsic wave functions which differ just by the "right" or "left" orientation of the pear-shape configuration,

$$|IMK;\Pi\rangle = \sqrt{\frac{2I+1}{8\pi}}\left[D^I_{MK}\chi_K + \Pi(-)^{I+K}D^I_{M-K}\chi_{-K}\right]. \qquad (13)$$

Such doublets in fact do not even require axial symmetry; the label $\pm K$ may have a more general meaning. The intrinsic partners are time-conjugate and, according to the Kramers theorem, they are degenerate in the adiabatic approximation.

In reality the doublets (13) are split by additional interactions. This can be accomplished by Coriolis forces (the body-fixed frame of the rotating nucleus is non-inertial) or by the tunneling between the two orientations. However such a splitting is not large and the closeness of intrinsic structure should help in increasing the mixing by the weak interactions. As explained in Refs. [21, 22, 25, 27, 28], only the interaction violating both $\mathcal{P}$- and $\mathcal{T}$-invariance can mix the doublet partners because

$$\langle IMK;-\Pi|W|IMK;\Pi\rangle = \frac{1}{2}\Big[\langle\chi_K|W|\chi_K\rangle - \langle\chi_{-K}|W|\chi_{-K}\rangle\Big]. \qquad (14)$$

The matrix elements of the pseudoscalar W change sign together with K which is possible only if the time-reversal invariance is violated, along with parity. The "normal" weak interaction is $\mathcal{T}$-invariant. Therefore it is capable of mixing the parity doublets only with the help of a mediator, a regular $\mathcal{P},\mathcal{T}$-conserving interaction, including that one responsible for the doublet splitting. This indirect mixing of parity doublets was suggested in Ref. [27] for explaining the well known "sign problem" in ^{232}Th (the same sign of asymmetry for all neutron resonances which display large parity non-conservation seemingly contradicts to the statistical mechanism of the enhancement). In contrast to this, the $\mathcal{P},\mathcal{T}$-violating interaction can mix the parity doublets directly, which is important for the enhancement of the Schiff moment.

4 Coherent mechanism: soft octupole mode

As was mentioned earlier, in a nucleus with the combination of developed quadrupole and octupole deformations, the intrinsic Schiff moment is determined by the collective octupole moment β_3, whereas the Schiff moment in the space-fixed frame is proportional to its square. Obviously, the sign of the octupole moment is irrelevant. This gives rise to the idea [29, 30] that, instead of static octupole deformation, the same role of the enhancing agent can be played by the *dynamic octupole deformation*. The soft octupole mode (low-lying collective 3^- "one-phonon" state) is observed in many nuclei and, for a small frequency ω_3 of this mode, the vibrational amplitude increases, $\langle\beta_3^2\rangle \propto 1/\omega_3$. If the Schiff moment is indeed enhanced under such conditons without static octupole deformation, this can provide a more broad choice for the experimental search. Numerically, the mean square amplitude $\langle\beta_3^2\rangle$ is close to the squared value $\langle\beta_3\rangle^2$ of static octupole deformation in pear-shaped nuclei. This value can be extracted from the reduced transition probability $B(\mathrm{E}3; 0 \to 3^-)$.

In the presence of the soft octupole mode, the octupole moment $Q_{3\mu}$ oscillates with the low frequency, and its intrinsic component along the axis defined by the static quadrupole deformation β_2 is phenomenologically given by

$$Q_3 = \frac{3}{4\pi}\, eZR^3\beta_3. \tag{15}$$

This implies, eq. (12), the slowly oscillating intrinsic Schiff moment,

$$S_{\rm intr} = \frac{3}{5\sqrt{35}}\, Q_3\beta_2. \tag{16}$$

As we have already stressed, the intrinsic Schiff moment is unrelated to the violation of fundamental symmetries.

Now we need to trigger into action the mechanisms converting the intrinsic Schiff moment into observable $\mathcal{P},\mathcal{T}$-violating effects. The description of the previous paragraph referred to the deformed even-even core. The space-fixed Schiff moment needs the non-zero nuclear spin so we proceed to the neghboring odd-A nucleus. The unpaired nucleon interacts with the octupole mode. This dynamic octupole deformation of the mean field can mix, still in the body-fixed frame, the single-particle orbitals of opposite parity. As suggested in Ref. [29], the mixing leads to the non-vanishing expectation value of the weak interaction $\langle W\rangle$ in the body-fixed frame. This process can be called *"particle excitation"*. In a parallel process of *"core excitation"* [30], the octupole component of the weak $\mathcal{P},\mathcal{T}$-violating field of the odd particle can excite the soft octupole mode in the core.

