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The Effective M1 Operator
and Magnetic Moments of sd-Shell Nuclei*

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Analysis of experimental magnetic dipole moments of A=17-39 nuclei with shell-model wave functions shows that a constant-valued effective M1 operator which differs only slightly from the free-nucleon specifications suffices to explain all known data at a reasonable level of accuracy.

The experimental values of nuclear magnetic dipole moments are explained theoretically by a combination of assumptions and/or calculations of the relevant wave function structures and of the magnetic properties manifested in nuclear matter by the neutron and proton. The values of the single-particle matrix elements of the magnetic dipole operator which are appropriate to use in a calculation of moments in the context of a finite model basis space (the effective M1 operator) are affected by contributions from admixtures of adjacent configurations excluded from the space.[1-3] In addition, mesonic effects can cause the magnetic properties exhibited by the neutron and proton in free space to be altered in the nuclear environment.[4-6] Unfortunately, a completely unambiguous separation of these two kinds of effects is not possible.

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Previous analysis of the effective M1 operator needed to explain observed magnetic moments have been restricted in the main to nuclei whose wave functions are simply describable within the confines of a given shell-model representation; that is to say only one particle or one hole systems and a few two-body states have been considered,[1-3] along with pairs of mirror states.[7,8] This has resulted from the necessity of excluding uncertainties about the correct multiparticle wave functions within the model basis. In this note we explore the question of the effective M1 operator from a different direction than that conventionally taken. We use full-space $d_{5/2}$-$s_{1/2}$-$d_{3/2}$ shell-model wave functions to analyze measured moments of essentially all A=17-39 nuclei. We thereby determine and set limits upon the amount of renormalization of the M1 operator, and investigate the state dependence of this renormalization, on the basis of the totality of data in this region.

The wave functions ψ^{NJT} we use in our analysis of states of A=N+16 nuclei are expansions over the full set of $0d_{5/2}$-$1s_{1/2}$-$0d_{3/2}$ basis vectors. They are derived from diagonalization of two Hamiltonians[9] which were empirically adjusted to give best fits to level energies of, respectively, the A=18-24 and A=32-38 regions. One-body density matrices,

$$\rho^{NJT}_{ijI} = \frac{\langle\psi^{NJT}|||(a_i^{\dagger}\otimes a_j)_{1I}|||\psi^{NJT}\rangle}{\sqrt{3(2I+1)}},$$

where $(a_i^{\dagger}\otimes a_j)_{1I}$ is the coupling to rank 1 in spin and rank I in isospin of the single-particle annihilation operator for orbit j and the single-particle creation operator for orbit i, are evaluated from the amplitudes of the shell-model wave functions and used as fixed input to the remaining analysis. The magnetic moments are given by

$$\mu = \sqrt{\frac{4\pi}{3}} \sum_{I=0}^{1} \frac{\langle JJ10|JJ\rangle\langle TT_z I0|TT_z\rangle}{\sqrt{2J+1}\quad\sqrt{2T+1}} \langle\psi^{NJT}|||M1|||\psi^{NST}\rangle_I ,$$

where $\langle\psi^{NJT}|||M1|||\psi^{NJT}\rangle_I = \sum_{ij} \langle i|||M1|||j\rangle_I \rho^{NJT}_{ijI}$.

Given the ρ^{NJT}_{ijI}, the magnetic moments are completely specified by the single-particle matrix elements $\langle i|||M1|||j\rangle_I$, of which there are eight (independent) in the sd-shell if the conventional ℓ-selection rule for M1 is assumed.

We first evaluate the magnetic moments for sd-shell nuclei by assuming the free-nucleon values (g_ℓ^p=1.0 n.m., g_ℓ^n=0.0, g_s^p=5.58 n.m., g_s^n=-3.82 n.m.) for the M1 operator. The results for A=17-25 nuclei are shown by the circle points in Figure 1. The Hamiltonian-independent results for A=17 of course just illustrate the long known fact that the A=17 moments fall close to the Schmidt lines. The uniformly good reproduction of the moments of the many-particle systems up through A=25 suggests that there are no significant changes in the form of the effective M1 operator in this whole region. All observed deviations from the Schmidt limits are encompassed in the variations determined by the density matrices and arise from the configuration mixing within the active sd-shell orbits. The small discrepancies for A=18 seem plausibly attributable to the known "4 particle-2 hole" contaminations to these states which are not contained in the present wave functions.

The magnetic moments of A=39 do not fall on the Schmidt lines. We find, consistent with this, that the moments of the multi-hole systems below A=39 are also not reproduced by combining the free-nucleon M1 operator with the density matrices. These results are indicated by the circle points in Figure 2. (It should be noted, however, that due to the smaller $d_{3/2}$ moments, caused by the cancellation of the spin and orbit contributions, magnitudes of moments in the A>30 region are naturally smaller, and the scale of Figure 2 has correspondingly been increased.)

These results thus establish that the bare M1 operator combined with the best available shell-model wave functions yields "good" agreement with measured moments in the lower half of the sd-shell and "mediocre" agreement with data in the upper half of the shell. The question remains whether the deviations between theory and experiment can be accounted for by a state-independent renormalization of the operator in the confines of the usual selection rules. The conventional specification of the renormalization of the M1 operator in terms of g-factors is redundant since there are only eight independent M1 matrix elements for our j-j shell-model representation. Hence we treated the 4 isoscalar and 4 isovector single-particle matrix elements as parameters and fitted them to 39 of the precisely measured M1 moments in the A=17-39 region. The results of this fit are compared to the free-nucleon values of the matrix elements in Table I. The uncertainties quoted are obtained by assuming a uniform error of 0.08 n.m. for each datum, a value equal to the rms deviation obtained in the fit. The results obtained for magnetic moments by using this empirically modified M1 operator are shown in Figures 1 and 2 by the triangle points. The good reproduction of the A=17-25 region has been maintained and the reproduction of A=29-39 data has been made equivalently good. Hence an effective operator which is suggested jointly by the A=17 and A=39 systems serves to reproduce all known moments to reasonable accuracy.

