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An Energy-Dependent, Lane-Model, Nucleon-Nucleus Optical Potential*

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ABSTRACT

An energy-dependent, Lane-model, nucleon-nucleus optical potential is presented. The isovector strength parameters of the potential have been determined by fitting (p,n)-IAS angular-distribution data between 25 and 45 MeV for targets from ^{48}Ca to ^{208}Pb. The isoscalar strength parameters have been obtained by requiring that the Lane-model potential reproduce the Becchetti-Greenlees Coulomb-corrected proton potential. The energy associated with the proton in all energy-dependent parts of the potential is reduced by the average Coulomb potential inside the nucleus. The main result of the parameter search, other than determining the strength of the isovector energy dependence, is to redistribute the isovector strength found by Becchetti and Greenlees between the real-volume and imaginary-surface terms. The Lane-model optical potential so obtained is reasonably successful in reproducing (p,p), (p,n)-IAS, and (n,n) scattering over a wide mass and energy range.

NUCLEAR REACTIONS: Energy-dependent, Lane-model, nucleon-nucleus optical potential. E_p reduced by average coulomb potential. ^{48}Ca, ^{90}Zr, ^{120}Sn, ^{208}Pb(p,n), E=25, 35, 45 MeV; measured IAS $\sigma(\theta)$. Deduced energy-dependent isovector parameters; DWBA parameter search; fit IAS $\sigma(\theta)$. Predict (n,n) $\sigma(\theta)$.

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I. INTRODUCTION

It has been pointed out by Lane[1] that the nuclear optical potential may be written in a charge-independent form. Such a potential may be used not only to describe both proton and neutron elastic scattering from nuclei, but also the (p,n) reaction to the isobaric analog of the target ground state (IAS). The Lane-model optical potential may be written as

$$U = -U_0 + 4U_1(\vec{t}\cdot\vec{T})/A, \qquad (1)$$

where $\vec{t}$ and $\vec{T}$ are the nucleon and nucleus isospins, respectively, A is the mass number of the nucleus, and U_0 and U_1 are functions that do not depend upon isospin in the isospin-independent and isospin-dependent parts of the potential, respectively. U_0 and U_1 should, in general, depend upon relative position and momentum,[2-9] and both may be complex.[7-12] If a sufficiently general parameterization of the Lane-model optical potential were available, one would be able to calculate proton elastic, neutron elastic, and (p,n)-IAS quasi-elastic scattering from a single potential. Thus, the Lane-model optical potential contains two principal parts, and can be used to describe three experimentally-feasible scattering processes. It should be possible to determine a reasonable parameterization of the potential using data for any two of the scattering processes. The potential so obtained should then be able to reproduce data for the third scattering process with no adjustable parameters. However, it is not necessarily true that any Lane-model potential parameterization that reproduces two of the scattering processes must also reproduce the third. A

possible source of such difficulty is the fact that nucleon optical potentials are not unique; thus, some care is required in obtaining a useful parameterization.

A Lane-model-consistent analysis is one in which the potentials for (p,p), (p,n)-IAS, and (n,n) are related by Eq. 1. Such an analysis of proton elastic and (p,n)-IAS data may yield an optical-model parameterization that is also capable of reproducing neutron elastic scattering data. Such analyses have been performed for individual targets at particular proton bombarding energies and corresponding neutron energies.[13,14] The results have been reasonably successful for targets up to Sn,[13] but have not been satisfactory in the Pb region.[14] Work has also been reported in which (p,n)-IAS data have been reasonably well reproduced by using existing proton and neutron optical parameters in a Lane-model framework,[15] even though the energy dependence of the isospin-dependent part of the potential was not handled in a consistent manner.[16] It should be noted that although (p,n)-IAS data are usually analyzed in terms of the Lane model, almost all previous analyses have not used the model in a consistent manner[17] (i.e., the potentials used are not related by Eq. 1). The usual procedure has been to vary the (p,n) form factor to fit the data, while holding the proton and neutron optical parameters fixed. Clearly, the proton and/or neutron optical parameters should change as the isospin-dependent part of the Lane-model potential is varied.

This paper presents the first attempt, to our knowledge, to obtain a global, energy-dependent, Lane-model-consistent, nucleon-nucleus optical potential. The parameters of the isovector part

of the potential have been determined by fitting (p,n)-IAS data measured by us[18] for targets of ^{48}Ca, ^{90}Zr, ^{120}Sn, and ^{208}Pb at proton bombarding energies of 25, 35 and 45 MeV. The respective Q-values are -7.2, -12.0, -13.6 and -18.8 MeV, giving a range of outgoing neutron energies from 6 to 38 MeV. Non-linear least-squares searches on the values of the isovector parameters have been performed with a computer program which incorporates the DWBA code DWUCK as a subroutine. The parameters of the isoscalar part of the potential have been obtained from proton optical-model parameters determined by Becchetti and Greenlees[20] (BG), with minor modifications that will be discussed. Global searches have been performed using all of our (p,n)-IAS data. The fits obtained for the (p,n)-IAS data are presented, along with a comparison of neutron elastic scattering calculations and data.

II. THEORY

When Eq. (1) is used to describe proton elastic scattering, it leads to a pair of coupled Schrödinger equations. One equation describes the proton scattering from the target ground state, and the other equation describes the outgoing neutron and the isobaric analog of the target ground state. The coupling term describes the (p,n) reaction that connects the target ground state to its isobaric analog. The (p,n)-coupling term is small compared to the elastic scattering terms. Thus, it is reasonable to use the distorted-wave Born approximation (DWBA) to calculate the (p,n)-IAS cross section.[9,21-23] Likewise, the proton-target and neutron-analog scatterings can be calculated with reasonable

accuracy using the homogeneous equations obtained by dropping the (p,n) coupling term. The equations used are:

$$(K - U_0 - 2TU_1/A + V_c - E_p)\chi_{pT} = 0 \quad (2a)$$

$$(K - U_0 + 2(T-1)U_1/A + \Delta_c - E_p)\,\chi_{nA} = 0, \text{ and} \quad (2b)$$

$$T_{pn} = \langle \chi_{nA} | -2\sqrt{2T}\, U_1/A | \chi_{pT} \rangle. \quad (2c)$$

In addition,

$$(K - U_0 + 2TU_1/A - E_n)\,\chi_{nT} = 0, \quad (2d)$$

where T=(N-Z)/2 of the target nucleus, K is the kinetic-energy operator, V_c is the Coulomb potential, Δ_c is the Coulomb displacement energy (Δ_c=-Q for the (p,n)-IAS reaction), E_p and E_n are the asymptotic proton and neutron energies, respectively, T_{pn} is the transition amplitude for the (p,n)-IAS reaction, and χ_{pT}, χ_{nA}, and χ_{nT} are the distorted waves describing the proton-target, neutron-analog, and neutron-target systems, respectively. Clearly, all of Eqs. (2) should have the same form for U_0 and U_1, if Lane-model consistency is to be maintained.

