sen, S. Steadman, K. Van Bibber, and G. Young, in *Proceedings of the International Conference on Reactions Between Complex Nuclei,* edited by R. L. Robinson *et al.* (North-Holland, Amsterdam, 1974).

[3]K. R. Chapman, Nucl. Instrum. Methods 124, 299 (1975).

[4]G. G. Frank, R. V. Elliott, R. H. Spear, and J. A. Kuehner, Can. J. Phys. 51, 1155 (1973).

[5]P. W. Green, J. A. Kuehner, and D. T. Kelly, Bull. Am. Phys. Soc. 19, 993 (1974), and Phys. Rev. C (to be published).

[6]A. E. Litherland and A. J. Ferguson, Can. J. Phys. 39, 788 (1961).

[7]P. M. Endt and C. Van der Leun, Nucl. Phys. A214, 62 (1973).

[8]R. A. Lindgren, R. G. Hirko, J. G. Pronko, A. J. Howard, M. W. Sachs, and D. A. Bromley, Nucl. Phys. A180, 1 (1971).

[9]H. V. Klapdor, G. Rosner, H. Reiss, and M. Schrader, Nucl. Phys. A244, 157 (1975).

[10]H. J. Rose and D. M. Brink, Rev. Mod. Phys. 39, 396 (1967).

Anomalous ϵ/β^+ Decay Branching Ratios: A Theoretical Explanation

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Anomalous ϵ/β^+ branching ratios for hindered allowed transitions in ^{145}Gd and ^{143}Sm decay are explained in terms of second-order corrections to normal allowed theory. These calculations lead to correction factors as large as 1000 in nuclei near $Z=80$, and explain a smaller anomaly for ^{22}Na decay. A simplified equation is presented to estimate skew ratios, $(\epsilon/\beta^+)_{\mathrm{expt}}/(\epsilon/\beta^+)_{\mathrm{theor}}$, for moderately hindered transitions.

In a previous Letter[1] we reported two large ϵ/β^+ branching-ratio anomalies relative to calculated values for allowed transitions.[2] Our subsequent Comment[3] on absolute measurements of these ratios showed that all twelve measurable transitions from ^{145g}Gd decay were substantially anomalous. These results are presented in Table I along with results from ^{143g}Sm decay which were relative measurements. A value for ^{22}Na decay[4] is also included in Table I. This is, perhaps, the most accurately measured ϵ/β^+-decay branching ratio in the literature, and, although it was studied looking for Fierz interference effects, the discrepancy was never adequately explained. We now believe that we can explain these anomalous ratios in terms of second-order (off-center) corrections to allowed decay. Calculations of these corrections are explained below, and they qualitatively describe the magnitude of the anomalies.

The second-order corrections to allowed β decay are proportional to $(pR)^2$ or $(pR)(v_N/c)$ and are normally about (1–2)% of the allowed matrix elements $\int 1$ and $\int\sigma$. For heavier nuclei these contributions become increasingly important because $R \approx 0.426 aA^{1/3}$. A general correction term to the positron-decay probability can be written as[5]

$$C(W_e)=[M_0(1,1)]^2+[m_0(1,1)]^2-\frac{2\mu_1\gamma_1}{W_e}[M_0(1,1)][m_0(1,1)]$$
$$+\lambda_1\left\{[M_1(1,1)]^2+[m_1(1,1)]^2-\frac{2\mu_1\gamma_1}{W_e}[M_1(1,1)][m_1(1,1)]\right\}$$
$$+\lambda_1\left\{[M_1(1,2)]^2+[M_2(1,2)]^2-\frac{2\mu_1\gamma_1}{W_e}[M_1(1,2)m_1(1,2)+M_2(1,2)m_2(1,2)]\right\}$$
$$+\lambda_2\left\{[M_1(2,1)]^2+[M_2(2,1)]^2-\frac{\mu_2\gamma_2}{W_e}[M_1(2,1)m_1(2,1)+M_2(2,1)m_2(2,1)]\right\}. \quad (1)$$

TABLE I. ϵ/β^+ decay branching ratios.

