Temperature effects in the nuclear isoscaling

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The properties of the nuclear isoscaling at finite temperature are investigated and the extent to which its parameter α holds information on the symmetry energy is examined. We show that, although finite temperature effects invalidate the analytical formulas that relate the isoscaling parameter α to those of the mass formula, the symmetry energy remains the main ingredient that dictates the behavior of α at finite temperatures, even for very different sources. This conclusion is not obvious as it is not true in the vanishing temperature limit, where analytical formulas are available. Our results also reveal that different statistical ensembles lead to essentially the same conclusions based on the isoscaling analysis, for the temperatures usually assumed in theoretical calculations in the nuclear multifragmentation process.

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I. INTRODUCTION

The scaling law obeyed by the ratio of the yields $Y_{A,Z}(i)$ of fragments of mass and atomic numbers A and Z observed in different reactions, labeled '1' and '2', *i.e.* the nuclear isoscaling [1, 2]

$$R_{21} = \frac{Y_{A,Z}(2)}{Y_{A,Z}(1)} = C\exp(\alpha N + \beta Z) , \qquad (1)$$

opens the possibility of accessing the properties of nuclear matter far from equilibrium. The parameter C in the above Eq. is just a normalization factor, but α and β hold valuable information on the nuclear interaction [3, 4]. The relevance of this property is due to the fact that, if the sources formed in the intermediate stages of the two reactions have the same temperature T, effects associated with the decay of the primary hot fragments should not distort this scaling law. Since it is difficult to tune the temperature of the sources experimentally, similar reactions are usually used so that one expects T to be very close in both sources. Therefore, the parameters α and β preserve information related to the stages at which the fragments are created. This assumption is supported by theoretical calculations [3, 5, 6].

The density dependence of the symmetry energy coefficient $C_{\rm sym}$ is of particular interest. In the vanishing temperature limit and for similar sources, it gives the main contribution to the parameter α [3, 4]

$$\alpha = 4\frac{C_{\rm sym}}{T}\left[\left(\frac{Z_1}{A_1}\right)^2 - \left(\frac{Z_2}{A_2}\right)^2\right] , \qquad (2)$$

where A_i and Z_i stand for the mass and atomic numbers of the sources formed in the two reactions. This close connection between α and $C_{\rm sym}$ has extensively been exploited in many studies [6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16].

In this work we examine the influence of the temperature on the relationship between α and the different contributions to the binding energy using the Statistical Multifragmentation Model (SMM) [17, 18, 19]. Although there are no analytic formulas relating α to these parameters at the temperatures involved in central collisions at intermediate energies, we show that the symmetry energy rules the behavior of α at these temperatures, even in the cases where the other terms dominate its behavior at vanishing temperatures. The extent to which the isoscaling analysis is sensitive to the statistical ensemble employed in the calculation is also investigated and we demonstrate that, in the temperature range relevant to nuclear multifragmentation, the micro-canonical (M.C.), canonical and the grand-canonical (G.C.) ensembles lead essentially to the same conclusions. The remainder of the paper is organized as follows. We briefly recall the main features of the SMM model in sect. II. The results are presented in sect. III. We summarize in sect. IV what we have learned.

II. THEORETICAL FRAMEWORK

The Helmholtz free energy F is the main ingredient of statistical models since, besides allowing for the calculation of the thermodynamical properties of the system, it holds the physical ingredients used in the model. In the SMM, F is written as:

$$F(T) = \frac{C_{\rm c}}{(1+\chi)^{1/3}}\frac{Z_0^2}{A_0^{1/3}} + F_{\rm trans}(T) \quad (3)$$
$$+\sum_{A,Z} N_{A,Z}\left[-B_{A,Z} + f^*_{A,Z}(T) - \frac{C_{\rm c}}{(1+\chi)^{1/3}}\frac{Z^2}{A^{1/3}}\right]$$

where

$$F_{\rm trans} = -T(M-1)\log\left(V_f/\lambda_T^3\right) + T\log\left(g_0 A_0^{3/2}\right) \quad (4)$$
$$-T\sum_{A,Z} N_{A,Z}\left[\log\left(g_{A,Z}A^{3/2}\right) - \frac{1}{N_{A,Z}}\log(N_{A,Z}!)\right] .$$