The estimate of the first mechanism can be based on the octupole-octupole part of the residual nucleon interaction. The original orbital $|\nu)$ acquires the octupole phonon admixture while the particle is scattered to some orbitals $|\nu'\rangle$ of opposite parity,

$$|\nu) \Rightarrow |\tilde{0}\rangle = |\nu;0\rangle + \sum_{\nu'} a_{\nu'}|\nu';1\rangle, \tag{17}$$

where the number after the semicolon in the state vector indicates the number of octupole phonons. The orthogonal one-phonon state is, in the same approximation,

$$|\nu;1\rangle \Rightarrow |\tilde{1}\rangle = |\nu;1\rangle + \sum_{\nu'} b_{\nu'}|\nu';0\rangle. \tag{18}$$

The mixing amplitudes between the orbitals with energies ϵ_ν are

$$a_{\nu'} = \frac{\beta_3(F_3)_{\nu'\nu}}{\epsilon_\nu - \epsilon_{\nu'} - \omega_3}, \quad b_{\nu'} = \frac{\beta_3(F_3)_{\nu'\nu}}{\epsilon_\nu - \epsilon_{\nu'} + \omega_3}, \tag{19}$$

where we assume the octupole forces in the form $\beta_3 F_3$, the octupole collective coordinate β_3 being defined by eq. (14), while F_3 is operator acting on the particle and having the form $-(dU/dr)Y_{30}$ with the radial factor usually taken as a derivative of the spherical mean field potential, a reasonable approximation for realistic deformations. The quantity β_3 in eq. (19) is the transition matrix element of this collective octupole coordinate between the ground and one-phonon states in the even-even core.

Now the states $|\tilde{0}\rangle$ and $|\tilde{1}\rangle$ are mixed by the $\mathcal{P},\mathcal{T}$-violating potential. This mechanism feels the coherent part of the weak interaction W_0 averaged over the core nucleons. The mixing matrix element is found as

$$\langle\tilde{0}|W_0|\tilde{1}\rangle = \beta_3 \sum_{\nu'} \frac{2(\epsilon_\nu - \epsilon_{\nu'})}{(\epsilon_\nu - \epsilon_{\nu'})^2 + \omega_3^2} (W_0)_{\nu\nu'}(F_3)_{\nu'\nu}. \tag{20}$$

In the adiabatic limit, when the octupole mode frequency ω_3 is small compared to the single-particle spacing between the orbitals of opposite parity, the weak interaction is essentially acting at a fixed octupole deformation and then it is averaged over the slowly evolving phonon wave function. Then the result practically coincides with that for the static octupole deformation discussed earlier. The only difference is the substitution of the static β_3^2 by the dynamic mean square average $\langle\beta_3^2\rangle$.

In the core excitation mechanism [30], the effective part of the weak interaction W_{ab} acts between the valence nucleon b and the paired nucleons a in the core. Because of pairing in the core, only the contribution proportional to the spin of the valence nucleon survives,

$$W_a = -\frac{G}{\sqrt{2}\,2m}\,\eta_{ba}\Big(\nabla_a \cdot \psi_b^\dagger(\mathbf{r}_a)\vec{\sigma}_b\psi_b(\mathbf{r}_a)\Big). \tag{21}$$

We need to extract from this interaction the octupole component W_3 proportional to the operator $Q_3 = r^3 Y_{30}$. The result [30] depends on the specific orbital of the external nucleon and can be presented in the form

$$(W_3)_a \approx k\frac{G}{mR^7}\,Q_3\eta_{ba} \tag{22}$$

(this operator has to be multiplied by the creation or annihilation operator of the 3^- phonon). Here k is the numerical factor determined by the spin-orbit structure of the valence orbital; in typical cases $|k| \approx 0.6$. The matrix element of this interaction exciting an octupole phonon (that contains both proton and neutron coherent components) is given by

$$\langle 1|W_3|0\rangle = k\frac{G}{mR^7}\,AR^3\,\frac{3}{4\pi}\,\langle\beta_3\rangle^2\eta_b, \tag{23}$$

where the coupling constant is $\eta_b = (Z/A)\eta_{bp} + (N/A)\eta_{bn}$, and the subscript b is $n(p)$ for the odd neutron (proton).