	Daughter Energy Level	ϵ(tot)/β^+ Experimental	Theoretical[a]	R, Skew Ratio (exp/theor)
^{145g}Gd	808.5	18 ±8	0.45±0.04	40
	1041.9	1.0 ±0.1	0.57±0.07	1.8
	1567.3	37 ±18	0.95±0.10	39
	1599.9	13 ±5	0.99±0.11	13
	1757.8	1.87±0.09	1.18±0.14	1.58
	1761.9	2.6 ±0.8	1.20±0.15	2.2
	1845.4	43 ±25	1.31±0.17	33
	1880.6	2.15±0.12	1.37±0.17	1.57
	2048.9	4.2 ±1.0	1.72±0.24	2.4
	2113.9	10 ±4	1.90±0.29	5.3
	2494.8	4.8 ±0.5	3.41±0.61	1.4
	2642.2	8.1 ±0.9	4.45±0.95	1.8
^{143g}Sm[b]	1056.6	≡9.7 ±0.7	≡9.7	≡1.0
	1173.1	63 ±10	13	4.9
	1403.1	35 ±5	29	1.2
	1515.0	30 ±7	49	0.61
^{22}Na	1274.5	0.1045±0.0005	0.1135	0.921

[a]N. B. Gove and M. J. Martin, Nucl. Data, Sect. A 10, 205 (1971). ^{145g}Gd calculated for $Q_\epsilon = 5311.120$ keV.

[b]These ϵ/β^+ ratios are calculated assuming that the theoretical ratio is correct for the transition to the 1056.6-keV level.

The Fermi matrix elements, $\int 1$, as well as the second- and third-rank tensor components characterized by the matrix elements A_{ij}, T_{ij}, R_{ij}, and B_{ijk} have been neglected here. Here the M_L's contain the lepton and nuclear matrix elements for decays where the leptons can carry away more than 1 unit of angular momentum. The notation used here is the same as that in Behrens and Jänecke,[5] where the terms are precisely defined and where the lepton part of the matrix elements is accurately calculated. The remaining nuclear form factors $F_{Lls}{}^{(n)}$ are described in general form there, but they are not easily calculated. They can be estimated by using Morita's approximate form factors[6]

$$\begin{aligned}
&\int \vec{\sigma} r^2 \approx \tfrac{3}{5} R^2 \int \vec{\sigma},\\
&i\int \gamma_5 \vec{r} \approx (2M)^{-1} \int \vec{\sigma} \mp \Lambda(\alpha Z/4R) \int (\vec{\sigma}\cdot\vec{r})\vec{r},\\
&\int (\vec{\sigma}\cdot\vec{r})\vec{r} \approx \tfrac{3}{5}\eta R^2 \int \vec{\sigma},\\
&\int \vec{\sigma}\times\vec{r} \approx M^{-1}[\int \vec{\sigma} + \int \vec{r}\times\vec{p}].
\end{aligned} \tag{2}$$

These equations are only good to tens of percent, but they allow order-of-magnitude estimates of the correction factors. The quantity η in $\int(\vec{\sigma}\cdot\vec{r})\vec{r}$ and thus also in $i\int\gamma_5\vec{r}$ relates the relative contributions of the allowed and second-order matrix elements. The factor $\Lambda \approx 1 \pm (W_0 \mp 2.5)A^{1/3}/Z$ is close to unity except for light nuclei. If the allowed matrix elements are small, the second-order corrections need not be similarly reduced. In this case η is large and $\int(\vec{\sigma}\cdot\vec{r})\vec{r}$ and $i\int\gamma_5\vec{r}$ may actually dominate the situation. An estimate η_{ft} of η can be given by the equation[7]

$$\log(\eta_{ft}{}^2) = \log ft - 3.6, \tag{3}$$

assuming 3.6 to be a "good" $\log ft$ value for an unhindered allowed transition, such as a mirror decay. Thus, for a transition with $\log ft = 7.5$, $\eta_{ft} = 89$. The equations for electron-capture decay differ slightly from those for positron decay and generally follow the formalism for β^- decay except that we must reverse the sign of the neutrino momentum q_x and replace the form factor $F_{Lls}{}^{(n)}$ by $(-1)^{L-s}F_{Lls}{}^{(n)}$. These differences are described more fully in Behrens and Jänecke.[5]

An approximate equation for the skew ratio, $R = (\epsilon/\beta^+)_{\rm expt}/(\epsilon/\beta^+)_{\rm theory}$, describing the extent of the anomaly relative to the allowed theory, is given by

$$R = \frac{C(W_x)}{C(W_e)} \approx \left[\frac{1+\frac{3}{20}(D_1+N_1)\alpha Z_\epsilon \eta}{1-\frac{3}{20}(D_1+N_1)\alpha Z_{\beta+}\eta}\right]^2, \tag{4}$$

where $D_1 + N_1 = \frac{1}{2}\alpha Z - \frac{1}{3}W_0 R$. Here Z_ϵ is the atomic number of the decaying nucleus and $Z_{\beta+}$ is the number of the daughter. Clearly this formulation is not continuous for all values of η and is only useful for small η. More complete calculations including all Gamow-Teller second-order corrections have been completed on the Michigan State University Sigma-7 computer. The results of these calculations for $Z = 20$ to $Z = 80$ and $W_0 = 2$ to $W_0 = 10$, $\eta > 0$, are presented in Fig. 1. For large Z, skew ratios greater than 1000 may be observed for reasonable values of η. These effects are greatest when the correction terms are of similar magnitude to the allowed terms, leading to maximum interference. In the extremes of very large or small η only small skew ratios are expected.