In the above expressions, $V_\chi/V_0 = 1+\chi$ denotes the ratio of the freeze-out volume V_χ to the ground state volume V_0 of the source, A_0 and Z_0 are its mass and atomic numbers, respectively. In this work, we use $\chi = 2$ in all the calculations. The fragment multiplicity in each fragmentation mode is represented by $N_{A,Z}$, $V_f = V_\chi - V_0$ is the free volume, $\lambda_T = \sqrt{2\pi\hbar^2/mT}$ corresponds to the thermal wavelength, where m is the nucleon mass. For $A \leq 4$ empirical values for the spin degeneracy factor $g_{A,Z}$ are used as well as for the binding energy $B_{A,Z}$. These light fragments, except for the alpha particle, are assumed to behave like point particles with no internal degrees of freedom. In this work, we adopt the simple Liquid Drop Mass (LDM) formula used in Ref. [15] to calculate the binding energies of heavier nuclei:

$$B_{A,Z} = C_v A - C_s A^{2/3} - C_c\frac{Z^2}{A^{1/3}} + C_d\frac{Z^2}{A} , \quad (5)$$

where

$$C_i = a_i\left[1 - k_i\left(\frac{A-2Z}{A}\right)^2\right] \quad (6)$$

and $i = v, s$ corresponds to the volume and surface terms, respectively. For the sake of clarity in the subsequent calculations discussed in sect. III, we have suppressed the pairing term. The coefficient C_c corresponds to the usual Coulomb term, whereas corrections associated with the surface diffuseness are taken into account by the factor proportional to C_d. Two simple versions of this LDM formula are used below, labeled LDM1 and LDM2 as in Ref. [15]. In the first one, $k_s = 0$ so that surface corrections to the symmetry energy are neglected and it reads $E^{(1)}_{\rm sym} = C_{\rm sym}(A-2Z)^2/A$, $C_{\rm sym} = a_v k_v$. The Coulomb correction proportional to C_d is also suppressed in the LDM1 formula, *i.e.* $C_d = 0$. In the case of the LDM2 these terms are preserved and one has $E^{(2)}_{\rm sym} = \left[C_{\rm sym} - a_s k_s/A^{1/3}\right](A-2Z)^2/A$. For the sake of the comparisons below, we kept the definition of $C_{\rm sym}$ used in the LDM1 instead of including the term $-a_s k_s/A^{1/3}$.

The internal free energy of the fragment $f^*_{A,Z}(T)$ is given by the standard SMM expression [17]:

$$f^*_{A,Z}(T) = -\frac{T^2}{\epsilon_0}A + \beta_0 A^{3/2}\left[\left(\frac{T_c^2 - T^2}{T_c^2 + T^2}\right)^{5/4} - 1\right] , \quad (7)$$

where $\beta_0 = 18.0$ MeV, $T_c = 18.0$ MeV, and $\epsilon_0 = 16.0$ MeV. This formula is used for all nuclei with $A > 4$. In the case of the alpha particles, we set $\beta_0 = 0$ in order to take into account, to some extent, the large gap between its ground state and the first excited state. The spin degeneracy factor for these nuclei is set to unity, due to the schematic treatment of their excited states.

In the translational contribution to the free energy $F_{\rm trans}$, the factor $M-1$ (rather than $M = \sum_{A,Z} N_{A,Z}$) and $T\log\left(g_0 A_0^{3/2}\right)$ arise due to the subtraction of the center of mass motion.

These ingredients are used in all the different ensembles employed in this work, *i.e.*, micro-canonical, canonical, and grand-canonical. For practical reasons, as it allows for extremely fast calculations, we used the McGill version of the canonical ensemble of SMM [20]. Since we adopt the same ingredients, it is equivalent to the traditional canonical Monte Carlo version of SMM [21]. We refer the reader to Ref. [15] for details on the grand-canonical ensemble calculations.