Using the mixing produced by the operator W_3 for calculating the effective Schiff moment operator and projecting to the space-fixed frame we come to the result [30] of the same order of magnitude as in the case of the particle excitation. Compared to the static octupole deformation, the difference is, apart from numerical factors of order one, just in the substitution of static $\langle\beta_3\rangle^2$ by the effective dynamic mean square value. Taking the limiting value in ^{199}Hg as a current standard, we can expect the enhancement in the interval of 100 - 1000 if the energy spacing Δ is of the order or less than 100 keV. The appropriate candidates are ^{223,225}Ra, ^{223}Rn, ^{223}Fr, ^{225}Ac, and maybe ^{239}Pu, where the estimates of Ref. [30] are lower than in Ref. [29].

5 Coherent mechanism: soft quadrupole and octupole modes

The results of the previous consideration point out a tempting possibility of searching for the significant enhancement of the Schiff moment in a broad class of *spherical* nuclei where both collective modes, quadrupole and ostupole, are clearly pronounced and have low frequencies. As an example, one can mention light spherical isotopes of radium and radon. The experimental data [31] for ^{218,220,222}Rn and for other even-even nuclei in this region show long quasivibrational bands of positive and negative parity, where energy intervals are far from the rotational rules. The phonon frequencies are quite low, and there are strong E1 transitions between the quadrupole and octupole bands. The softness of the modes and large phonon transition probabilities $B(\text{E2}; 0 \rightarrow 2^+)$ and $B(\text{E3}; 0 \rightarrow 3^-)$, along with strong dipole interband coupling, indicate that the situation might be favorable for the enhancement of the Schiff moment.

The mixing of the 2^+ and 3^- phonons with the valence particle in a neighboring odd-A nucleus can be considered as a slow (adiabatic) process of adjustment of the valence orbitals to the oscillating mean field, as we argued in the previous section. If the particle can form states with the same spin in both types of mixing, these states should be rather close in energy and can be mixed among themselves by the weak interaction. Here we do not introduce any body-fixed frame so the angular momentum must be strictly conserved in those mixing processes. Thus, in our main eq. (6), we can have in the odd nucleus states of both parities with the same I, M quantum numbers like

$$|IM\rangle = \left[C_0 \alpha^\dagger_{jM} \delta_{jI} + \sum_{\lambda j'} C_2(j'\lambda : I)(\alpha^\dagger_{j'} Q^\dagger_\lambda)_{IM} \right] |0\rangle. \qquad (24)$$

Here α_{jm} and $Q_{\lambda\mu}$ are quasiparticle and phonon operators, respectively, whereas $|0\rangle$ represents the ground state of the even nucleus.

The detailed microscopic calculations along these lines were performed in Ref. [32]. In the neutron-odd nucleus, the proton contribution needed for the

Schiff moment comes from the transition matrix element of the Schiff operator between the appropriate states (24) of the same spin I and opposite parity,

$$\langle I^{\pm}, M=I|S_z|I^{\mp}, M=I\rangle = \sum_{\lambda\lambda' j} X(jI;\lambda\lambda')C_2(j\lambda;I^{\pm})C_2(j\lambda';I^{\mp})(\lambda||S||\lambda'). \tag{25}$$

where $X(jI;\lambda\lambda')$ are geometric coefficients resulting from vector coupling of angular momenta. The reduced matrix element of the Schiff momentum, $(\lambda||S||\lambda')$, is taken between the phonon states in the even-even core. Because of the strong dipole coupling between the corresponding bands in the candidate nuclei, we expect that this matrix element should not cause an additional reduction.