The values of η necessary to obtain the ^{145g}Gd skew ratios are presented in Table II. These values are compared with η_{ft} and are always smaller than η_{ft} which should be a maximal value for any transition. In some cases, the second-order corrections must be nearly unhindered for

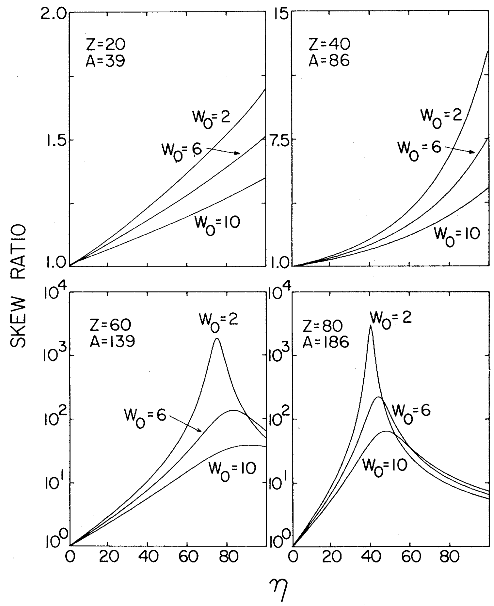

FIG. 1. Skew ratios required to correct the simple allowed calculations of ϵ/β^+ ratios by including second-order corrections.

TABLE II. Experimental and calculated Morita parameters.

	Daughter energy level	η_{ϵ/β^+} [a]	$\eta_{\beta-\gamma}$ [b]	η_{ft} [c]
^{145g}Gd	808.5	59(94)	···	71
	1041.9	11	···	35
	1567.3	55(95)	···	89
	1599.9	42	···	63
	1757.8	8	···	13
	1761.9	14	···	63
	1845.4	52(97)	···	100
	1880.6	8	···	13
	2048.9	15	···	56
	2113.9	28	···	100
	2494.8	6	···	45
	2642.2	10	···	35
^{22}Na	1274.6	−62	−60	−79

[a] η_{ϵ/β^+} determined from ϵ/β^+ ratios. In some cases this is a double-valued function for a given skew ratio.
[b] $\eta_{\beta-\gamma}$ determined from β-γ angular correlations using the same theoretical assumptions used to calculate the previous column.
[c] Calculated using Eq. (3).

such good agreement between η and η_{ft}. This agreement is excellent considering the assumptions in these calculations. The value of η can also be calculated from β-γ angular correlation coefficients. To first order these correlations are isotropic, with the anisotropies resulting from higher-order corrections. Unfortunately, the ^{145g}Gd decays frequently proceed through spin-$\frac{1}{2}$ states, yielding only isotropic angular correlations. Using the same assumptions discussed here, Subotowicz *et al.*[8] have presented equations predicting these correlation coefficients as a function of η. Results derived from these experiments should give a model-independent comparison of η derived in two ways. Such measurements have been completed[7] for ^{22}Na and are included in Table II. The values of η derived from both methods agree identically, including the sign, giving us strong confidence in the correctness of this approach. Unfortunately, no other unambiguous comparisons can be made at this time.

Although we seem to have strong evidence that our approach is essentially correct, the numerous assumptions we have made prevent us from calculating the skew ratios more accurately at this time. We assume here that $\Lambda = 1$ as was originally suggested by Morita.[6] Recent communication with Morita[9] has indicated that this may not be valid. In that case the substitution $\eta \rightarrow \Lambda\eta$ must be performed for our discussions. Also, Morita has suggested that $\int(\vec{\sigma}\cdot\vec{r})\vec{r} \approx \frac{3}{5}\eta R^2 \int\vec{\sigma}$ is least valid for strongly hindered decays. This effect may also be absorbed into η, but it removes much of the physical insight that η might contain. Having good shell-model wave functions should allow us to calculate the second-order matrix elements and hence η directly, and we are currently working in this direction. Most important, however, is the fact that even slightly hindered allowed transitions in other than very light nuclei should be expected to show anomalous ϵ/β^+ branching ratios. Thus, analyses such as determining Q_ϵ from ϵ/β^+ branching ratios are suspect and not to be relied on. Finally, these interference effects should be related to second-class currents, and the exact relationships should be investigated further.