III. RESULTS AND DISCUSSION

The isoscaling parameter α is associated with the baryon chemical potentials $\mu_B^{(i)}$, $i = 1, 2$, of the sources '1' and '2', and $\alpha = (\mu_B^{(2)} - \mu_B^{(1)})/T$ [4]. In the vanishing temperature limit, it has been demonstrated in Ref. [4] that

$$\mu_B^{(i)} = -a_v + \frac{a_s}{A_i^{1/3}} - C'_c\frac{Z_i^2}{A_i^{4/3}} + C_{\rm sym}\left[1 - \frac{4Z_i^2}{A_i^2}\right] , \quad (8)$$

where $C'_c = C_c[1 - 1/(1+\chi)^{1/3}]$. It is easy to extend this formula to the LDM2, and one obtains, in the $T \to 0$ limit:

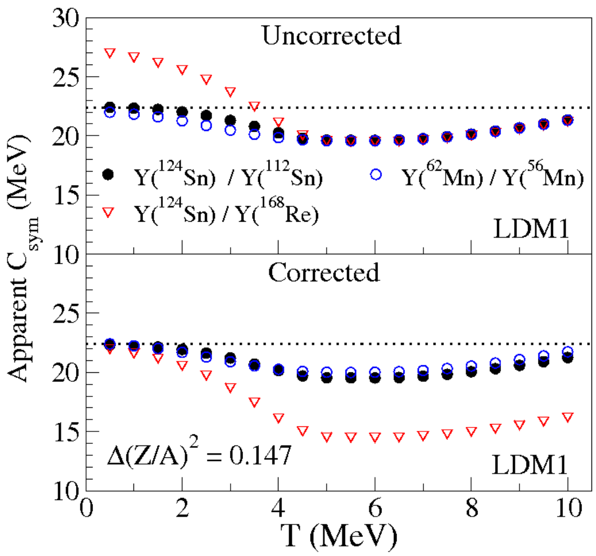

FIG. 1: (Color online) Apparent symmetry energy coefficient for different systems (with $\Delta(Z/A)^2 = 0.147$) calculated with the G.C. ensemble for the LDM1 formula. The dashed lines correspond to $a_v k_v = 22.39$ MeV. For details, see the text.

$$T\alpha = C_{\rm sym}\Delta(Z/A)^2 + a_s\left[\frac{1}{A_2^{1/3}} - \frac{1}{A_1^{1/3}}\right] - \frac{k_s a_s}{A_2^{1/3}}\left[1 - \frac{4Z_2^2}{A_2^2}\right] + \frac{k_s a_s}{A_1^{1/3}}\left[1 - \frac{4Z_1^2}{A_1^2}\right] - C_c'\left[\frac{Z_2^2}{A_2^{4/3}} - \frac{Z_1^2}{A_1^{4/3}}\right] + C_d\frac{\Delta(A/Z)^2}{4}\;, \qquad (9)$$

where $\Delta(Z/A)^2 \equiv 4(Z_1^2/A_1^2 - Z_2^2/A_2^2)$. If the sources are similar, the symmetry energy dominates the sum and, by neglecting factors involving $a_s k_s$, it gives the uncorrected value of the symmetry coefficient:

$$C_{\rm sym}^{\rm Unc} = T\alpha/\Delta(Z/A)^2\;. \qquad (10)$$

Otherwise, the corrections introduce a fixed shift to $C_{\rm sym}$ for all temperatures, whose value depends on the sources considered:

$$C_{\rm sym}^{\rm Cor} = \left\{T\alpha + C_c'\left[\frac{Z_2^2}{A_2^{4/3}} - \frac{Z_1^2}{A_1^{4/3}}\right] - C_d\frac{\Delta(Z/A)^2}{4} + \frac{k_s a_s}{A_2^{1/3}}\left[1 - \frac{4Z_2^2}{A_2^2}\right] - \frac{k_s a_s}{A_1^{1/3}}\left[1 - \frac{4Z_1^2}{A_1^2}\right] - a_s\left[A_2^{-1/3} - A_1^{-1/3}\right]\right\}/\Delta(Z/A)^2\;. \qquad (11)$$