Concrete calculations [32] used the random phase approximation (RPA) in the form of the quasiparticle-phonon model [33]. The multipole-multipole forces are fixed in even nuclei by the phonon parameters. The result for the Schiff moment can be expressed in terms of the single-particle Schiff matrix elements, $(j_1||S||j_2)$, standard pairing amplitudes, (u, v), and the RPA phonon amplitides of two-quasiparticle and two-quasihole components, (A, B),

$$(\lambda||S||\lambda') = \sqrt{35}\sum_{123}(u_1u_2 - v_1v_2)\left\{\begin{array}{ccc} \lambda & \lambda' & 1 \\ j_1 & j_2 & j_3 \end{array}\right\}$$

$$\times (j_1||S||j_2)\Big[A_\lambda(23)A_{\lambda'}(31) + B_\lambda(23)B_{\lambda'}(31)\Big]. \tag{26}$$

In the conventional RPA framework, the three-phonon couplings, as in eq. (26), are expressed by triangular diagrams, which come with a considerable reduction due to the combinations $u_1u_2 - v_1v_2$ of the pairing coherence factors. This combination is antisymmetric with respect to the single-particle Fermi surface and would vanish in the case of full symmetry around the Fermi surface. This can be understood in analogy with the well known Furry theorem of quantum electrodynamics. In that case three-photon diagrams vanish exactly because of the precise cancellation of electron and positron contributions to the loop with three photon tails. In the discrete nuclear spectrum, there is no full symmetry and the result does not vanish but still it is partially suppressed.

The weak interaction was taken in the mean field form, eq. (5),

$$\overline{W}_b(\mathbf{r}) = \frac{G}{\sqrt{2}2m}\,\eta(\vec{\sigma}\cdot\mathbf{r})\,\frac{1}{4\pi r}\,\frac{d\rho(r)}{dr}, \tag{27}$$

where $\rho(r)$ is determined by the pairing occupancy factors in the core. There are several contributions of the interaction (27) into various parts of the complicated calculation: in the wave functions of the unpaired quasiparticle, in the matrix elements of quasiparticle-phonon coupling. in the intermediate particle and phonon propagators, and in the phonon loops. Combining these calculations with the energy denominators we come to the final results.

At this stage we could not find an enhancement of the nuclear Schiff moment. For example, for the ^{219}Rn isotope the matrix element of the weak interaction equals -1.3 $\eta \cdot 10^{-2}$ eV, and the final value of the ground state Schiff moment was 0.30 $\eta \cdot 10^{-8} e\cdot$fm^3. Typically, the reduced matrix elements $(2^+|S|3^-)$ in the even nucleus are of the order (1-2) $e\cdot$fm^3, and the matrix elements of the Schiff operator between the ground state in the odd nucleus and its parity partner are around 0.1-0.2 $e\cdot$fm^3. Final results for the Schiff moment are of the same order as in pure single-particle models (the single-particle contribution unrelated to the soft modes [34, 13, 14] has to be added).

These calculations seemingly contradict to the idea of a possible enhancement by soft collective modes. Nevertheless, a useful exercise [32] confirms that the effect indeed exists but, in the RPA framework, requires artificially low collective frequencies when the dynamic deformation amplitudes increase as $\beta \propto 1/\omega$. One can consider the theoretical RPA limit of collapsing frequencies,

$$\omega_{2,3} \Rightarrow y\omega_{2,3}, \quad y \ll 1, \tag{28}$$

and accurately separate the singular part of the RPA solutions. As the collective frequencies go down, the reduced matrix element $(2^+|S|3^-)$ in the even nucleus, the mixing matrix element of the weak interaction in the odd nucleus and the final Schiff moment grow large. These trends are seen in the following Table.

Nucleus	y	$(2^+\|S\|3^-)$	m.e. W	m.e. S	S
^{219}Ra	1	1.7	-1.3	-0.1	0.3
	0.1	20	1.1	-0.2	-0.2
	0.01	195	53	-0.2	6.2
^{221}Ra	1	2.2	0.2	-0.2	-0.1
	0.1	23	-19	-0.5	6
	0.01	235	-253	-2.7	560

(29)

It is clear from the Table that the matrix element $(2^+|S|3^-)$ increases $\propto 1/\omega$. Other matrix elements are also sensitive to the level spacing in the odd nucleus. Here we need to mention that the RPA results with the parameters fitted to the phonon frequencies do not produce a satisfactory description of spectra in odd nuclei.