Since it appears that the isovector, as well as the isoscalar, part of the optical potential varies with energy,[2-9] we wish to include a linear energy dependence in the parameterization of Eq. (1). The appropriateness of a Coulomb "correction" in the energy dependence of the proton optical potential has been recognized for over a decade.[7,20,24] The usual procedure is to introduce a term proportional to $E_p - \bar{V}_c$, where $\bar{V}_c$ represents the Coulomb energy of the scattered proton averaged over the region of nuclear interaction. Since $\bar{V}_c \approx \Delta_c$, the Coulomb-corrected proton energy is approximately equal to the outgoing neutron energy for the (p,n)-IAS reaction, and either might be used to evaluate the DWBA form factor. In this work, we simply use

their average. Thus, an explicitly energy-dependent form of Eq. (1) may be written as

$$U(r,E) = -(U_{0C}+U_{0E}E) + 4(U_{1C} + U_{1E}E)(\vec{t}\cdot\vec{T})/A \qquad (3)$$

where E is equal to $E_p-\bar{V}_c$ for the proton-target system, $E_p-\Delta_c$ for the neutron-analog system, $1/2(2E_p-\bar{V}_c-\Delta_c)$ for the (p,n)-IAS form factor, and E_n for the neutron-target system. Equation (3) has been used to calculate the proton, neutron, and (p,n)-IAS potentials; thus, Lane-model consistency has automatically been maintained in all calculations, to within the approximations previously mentioned. In particular, every time any of the parameters changed, whether for numerical derivatives or as the final result of a search iteration, all three potentials have been recalculated so that the changes would be reflected not only in the (p,n)-IAS form factor, but also in the proton and neutron distorted waves.

III. PARAMETER SEARCH

Parameterization Considerations

By inspecting the form of Eqs. (2), it is clear that the isovector part of the potential should be determined primarily by the (p,n)-IAS data, while the isoscalar part should be determined primarily by the proton elastic data (since 2T/A is at most about 0.2). Ideally, one would simultaneously fit (p,n)-IAS data and proton elastic scattering data in the parameter search. Unfortunately, proton elastic scattering data are not available for all of our targets and energies. Thus, some other way is needed to include proton scattering information in the parameter searches. It has been decided to rely

upon a global optical potential used to describe proton elastic scattering. In particular, the proton elastic data have been replaced by an optical potential obtained by Becchetti and Greenlees[20] (BG). In doing this, we have assumed that the optical potential obtained by BG gives an adequate description of proton elastic scattering. In addition, it has been assumed that the isoscalar part of the Lane-model potential corresponds to the non-(N-Z)/A-proportional terms of the BG potential. The isovector part of the Lane-model potential has been determined by fitting our (p,n)-IAS data.

This procedure has the obvious problem that changing the isovector part of the potential from that found by BG also changes the proton optical potential. Thus, there would be no reason to believe that the resulting proton optical potential would still provide an adequate description of proton elastic scattering data. The easiest way to overcome this problem is to constrain the Lane-model potential so that the proton potential obtained from it reproduces the BG proton potential as closely as possible. This is done by changing the isoscalar part of the potential, as the isovector part is changed, so that the resulting proton potential is the same, or nearly the same, as the BG proton potential. Exactness cannot be maintained globally, of course, since the isoscalar part of the potential does not depend upon isospin. However, for a fixed value of (N-Z)/A, the BG proton potential can be reproduced exactly by an energy-dependent Lane-model potential, if the same value of $\bar{V}_c$ is used in the two parameterizations. We have included constraints in our parameter searches so that the proton optical potential obtained from our Lane-model

potential parameterization is the same as the BG proton potential if 2T/A=(N-Z)/A=0.1. For example, the real volume isoscalar strength is given by

$$V_{0C}(L) = V_{0C}(BG) + 0.1[V_{1C}(BG)-V_{1C}(L)] \text{ and} \quad (4a)$$

$$V_{0E}(L) = V_{0E}(BG) - 0.1V_{1E}(L), \quad (4b)$$

where the subscripts denote energy-independent (C), energy-dependent (E), isoscalar (0), and isovector (1) strength parameters, and L and BG denote the Lane-model and Becchetti-Greenlees parameters, respectively. There is no $V_{1E}(BG)$ term in Eq. (4b), because the BG isovector potential is energy independent. Corresponding equations have been used for other isoscalar strength parameters in the Lane-model potential.

There is one other constraint that has been imposed upon our parameterization due to the fact that we are trying to reproduce as closely as possible the BG proton parameters. This constraint is that the isoscalar and isovector potentials have the same geometry parameters. Physically, this corresponds to assuming that the proton and neutron distributions in the nucleus have the same shape, which may be a rather poor assumption for nuclei with large neutron excess,[7] such as our targets. The radial form of the isoscalar part of the potential should be roughly proportional to the sum, and the isovector part, to the difference of the neutron and proton distributions. Fixing the isovector geometry also yields the practical benefit of removing at least four parameters from the search. This is

a non-trivial consideration, since each parameter adds almost 20 minutes per iteration to the computer time required.