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dation.

[1]R. B. Firestone, R. A. Warner, Wm. C. McHarris, and W. H. Kelly, Phys. Rev. Lett. 33, 30 (1974).

[2]E.g., N. B. Gove and M. J. Martin, Nucl. Data, Sect. A 10, 205 (1971).

[3]R. B. Firestone, R. A. Warner, Wm. C. McHarris, and W. H. Kelly, Phys. Rev. Lett. 35, 401 (1975).

[4]M. L. Fitzpatrick, K. W. D. Ledingham, J. Y. Gourlay, and J. G. Lynch, in *Proceedings of the International Conference on Inner Shell Ionization Phenomena and Future Applications, Atlanta, Georgia, 1972*, edited by R. W. Fink, J. T. Manson, I. M. Palms, and R. V. Rao, CONF-720 404 (U. S. Atomic Energy Commission, Oak Ridge, Tenn., 1973), p. 2013.

[5]H. Behrens and J. Jänecke, *Numerical Tables for Beta-Decay and Electron Capture*, edited by H. Schopper, in *Landolt-Börnstein Numerical Data and Functional Relationships in Science and Technology*, edited by K. H. Hellwege (Springer, Berlin, 1969), Group I, Vol. 4.

[6]M. Morita, Phys. Rev. 113, 1584 (1959).

[7]M. Subotowicz, Izv. Akad. Nauk SSSR, Ser. Fiz. 33, 1631 (1969) [Bull. Acad. Sci. USSR, Phys. Ser. 33, 1496 (1969)].

[8]M. Subotowicz, D. Bazyluk, M. Budzynski, I. Ptaszec, and J. S. Sarzinski, Izv. Akad. Nauk SSSR, Ser. Fiz. 37, 53 (1973) [Bull Acad. Sci. USSR, Phys. Ser. 37, 46 (1973)].

[9]M. Morita, Osaka University (private communication).

Isotopic Differences in the Charge Distribution of Even Molybdenum Isotopes from Elastic Electron Scattering

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Elastic-electron-scattering cross sections for ^{92,94,96,98,100}Mo in an effective momentum transfer range of 0.6–2.1 fm^{-1} are analyzed in a practically model-independent way, using a Fourier-Bessel expansion for the charge distribution. The ratios of cross sections of neighboring isotopes yield differences of charge distributions, which exhibit pronounced shell effects.

As part of a systematic investigation of ground-state charge distributions of nuclei near the magic neutron number $N=50$ by elastic electron scattering, cross sections for the even molybdenum isotopes ^{92,94,96,98,100}Mo have been measured at the Universität Mainz electron accelerator. Data were taken at approximate electron energies of 120, 200, and 274 MeV. They cover an effective momentum-transfer interval of $0.6 \leq q_{\text{eff}} \leq 2.1$ fm^{-1} with typical errors of (1.5–2.5)%, except for the data at the highest momentum transfers. The absolute cross sections have been determined by comparing the results for the molybdenum isotopes with the scattering on ^{12}C for each individual datum point. As ^{12}C reference cross sections I used the results of Merle.[1] Because of a special experimental setup which allowed quasisimultaneous measurements for all targets, the experimental uncertainties were kept very small, especially for the comparison of neighboring isotopes. Further details about the experimental apparatus are given in Ehrenberg *et al.*[2] As an example of the data, Fig. 1 shows the measured cross-section ratios of ^{92}Mo/^{94}Mo and of ^{92}Mo/^{100}Mo.

It was found to be impossible to describe the data with the usual three-parameter model charge distributions of the Fermi or modified-Gaussian type. Such functional forms are not flexible enough to fit electron scattering data with the precision which is nowadays available in the momentum-transfer range of this experiment.

A much better agreement is achieved, however, if one uses a Fourier-Bessel expansion for the charge distribution up to a certain cutoff radius R:

$$\rho(r) = \begin{cases} \sum_{\nu=1}^{\infty} a_\nu j_0(q_\nu r), & r \leq R, \\ 0, & r > R, \end{cases} \tag{1}$$

with normalization

$$4\pi \int \rho(r) r^2 \, dr = Z.$$

As has been described in detail in Dreher *et al.*,[3] in this parametrization the first coefficients a_ν are determined by the experimental data at momentum-transfer values $q_\nu = \pi\nu/R$ $[a_\nu \sim F(q_\nu)]$. Limits on the higher coefficients may be derived from estimates of the asymptotic behavior of the form factor $F(q)$.

I have improved on the analysis of Ref. 3, however, by performing all calculations in the frame-