To summarize the situation, we can conclude that in the situation when the phonon-quasiparticle coupling becomes strong, the standard RPA approach that accounts for a single-phonon admixture to quasiparticle wave fuinctions, is unreliable. The effect of enhancement appears either with static deformation or in the strong coupling limit when effectively the condensate of phonons emerges that mimics the deformed field. In the exactly solvable particle-core model [35] with the soft *monopole* model, $\lambda = 0$, the ground state of the odd-A nucleus contains a coherent phonon state with the average number of phonons defined by the coupling constant. The quasiparticle strength in this regime is strongly fragmented over many excited states. Similar effects should take place for quadrupole and octupole modes [36, 37, 38] when the coherence finally leads to the phase transition to static deformation.

In agreement with above arguments, the calculations [32] with artificially quenched frequencies show that the wave function of the odd nucleus becomes exceedingly fragmented. For example, in the realistic case, $y = 1$, for the ground state $I = 7/2$ in ^{219}Ra, there exists only one large combination of amplitudes required for the mixing, namely there are particle-phonon states $7/2^+$ with the wave function $(2g_{9/2}, 2^+)_{7/2}$ and $7/2^-$ with the wave function $(2g_{9/2}, 3^-)_{7/2}$; their weights in the full RPA wave functions are 98% for negative parity but only 8% for positive parity. With quenching of frequencies, these amplitudes are getting drastically reduced, up to 2% for negative parity and 1% for positive parity. Only after the spreading of the single-particle strength reached saturation in the orbital space under consideration, one can indeed see the enhancement of the Schiff moment.

Thus, the conventional RPA ansatz for the wave function of the odd nucleus as a superposition of particle-phonon components is invalid under conditions of soft collective modes. Many-phonon components take over a large fraction of the total wave function. Moreover, soft modes become mutually correlated. The correlation between soft quadrupole and octupole excitations was suggested in the global review of octupole vibrations [39]. The presence of the octupole phonon singles out the axis and triggers the spontaneous symmetry breaking with effective quadrupole condensate emerging. The predicted correlation of the two modes was confirmed by the recent experiments [40] for a chain of xenon isotopes. A similar effect of condensation is brought in by the odd particle. We gave a schematic derivation of arising correlations in Ref. [32]. Recently a model of two single-particle levels of the same large j and opposite parity with n particles interacting with quadrupole and octupole collective modes was considered using the exact diagonalization instead of the RPA [41]; the results will be reported elsewhere.

6 Conclusion

In this short review we tried to demonstrate the abundance of ideas and physical images related to the search of the effects of $\mathcal{P}, \mathcal{T}$- violating forces in atomic nuclei. Of course, there is immediate interest in measuring such effects which would lead us beyond the Standard Model, while currently we know only the upper limits. Because of extreme difficulty of such experiments and their time-consuming nature, it is important to try to establish the most promising path and to select nuclei where we can expect the most pronounced effects.

Along with that, it turns out that the wealth of physics related to the violation of fundamental symmetries in nuclei elucidates also many particular problems of nuclear structure which until now do not have definite answers. These problems are related to various manifestations of quantum-mechanical symmetries in a strongly interacting self-bound many-body system as the complex nucleus. (There are also ideas in the literature of using molecular and condensed matter systems [42, 43, 44].)

Parity violation is enhanced by the orders of magnitude by statistical (chaotic) properties of compound state neutron resonances. In the search for the $\mathcal{P},\mathcal{T}$-violation we are looking for coherent effects. The EDM of the atoms is induced by the nuclear Schiff moment through its $\mathcal{P},\mathcal{T}$-violating potential. The best perspectives for a significant enhancement of the nuclear Schiff moment are currently seen in the nuclei with static octupole deformation in the ground state. We argued that the soft octupole mode in a combination with well developed quadrupole deformation is expected to display similar enhancement. Finally, we came to soft nuclei with slow quadrupole *and* octupole motion of large amplitude. Although the direct attempt in this direction did not yet bring desired results, we need to better understand nuclear physics of such nuclei where the shape is in fact ill-defined and the routine theoretical methods, such as the RPA, are probably not sufficient. This leads to new problems of structure of mesoscopic systems on the verge of shape instability. Another interesting question is that of the three-body residual forces (coming from bare three-body forces or induced by the nucleon correlations). Such forces may give stronger mode-mode coupling not limited by the Furry theorem discussed above. In general, the entire area of research is very promising for understanding the fundamental symmetries at work in a many-body environment.

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