Optical Potential

We base our Lane-model optical-potential parameterization on the Becchetti-Greenlees Coulomb-corrected proton potential (BGVC) (the parameter set labeled "V_{cc}=0.84" in Ref. 20). This potential has been chosen because it includes a Coulomb-corrected effective proton energy in all energy-dependent terms of the parameterization, just like that obtained from Eq. (3) when it is applied to proton elastic scattering. However, we have expanded the BGVC parameter set to include a linear energy dependence in the isovector part of the potential (as opposed to the constant isovector strengths used by BG).

The optical potential adopted in this work contains an isospin independent spin-orbit term and may be written as

$$U(r,E) = -U_0'(r,E) + 4U_1'(r,E)(\vec{t}\cdot\vec{T})/A + U_{so}(r), \quad (5$$

the isoscalar and isovector parts of which are related to the corresponding terms in Eqs. 1 and 2 by

$$U_0 = U_0'-U_{so} \text{ and} \quad (6$$

$$U_1 = U_1'. \quad (6$$

U_0' and U_1' have the same form, with different strength coefficients and are given by

$$U_k'(r,E) = (V_{kC}+V_{kE}E)f(r,R_R,a_R)+i(W_{kC}+W_{kE}E)f(r,R_I,a_I) \quad (7$$

$$-i\,a_I(WS_{kC}+WS_{kE}E)\frac{d}{dr}f(r,R_I,a_I) \quad (k=0,1),$$

where

$$f(r,R_j,a_j) = (1+\exp((r-R_j)/a_j))^{-1} \text{ and}$$
$$R_j=r_jA^{1/3}.$$

Also,

$$U_{so}(r) = V_{so}\lambda\!\!\!^{-2}\vec{\sigma}\cdot\vec{\ell}\frac{1}{r}\frac{d}{dr}f(r,R_{so},a_{so}), \quad (8)$$

where $\vec{\sigma}$ and $\vec{\ell}$ are the spin and orbital angular momentum operators, respectively, and $\lambda\!\!\!^{-}$ is the pion Compton wavelength; and

$$V_c(r) = (Zze^2/2R_c)(3-(r/R_c)^2), \text{ for } r\leq R_c, \quad (9)$$
$$= Zze^2/r \quad \text{, for } r\geq R_c,$$

where Z and z are the target and projectile charge numbers, respectively, e is the electron charge, and the Coulomb radius is given by the semiempirical formula[25]

$$R_c=1.149A^{1/3}+1.788A^{-1/3} - 1.163/A.$$

Parameter Search

The general search procedure has been to optimize the fit to our (p,n)-IAS data by varying the strengths of the isovector terms in the Lane potential. If the form of the isoscalar part of the BGVC potential is also used for the isovector part of the potential, there can be up to six free parameters. These are the strength coefficients for the energy-independent and energy-dependent terms of the real volume, imaginary volume, and imaginary surface isovector terms. While some work has been done using six free parameters, the majority of our searches involve three or four parameters. The quality of fit obtained with a six-parameter search, as compared to a four-parameter search, does not justify either the additional complexity or

the additional computer time required. Chi-squared per degree of freedom only improves by about 10% in going from a four-parameter to a six-parameter search, which is not enough to make an obvious visual improvement in the quality of fit. In the four-parameter searches, the imaginary volume isovector terms have been set equal to zero. In most of the three-parameter searches, one of the two remaining energy-dependent isovector coefficients has also been set equal to zero.

The fit criterion used by the search program is the minimization of χ^2 defined by

$$\chi^2 = \sum_i[(\sigma_{OM}(\theta_i)-\sigma_{exp}(\theta_i))/\Delta\sigma(\theta_i)]^2, \quad (10$$

where $\sigma_{OM}(\theta_i)$ is the calculated differential cross section, $\sigma_{exp}(\theta_i)$ is the experimental value, and $\Delta\sigma(\theta_i)$ is the experimental error for the i^{th} data point. For the majority of our searches, $\Delta\sigma(\theta_i)$ is given by the square-root of the sum of the squares of the statistical error and a systematic error of 11%. The inclusion of the systematic error in the fit-criterion function tends to weight all of the data points more equally, which is not always desirable when doing optical-model fits. In the present work we are primarily interested in the strength and energy dependence of the isovector part of the optical potential, which are presumably most sensitive to the magnitude, and perhaps the gross structure, of the data. If only the statistical error were used, the forward-angle data would be weighted much more strongly than the back-angle points. This would make the quality of fit more sensitive to the detailed shape of the forward-angle data, which is probably determined more by the geometry parameters than by the strength parameters.[9,21]

Since we recognize the limitations of the geometry parameterization used, we feel that it is desirable to have the fit criterion reasonably insensitive to the detailed structure of the data.

Table I contains parameter sets obtained by fitting our (p,n)-IAS data, along with two sets of parameters from Ref. 20. The parameter sets labeled "A" through "F" have been obtained by weighting the data with the total errors, while sets "AR", "BR", and "FR" have been obtained by using only the relative errors in the data. The parameters labeled "BGVC" are the Becchetti-Greenlees Coulomb-corrected proton parameters that have been used as the basis for the search parameterization. The form of set BGVC in Table I is obtained from set "V_{cc}=0.84" in Ref. 20 by combining the energy-dependent and Coulomb-correction terms to produce our effective energy $E=E_p-0.84Z/A^{1/3}$. The BG-Common parameter set ("Common" in Ref. 20) is the BG neutron parameter set that most closely corresponds to the BGVC set. The parameter sets result from:

(sets A and AR) a 4-parameter search with a real volume, imaginary surface, energy-dependent, isovector potential;

(sets B and BR) a 3-parameter search with a real volume energy-independent, imaginary surface energy-dependent, isovector potential;

(set C) a 3-parameter search with a real volume energy-dependent, imaginary surface energy-independent, isovector potential;

(set D) a 3-parameter search with a real volume, imaginary surface, energy-dependent, isovector potential with its energy-dependent strength coefficients constrained to be equal;

(set E) a 2-parameter search with a real volume, imaginary surface, energy-independent, isovector potential; and

(sets F and FR) a 6-parameter search with a real volume, imaginary volume, energy-dependent, isovector potential.