The parameter α is calculated from fits based on Eq. (1) and the yields are provided by the G.C. ensemble (except where stated otherwise) in all the calculations presented below. It is used in Eqs. (10) and (11) to obtain the apparent $C_{\rm sym}$. If either of these Eqs. is valid at finite temperatures, one should obtain a constant value equal to $a_v k_v$ for all T.

Figure 1 displays the apparent $C_{\rm sym}$ as a function of the temperature, obtained with the expressions above, for the LDM1 ($k_s = 0$) and the (^{112}Sn,^{124}Sn), (^{168}Re,^{124}Sn), and (^{56}Mn,^{62}Mn) pairs of sources. One should notice that $\Delta(Z/A)^2$ is fixed for these pairs of sources, so that the main contribution associated with the symmetry energy is the same in all the cases. The results corresponding to Eq. (10) are displayed in the upper panel of this picture and are very similar for the (^{112}Sn,^{124}Sn) and (^{56}Mn,^{62}Mn) sources. As expected, the apparent $C_{\rm sym}$ approaches $a_v k_v = 22.39$ MeV at low temperatures, but important deviations are observed in this limit for the (^{168}Re,^{124}Sn) sources. This is because the contributions associated with a_s and the Coulomb terms in Eq. (9) cannot be disregarded in this case. The results displayed at the bottom panel of Fig. 1 show that the expected value of $C_{\rm sym}$ is obtained in the low temperature limit when the corresponding shifts, Eq. (11), are taken into account.

One striking feature of the upper panel of Fig. 1 is that, in spite of the large differences observed at low temperatures between the (^{168}Re,^{124}Sn) sources and the others, the three curves merge for $T > 4.0$ MeV. This means that Eqs. (10) and (11) break down at large temperatures and α can no longer be approximated by Eq. (2) or (9). However, the fact that the curves given by Eq. (10) agree for $T > 4.0$ MeV implies that, although it is not valid in this regime, α is indeed strongly correlated with $C_{\rm sym}$, despite the fact that the analytical temperature dependence of α is unknown. Since the nuclear multi-

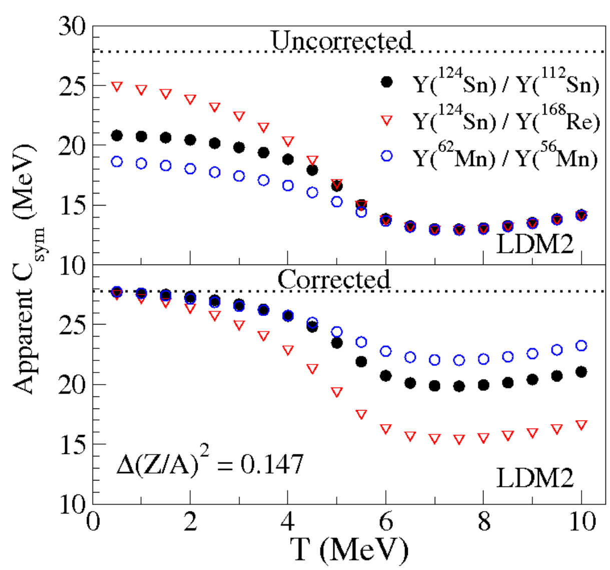

FIG. 2: (Color online) Same as Fig. 1 for the LDM2 formula. The dashed lines correspond to $a_v k_v = 27.80$ MeV. For details, see the text.

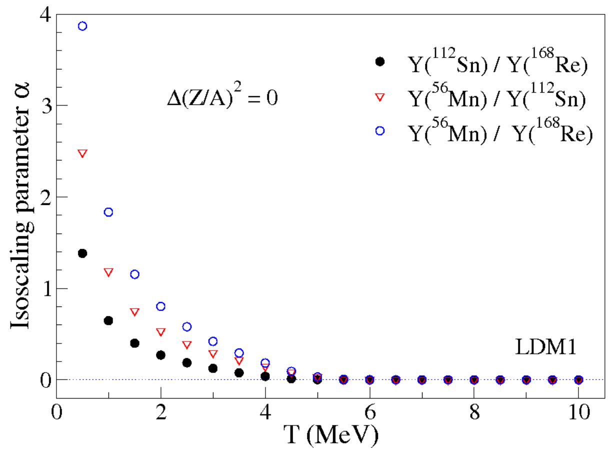

FIG. 3: (Color online) Isoscaling parameter α calculated with the LDM1 formula. For details, see the text.

fragmentation process is expected to take place from this temperature value on, these results suggest that α remains a valuable observable in these studies.

In order to check whether these conclusions are not biased by the simple form of the LDM1 formula, we also performed calculations using the LDM2 model. The results are shown in Fig. 2. One sees that the differences are much more pronounced in this case at low temperatures but the qualitative conclusions above remain true. For $T > 5.0$ MeV, $C_{\rm sym}^{\rm Unc}$ is the same for all the systems, in spite of the very large differences in the vanishing temperature limit. This suggests that our findings will not be affected by the use of more involved mass formulas.

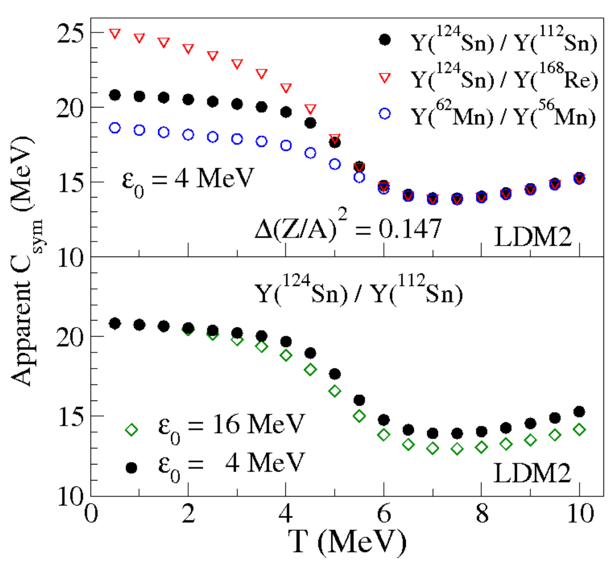

FIG. 4: (Color online) Uncorrected apparent symmetry energy coefficient calculated from the LDM2 formula for $\epsilon_0 =$ 4 MeV (upper panel). The bottom panel shows a comparison between the results obtained with $\epsilon_0 = 4$ MeV and $\epsilon_0 = 16$ MeV. For details, see the text.

To see that the symmetry energy is, in fact, the essential ingredient that dictates the behavior of the isoscaling parameter α at high temperatures, we consider the (^{168}Re,^{112}Sn), (^{112}Sn,^{56}Mn), and (^{168}Re,^{56}Mn) sources, for which $\Delta(Z/A)^2 = 0$. Equation (9) predicts that $\alpha \neq 0$ due to contributions from the Coulomb and surface terms whose relevance increases as the differences between the mass and/or the atomic numbers of the sources become larger. The results displayed in Fig. 3 show that this is true only for $T \to 0$. The effect of these terms vanishes at high temperatures and $\alpha \to 0$ for $T \gtrsim 5.0$ MeV, *i.e.* the Coulomb and surface terms have very little influence on the behavior of α at high temperatures. The symmetry energy is the key ingredient in this temperature domain, despite the formulas derived at $T \to 0$.