All of the parameter sets, except BG-Common, have the same geometry as BGVC, which is: r_R=1.17 fm, a_R=0.75 fm, r_I=1.32 fm, and a_I=0.51+0.7(N-Z)/A fm. The BG-Common set has the same real geometry, but the imaginary geometry is r_I=1.26 fm and a_I=0.58 fm. All of the parameter sets include an isoscalar spin-orbit term specified by V_{so}=6.2 MeV, r_{so}=1.01 fm, and a_{so}=0.75 fm. All parameter sets, except BG-Common, use a Coulomb correction $\bar{V}_c=0.84Z/A^{1/3}$ MeV. The BG-Common set has no Coulomb correction since it was determined for neutron scattering only.

The average χ^2 per data point (χ^2/N) is given in Table I for each parameter set. The values of χ^2/N obtained with relative errors only are larger since the errors are smaller. The numbers in parentheses result from using the parameters obtained by fitting data with total (relative) errors to calculate the fit to data using relative (total) errors. The (p,n) χ^2/N values listed between sets BGVC and BG-Common have been obtained by using the BGVC and BG-Common parameters to calculate the proton and neutron distorted waves, respectively, and to calculate the (p,n)-IAS form factor with the prescription of Hoffmann.[15] The smaller (larger) number has been obtained with the total (relative) experimental errors.

The parameter set that appears to produce the best overall results is set B, which may be written for proton or neutron scattering as:

$$V=55.8-0.32E\pm17.7(N-Z)/A \text{ MeV},$$
$$r_R=1.17 \text{ fm}, \; a_R=0.75 \text{ fm},$$
$$WS=9.6-0.22E\pm(18.1-0.31E)(N-Z)/A \text{ MeV}, \qquad (11)$$
$$W=-1.4+0.22E \text{ MeV}$$
$$r_I=1.32 \text{ fm}, \; a_I=-.51+0.7(N-Z)/A \text{ fm}$$
$$V_{so}=6.2 \text{ MeV},$$
$$r_{so}=1.01 \text{ fm}, \; a_{so}=0.75 \text{ fm},$$

where the plus (minus) sign in V and WS is used for proton (neutron) scattering and E is given by $E=E_p-0.84Z/A^{1/3}$ for protons and by $E=E_n$ for neutrons. E_p and E_n are the incident proton and neutron energies, respectively. The (p,n)-IAS form factor (or potential) obtained from set B may be written as:

$$V=(2\sqrt{N-Z}/A)x17.7 \text{ MeV},$$
$$r_R=1.17 \text{ fm}, \; a_R=0.75 \text{ fm}, \qquad (12)$$
$$WD=(2\sqrt{N-Z}/A)(18.1-0.31E),$$
$$r_I=1.32 \text{ fm}, \; a_I=0.51+0.7(N-Z)/A \text{ fm},$$

where $E=1/2(E_p-0.84Z/A^{1/3}+E_n)$, $E_n=E_p-\Delta_c$, $-\Delta_c$ is the (p,n)-IAS Q-value, and E_p is the incident proton energy. The shape of the potential is given in Eq. 7. If it is desirable to have an energy dependence in the real isovector term, set D would probably produce the best overall result.

RESULTS

The parameter sets listed in Table I generally reproduce the potential obtained from set BGVC for elastic proton scattering. For most of our targets and proton bombarding energies, the total difference between the proton potential strengths obtained using our parameters and those obtained using the BGVC

parameters is less than 0.5 MeV. The maximum difference, around one MeV, occurs for 25-MeV protons on ^{208}Pb. Figure 1 shows the results of proton elastic scattering calculations using sets B and BGVC. As can be seen from the figure, the two calculations agree quite well at forward angles and tend to deviate some at back angles, especially for ^{48}Ca and ^{120}Sn at 45 MeV. In general, it would seem reasonable to expect parameter set B to reproduce proton elastic scattering data almost as well as the BG Coulomb-corrected proton parameters (BGVC). It might be noted that the BG Coulomb-corrected and "Best-fit" proton parameters have the same values of χ^2/N in Ref. 20.

Figure 2 shows our (p,n)-IAS data along with the results of DWBA calculations using parameter set B (solid lines). The dashed lines in Fig. 2 are obtained using the BGVC proton parameters, the BG-Common neutron parameters, and the (p,n) form-factor prescription of Hoffmann.[15] The results obtained with the BG parameters are rather impressive since no free parameters have been adjusted to fit the (p,n)-IAS data. This not only indicates that the BG Coulomb-corrected proton and "Common" neutron parameters are reasonable, but further demonstrates the utility of the Lane model of the optical potential. The fits obtained using set B are clearly as good as, or better than, those obtained with the BG proton and neutron parameter sets. This is a fairly impressive result for a 3-parameter search. It demonstrates that it is possible to obtain reasonable fits to (p,n)-IAS data over a large incident-energy and target-mass range with a simple Lane-model optical-potential parameterization. (p,n)-IAS calculations have also

been performed using the BG "Best-fit" proton and neutron parameters. The cross sections are lower in magnitude than those obtained with the BG Coulomb-corrected parameters, resulting in an increase by a factor of 2.3 in χ^2/N compared to that for the Coulomb-corrected set. This may be interpreted as an indication of the desirability of using the Coulomb correction in all energy-dependent terms of the potential, at least for (p,n)-IAS calculations. It might also be noted that if set BGVC is used by itself as a Lane-model optical potential (in the same manner as set B) to calculate (p,n)-IAS cross sections, the χ^2/N obtained for our data is about a factor of 6.5 larger than that obtained by using set B. This is perhaps the best measure of the amount of improvement obtained in the fit to our data by doing the parameter search.

The most stringent test of our Lane-model parameterization is whether or not it is capable of reproducing neutron elastic scattering data. Figure 3 shows the results of optical-model calculations using set B (solid lines) and the BG-Common neutron parameters (dashed lines), along with neutron elastic scattering data.[26] The targets selected are all those for which there are (n,n) data from 7 to 24 MeV. Once again, the set B parameters reproduce the data as well as, if not better than, the BG-Common neutron parameters. It should be noted that no compound-elastic-scattering contribution has been included in the calculations,[27] although a significant compound-elastic correction was included by BG in their (n,n) fits below 10 MeV.[20] The results obtained using the BG "Best-fit" neutron parameters are slightly poorer than those obtained

using the BG-Common set. This is probably due to the fact that BG did include a compound-elastic contribution which has been omitted in the present calculations. Because natural targets were used for the Fe and Sn data, the curves for Fe and Sn are calculated for the natural isotopic abundances. It is important to note that, while the BG neutron parameters were obtained by fitting neutron elastic scattering data, the values for the other parameter sets in Table I have been determined by fitting (p,n)-IAS data in a Lane-model-consistent manner while maintaining the proton optical potential of BG. Thus, no free parameters have been involved in obtaining the solid curves in Fig. 3. It is impressive that set B reproduces neutron elastic scattering data over a large range of neutron energy and target mass, even though the neutron energies and most of the targets are different from those included in the (p,n)-IAS data used to determine the parameters.

DISCUSSION

In addition to set B, there are other parameter sets in Table I which give good fits to (p,p), (p,n)-IAS, and (n,n) data, and some which do not. The individual results suggest several observations about the fitting procedure and the nature of the isovector potential.

The parameter sets that have been obtained with total-error weighting in the (p,n)-IAS data consistently reproduce the neutron elastic scattering data better than do parameter sets obtained with only relative-error weighting. This is indicated by the roughly 50% increase obtained in the (n,n) χ^2/N for sets AR and FR

compared to sets A and F, respectively. It is interesting that there is relatively little difference between parameter sets B and BR. This suggests that perhaps the 3-parameter searches are significantly more stable than the searches with more than 3 free parameters. Set B is probably the most useful parameter set in Table I, since it gives the best overall results with a minimum of parameters.

Previous work has indicated that, in general, (p,n)-IAS data can be fit better with a complex, rather than a real, isovector potential.[7-12] In addition, by using a complex isovector potential, it has been possible to construct a Lane-model optical potential that (when applied to proton elastic scattering) essentially reproduces the Becchetti-Greenlees Coulomb-corrected proton optical potential. In order to see if a complex isovector potential is preferred in calculations for our (p,n)-IAS data, a search has been performed that corresponds to set A, but with a real surface isovector term instead of an imaginary surface isovector term. The geometry of the real surface term is the same as the geometry of the imaginary term in set A. The result is an increase by a factor of 1.5 in the value of χ^2/N, compared to that of set A, thus confirming a preference for a complex isovector potential.

The improvement in the quality of fit obtained in going from an energy-independent isovector potential to an energy-dependent isovector potential is indicated by comparing sets A and E. The χ^2/N for set E is a factor of 1.5 larger than that obtained for set A, indicating that a moderately-significant improvement in the quality of fit is obtained by including

the energy dependence. It is interesting that by removing the real volume isovector energy-dependent term, as for set B, χ^2/N increases by only a factor of 1.03 compared to that of set A. A greater effect is obtained by deleting the imaginary surface isovector energy dependence. The χ^2/N for set C, which has this deletion, is 1.18 times the χ^2/N for set A. This may be an indication that the imaginary surface isovector energy dependence is stronger than the real volume isovector energy dependence. However, such behavior of the energy dependence may be sensitive to the choice of geometry parameters.

The variations in the isovector strength parameters obtained for sets A through D indicate that the distribution of the isovector strength and energy dependence between the real volume and imaginary surface terms is not well determined in the present work. The problem is most likely caused by correlations between the various strength parameters similar to the continuous ambiguities found in most optical-model parameterization studies. In order to obtain some information on what quantities are well determined by our search procedure, the eigenvectors in parameter space have been found by diagonalizing the error matrix produced by the search program. Since the eigenvectors are linear combinations of the search parameters, this technique will only give linear approximations to any underlying ambiguities. While the actual coefficients vary somewhat from set-to-set, the general result is that two linear combinations of the isovector parameters are well determined. Roughly, they are the sum of the two constant coefficients and the sum of the two energy-dependent coefficients. Parameter

sets A through D are consistent with this result in that they all have approximately the same sums of the constant and energy-dependent isovector coefficients, about 35 MeV and -0.3, respectively. The positive slope of the real volume isovector energy-dependence in set A should not be taken at face value, but as an indication of the ambiguities in the parameter space or, perhaps, of an improper choice of the form and/or the geometry of the isovector potential.

The results of the two 6-parameter searches, sets F and FR, also indicate that an excess of free parameters only leads to meaningless numbers. While the fit to the (p,n)-IAS data is improved by the additional search parameters, the neutron predictions are much poorer than those obtained with set B.

SUMMARY

The purpose of this work has been to develop an energy-dependent Lane-model optical potential that can be used to describe proton elastic, neutron elastic, and (p,n)-IAS scattering over a large energy and target mass range. A straightforward parameterization of such a potential has been presented.

The isovector strength parameters of the optical potential have been obtained by fitting (p,n)-IAS data for targets of ^{48}Ca, ^{90}Zr, ^{120}Sn, and ^{208}Pb at proton bombarding energies of 25, 35, and 45 MeV. The isoscalar strength parameters have been obtained from the Coulomb-corrected proton parameter set of Becchetti and Greenlees, as have the geometries for both the isoscalar and isovector parts of the potential. The isoscalar strengths have been adjusted slightly to compensate, in an average manner, for changes in the isovector strengths so that the proton optical

potential obtained from our Lane-model optical potential closely reproduces the Becchetti-Greenlees Coulomb-corrected proton optical potential. Thus, the Lane-model optical potential should do an adequate job of reproducing proton elastic scattering data. The Lane-model optical potential also gives a reasonable account of the (p,n)-IAS data used in the parameter search.

The Lane-model optical potential has also been used to predict neutron elastic scattering. The calculations agree rather well with available neutron data for targets from Al to Bi for neutron energies from 7 to 24 MeV. These results demonstrate that a properly formulated energy-dependent Lane-model optical potential, determined by fitting proton elastic scattering and (p,n)-IAS data, is capable of reproducing neutron elastic scattering data over a large range of energies and targets.