Since there are no analytical expressions for α at finite temperatures, we investigate the influence of the internal excitation terms of the free energy by increasing the level density of the bulk term of Eq. (7) and we set $\epsilon_0 = 4$ MeV. One should notice that this leads to an important decrease of $f^*(T)$. For instance, for $A = 100$ and $T = 6.0$ MeV, the standard value is $f^*(T) = -319.4$ MeV whereas for $\epsilon_0 = 4$ MeV one has $f^*(T) = -994.4$ MeV. The uncorrected $C_{\rm sym}$ is shown in the upper panel in Fig. 4 for $\epsilon_0 = 4$ MeV and the LDM2 formula. One sees that the qualitative features observed previously remain the same and that all the curves merge for $T \gtrsim 5.0$ MeV. Furthermore, the results displayed in the bottom panel of this picture show that, in spite of the large differences in $f^*(T)$, the effect on the apparent $C_{\rm sym}$ is very small. This reveals a fair insensitivity of α to the entropic terms of the internal free energy of the nuclei. We have checked that similar results are obtained if one reduces β_0 in Eq. (7), which gives the surface contribution to $f^*(T)$.

Finally, we check whether our conclusions depend on

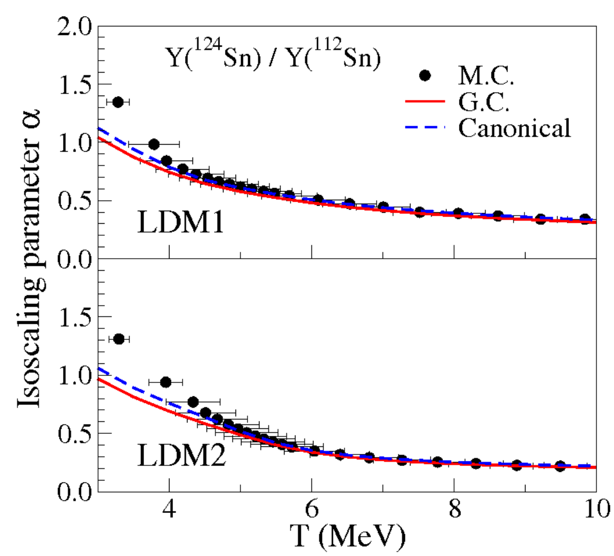

FIG. 5: (Color online) Isoscaling parameter α calculated with the M.C., G.C and canonical ensembles for the LDM1 and LDM2 formulas. For details, see the text.

the statistical ensemble employed in the calculations. Figure 5 shows a comparison between the values of α obtained with the M.C., canonical, and G.C. ensembles, using the two LDM formulas discussed in this work. One observes very small differences at large temperatures, *i.e.* for $T > 4 - 5$ MeV, which is the temperature domain in which these models are usually employed in the study of the nuclear multifragmentation. The discrepancies at low temperatures between the canonical and the G.C. ensembles are due to finite size effects, since these systems are far from the thermodynamical limit. The deviations are more important in the case of the M.C. calculations at low temperatures where the excitation energy of the system is very small and the strict energy conservation constraint plays an important role in determining the most important partitions. These results suggest that our conclusions should not be affected by the use of different ensembles in the temperature domain which is relevant to the multifragment emission.

IV. CONCLUDING REMARKS

We have demonstrated that the isoscaling parameter α is not sensitive to the Coulomb and the surface ($a_s A^{2/3}$) terms of the nuclear binding energy in the temperature domain in which the multifragment emission is expected to take place, although they are very important in the vanishing temperature limit. The symmetry energy dictates the behavior of α for $T \gtrsim 5.0$ MeV, despite the fact that the simple analytical formulas that relate α to $C_{\rm sym}$, Eqs. (2) or (9), are not valid in this temperature domain. The weak dependence of α on the entropic terms of the internal excitation energy of the fragments suggests that, at high temperatures, α is essentially governed by the symmetry energy and T, although its actual functional dependence is rather complex in this regime. Therefore, our results suggest that α is indeed a good probe for the symmetry energy coefficient but interpretations based on the isoscaling analysis should be taken with care. We also found that, for $T \gtrsim 5$ MeV, the isoscaling analysis is not sensitive to the statistical ensemble employed.

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