CONCLUSION

In this work, we have added one parameter, an imaginary surface energy-dependent isovector strength, to the Becchetti-Greenlees Coulomb-corrected proton parameter set, and then varied the potential to optimize the fit to our (p,n)-IAS data. The result of this search is primarily to redistribute the isovector strength between the real and imaginary terms, yielding parameter set B. Thus, with relatively minor modifications of the BG Coulomb-corrected proton potential, we have obtained a Lane-model optical potential that may be used with a reasonable amount of confidence to calculate (p,p), (p,n)-IAS, and (n,n) cross sections over a wide range of projectile energy and target mass.

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TABLE I--Parameter sets obtained by searching the values of the isovector strength parameters in order to optimize the fit to our (p,n)-IAS data (shown in Fig. 2). Two parameter sets obtained from Ref. 20 are also included (BGVC and BG-Common).

TABLE I

SET[a]	COMMENTS		V (MeV)	WS (MeV)	W (MeV)	(p,n) χ^2/N	(n,n) χ^2/N
A[b]	4-param. search	U_0	56.4-0.34E	9.2-0.20E	-1.4+0.22E	3.7 (23.0)	10.7
		U_1	12.3+0.25E	22.0-0.50E			
B[b,c]	3-param. search	U_0	55.8-0.32E	9.6-0.22E	-1.4+0.22E	3.8 (25.0)	9.5
	$V_{1E} = 0$	U_1	17.7	18.1-0.31E			
C[b]	3-param. search	U_0	54.9-0.28E	10.4-0.25E	-1.4+0.22E	4.4 (30.0)	12.0
	$WS_{1E} = 0$	U_1	27.4-0.36E	9.8			
D[b,c]	3-param. search	U_0	55.4-0.30E	9.9-0.23E	-1.4+0.22E	3.9 (27.0)	9.8
	$V_{1E} = WS_{1E}$	U_1	21.9-0.18E	15.2-0.18E			
E[b]	2-param. search	U_0	55.6-0.32E	10.5-0.25E	-1.4+0.22E	5.7 (33.9)	13.0
	$V_{1E} = WS_{1E} = 0$	U_1	20.1	8.5			
F[b]	6-param. search	U_0	56.7-0.35E	10.0-0.23E	-2.3+0.25E	3.3 (20.6)	18.7
		U_1	9.0+0.33E	13.5-0.23E	8.6-0.25E		
AR[d]	4-param. search	U_0	56.9-0.37E	8.9-0.19E	-1.4+0.22E	21.2 (4.1)	16.9
		U_1	6.5+0.46E	25.4-0.61E			
BR[d]	3-param. search	U_0	55.9-0.32E	9.6-0.22E	-1.4+0.22E	24.0 (4.1)	9.8
	$V_{1E} = 0$	U_1	16.8	18.3-0.27E			
FR[d]	6-param. search	U_0	56.9-0.36E	9.9-0.22E	-2.2+0.24E	18.2 (3.9)	26.0
		U_1	6.8+0.39E	15.5-0.31E	7.6-0.15E		
BGVC[e]	"Vcc = 0.84"	U_0	55.2-0.32E	10.2-0.25E	-1.4+0.22E	24.7 130.9	10.7
		U_1	24.0	12.0		6.3 36.0[f]	
BG-Common[e,g]	"Common"	U_0	55.2-0.32E	12.0-0.25E	-1.4+0.22E	9.2 47.4	13.2
		U_1	24.0	12.0			

GEOMETRY PARAMETERS: $r_R = 1.17$ fm $\quad r_I = 1.32$ fm

$a_R = 0.75$ fm $\quad a_I = 0.51+0.7(N-Z)/A$ fm

ENERGY PARAMETER:

(p,p)----$E = E_p - 0.84Z/A^{1/3}$

(n,n)----$E = E_n$

(p,n)----$E = 1/2(E_p - 0.84Z/A^{1/3} + E_n)$

Table 1 Footnotes

a) All parameter sets have an isoscalar spin-orbit term given by V_{so} = 6.2 MeV, r_{so} = 1.01 fm, and a_{so} = 0.75 fm.

b) During the search, the (p,n)-IAS data were weighted by the square root of the sum of the squares of the statistical errors and a systematic error of 11%.

c) Set B is our preferred parameter set. Set D is our preferred parameter set if a real isovector energy dependence is desired.

d) During the search, the (p,n)-IAS data were weighted by the statistical errors.

e) These parameter sets are from Ref. 20.

f) These χ^2/N values have been obtained with the BGVC and BG-Common parameters and the form-factor prescription of Hoffmann[15].

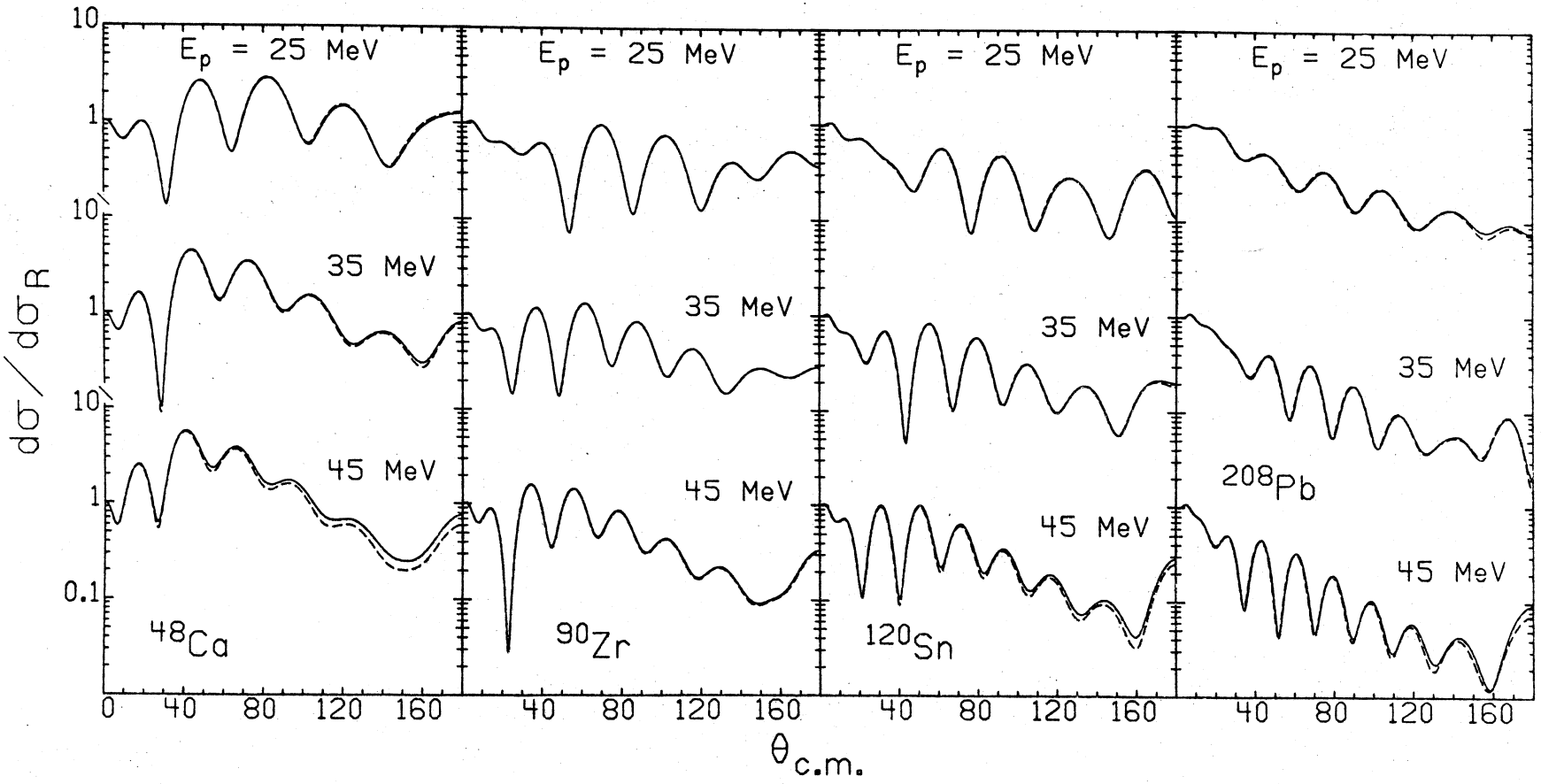
g) The imaginary geometry for set BG-Common is given by r_I = 1.26 fm and a_I = 0.58 fm.

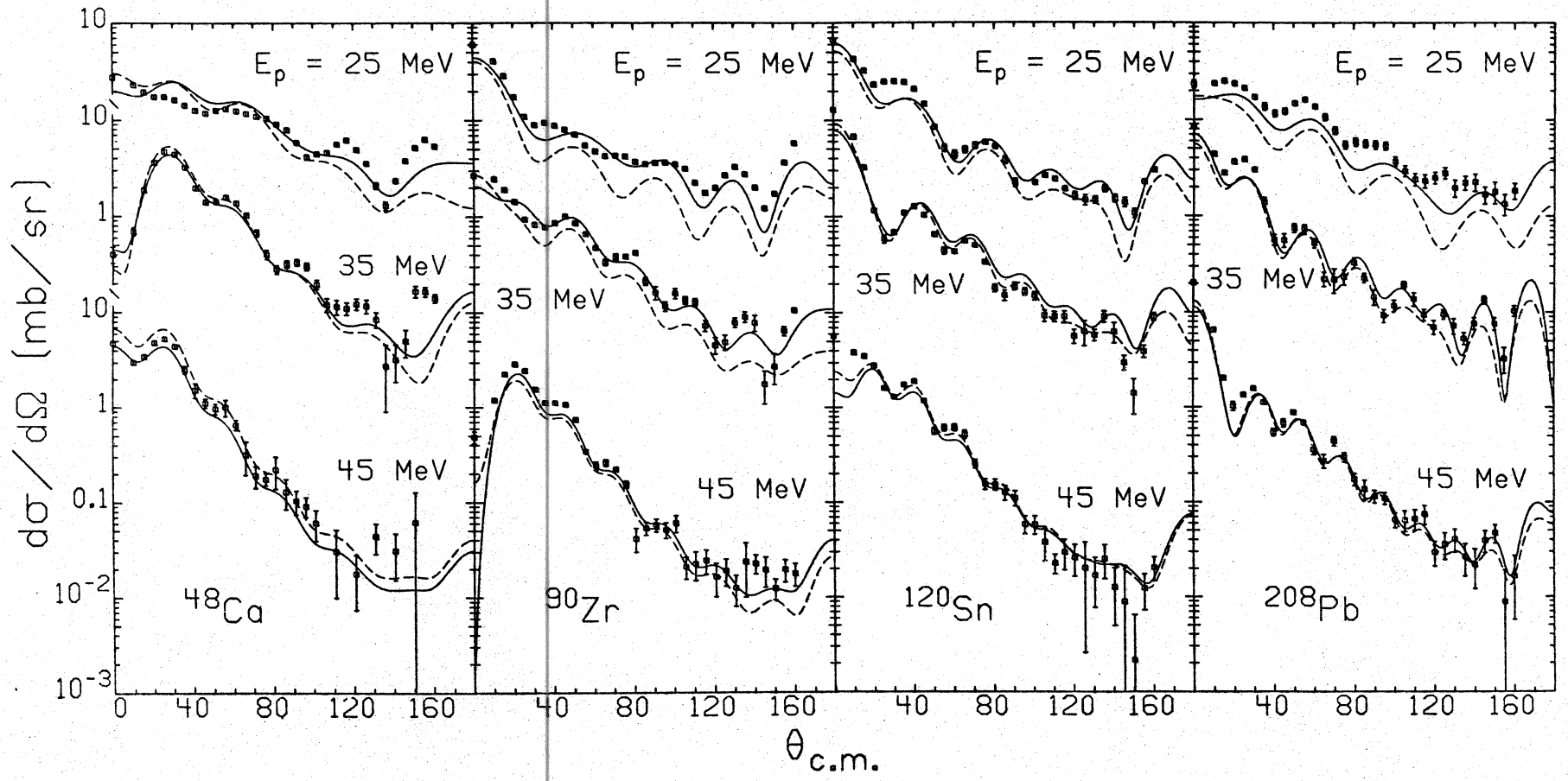
FIGURE CAPTIONS

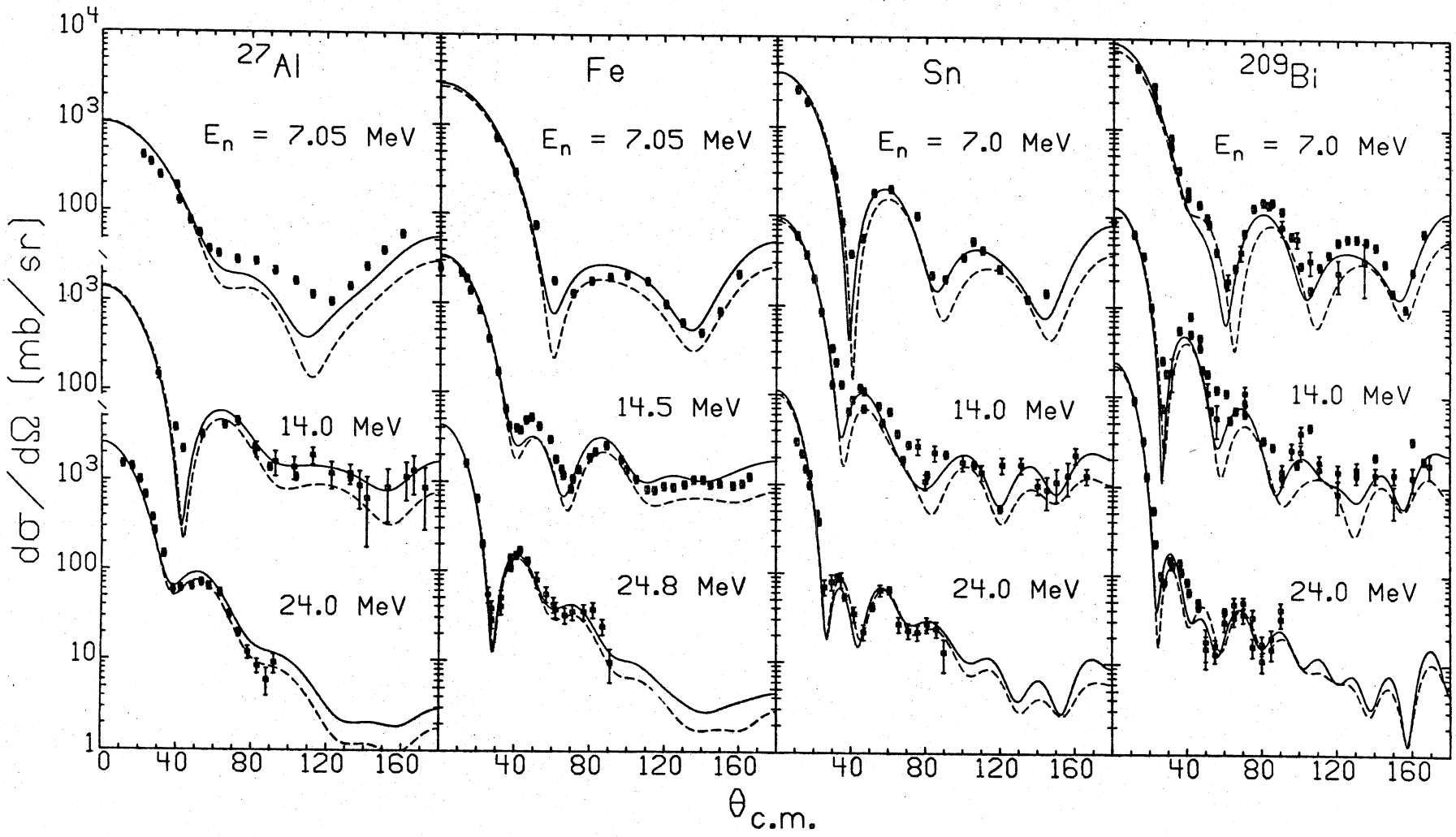
Fig. 1.--Comparison of optical-model calculations for (p,p) elastic differential cross-sections (ratio to Rutherford). The solid curves have been obtained with parameter set B of Table I. The dashed curves have been obtained with the BGVC proton parameters of Table I (obtained from Ref. 20.).

Fig. 2.--Comparisons of DWBA calculations with (p,n)-IAS differential cross-section data. The solid curves have been obtained with the Lane-model-consistent parameters, set B, of Table I, which result from a global search on these data. The dashed curves have been obtained with the BGVC proton and BG-Common neutron parameters of Table I (obtained from Ref. 20) and the (p,n) form-factor prescription of Ref. 15.

Fig. 3.--Comparisons of optical-model calculations with (n,n) elastic differential cross-section data. The solid curves have been obtained with parameter set B of Table I. The dashed curves have been obtained with the BG-Common neutron parameters of Table I (obtained from Ref. 20). The neutron data are from Ref. 26.


E_p = 25 MeV
35 MeV
45 MeV
^{48}Ca
^{90}Zr
^{120}Sn
^{208}Pb
$d\sigma / d\sigma_R$
$\theta_{c.m.}$
10
1
10/
1
10/
1
0.1
0
40
80
120
160


E_p = 25 MeV
35 MeV
45 MeV
^{48}Ca
^{90}Zr
^{120}Sn
^{208}Pb
dσ/dΩ (mb/sr)
θc.m.
10
1
0.1
10^{-2}
10^{-3}
0
40
80
120
160


27Al
Fe
Sn
209Bi
En = 7.05 MeV
En = 7.05 MeV
En = 7.0 MeV
En = 7.0 MeV
14.0 MeV
14.5 MeV
14.0 MeV
14.0 MeV
24.0 MeV
24.8 MeV
24.0 MeV
24.0 MeV
dσ/dΩ (mb/sr)
θc.m.
10^4
10^3
100
10
1
0
40
80
120
160