The net renormalization of the M1 operator from the free-nucleon values which we obtain is small (see Table I) and principally involves a quenching of the $d_{3/2}$ isovector term. No evidence for any state dependence of the renormalization can be found after the appropriate sd-configuration mixing is taken care of by the shell-model wave functions. The effective M1 matrix elements of Table I can be related to effective g-factors, although not, of course, in a one-to-one fashion. Values of the g-factors which best approximate the effective matrix elements (excluding the $s_{1/2}$-$s_{1/2}$ matrix elements) are $g_{\ell}^{p}=1.09$, $g_{\ell}^{n}=-0.02$, $g_{s}^{p}=5.16\pm0.02$, and $g_{s}^{n}=-3.58\pm0.02$ in n.m. The quenching of the spin-g-factors are consistent with the suggested mesonic corrections of Miyazawa[4] and Drell and Walecka[5]. The renormalizations implied for the orbital g-factors are also consistent with those of Chemtob[6] and Nagamiya and Yamazaki[8]. The second-order configuration-mixing corrections of Mavromatis and collaborators[3] are considerably larger. The $s_{1/2}$-$s_{1/2}$ M1 matrix elements, on the other hand,

imply an anti-quenching of the spin-g-factors, $g_{s}^{p}=5.70\pm0.34$ and $g_{s}^{n}=-4.18\pm0.34$ in n.m., which however is not inconsistent with the observed deviations of the ^{3}H and ^{3}He magnetic moments from the Schmidt lines.

Overall, the net renormalization determined for the M1 operator is smaller than the values currently predicted. The lack of state dependence in the renormalization also seems anomalous. Our results suggest either that many of the contributions to the M1 renormalization in this region are fortuitiously and stably self canceling or that current theory significantly over estimates such contributions. The present evidence for the existence of a well-defined and universal M1 operator for the sd-shell, one which accounts for an extensive body of data, will hopefully stimulate further attempts to explain the properties of this operator in fundamental terms. At the same time, the existence of this operator should stimulate and give meaning to further experimental efforts to measure magnetic moments, since there is now adequate reason to interpret observed values as good tests of theoretical wave functions.

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REFERENCES

1. A. Arima and H. Horie, Progr. Theor. Phys. 11(1954)509; 12(1954)623.
2. R.J. Blin-Stoyle, Proc. Phys. Soc. A66(1953)1158.
3. H.A. Mavromatis and L. Zamick, Nucl. Phys. A104(1967)17; Phys. Lett. 20(1966)171; H.A. Mavromatis, L. Zamick, and G.E. Brown, Nucl. Phys. 80(1966)545.
4. H. Miyazawa, Progr. Theor. Phys. 6(1951)801.
5. S.D. Drell and J.D. Walecka, Phys. Rev. 120(1960)1069.
6. M. Chemtob, Nucl. Phys. A123(1969)449.
7. K. Sugimoto, Phys. Rev. 182(1969)1051.
8. R. Leonardi and M. Rosa-Clot, Phys. Rev. Lett 24(1970)407.
9. Wilton Chung and B.H. Wildenthal, to be published.
10. S. Nagamiya and T. Yamazaki, Phys. Rev. C4(1971)1961.

TABLE I.--Single-particle M1 matrix elements (n.m.) derived from free-nucleon g-factors and from the fit to sd-shell magnetic-moment data.

$<\ell j \vert\vert\vert M1 \vert\vert\vert \ell' j'>_I^a$	free-nucleon	fitted^b
$<d_{5/2} \vert\vert\vert M1 \vert\vert\vert d_{5/2}>_0$	2.88	2.94±0.06
$<d_{5/2} \vert\vert\vert M1 \vert\vert\vert d_{3/2}>_0$	- 0.41	-0.19±0.15
$<s_{1/2} \vert\vert\vert M1 \vert\vert\vert s_{1/2}>_0$	0.74	0.64±0.14
$<d_{3/2} \vert\vert\vert M1 \vert\vert\vert d_{3/2}>_0$	1.13	1.29±0.04
$<d_{5/2} \vert\vert\vert M1 \vert\vert\vert d_{5/2}>_1$	11.62	11.49±0.18
$<d_{5/2} \vert\vert\vert M1 \vert\vert\vert d_{3/2}>_1$	- 7.79	-6.52±0.28
$<s_{1/2} \vert\vert\vert M1 \vert\vert\vert s_{1/2}>_1$	6.89	7.25±0.25
$<d_{3/2} \vert\vert\vert M1 \vert\vert\vert d_{3/2}>_1$	- 1.58	-0.97±0.13

[a] I equals 0 or 1 for isoscalar and isovector components, respectively.

[b] See text for description on uncertainties.

FIGURE CAPTIONS

Fig. 1.--Comparison of experimentally measured values (crosses) of magnetic dipole moments for A=17-25 nuclei with $d_{5/2}$-$s_{1/2}$-$d_{3/2}$ shell-model calculations which assume alternatively the free-nucleon g-factors (circles) and empirical M1 single-particle matrix elements (triangles). The A-J-T values indicate the nuclear states involved.

Fig. 2.--Comparison of experimentally measured values (crosses) of magnetic dipole moments for A=29-39 nuclei with $d_{5/2}$-$s_{1/2}$-$d_{3/2}$ shell-model calculations which assume alternatively the free-nucleon g-factors (circles) and empirical M1 single-particle matrix elements (triangles). The A-J-T values indicate the nuclear states involved.

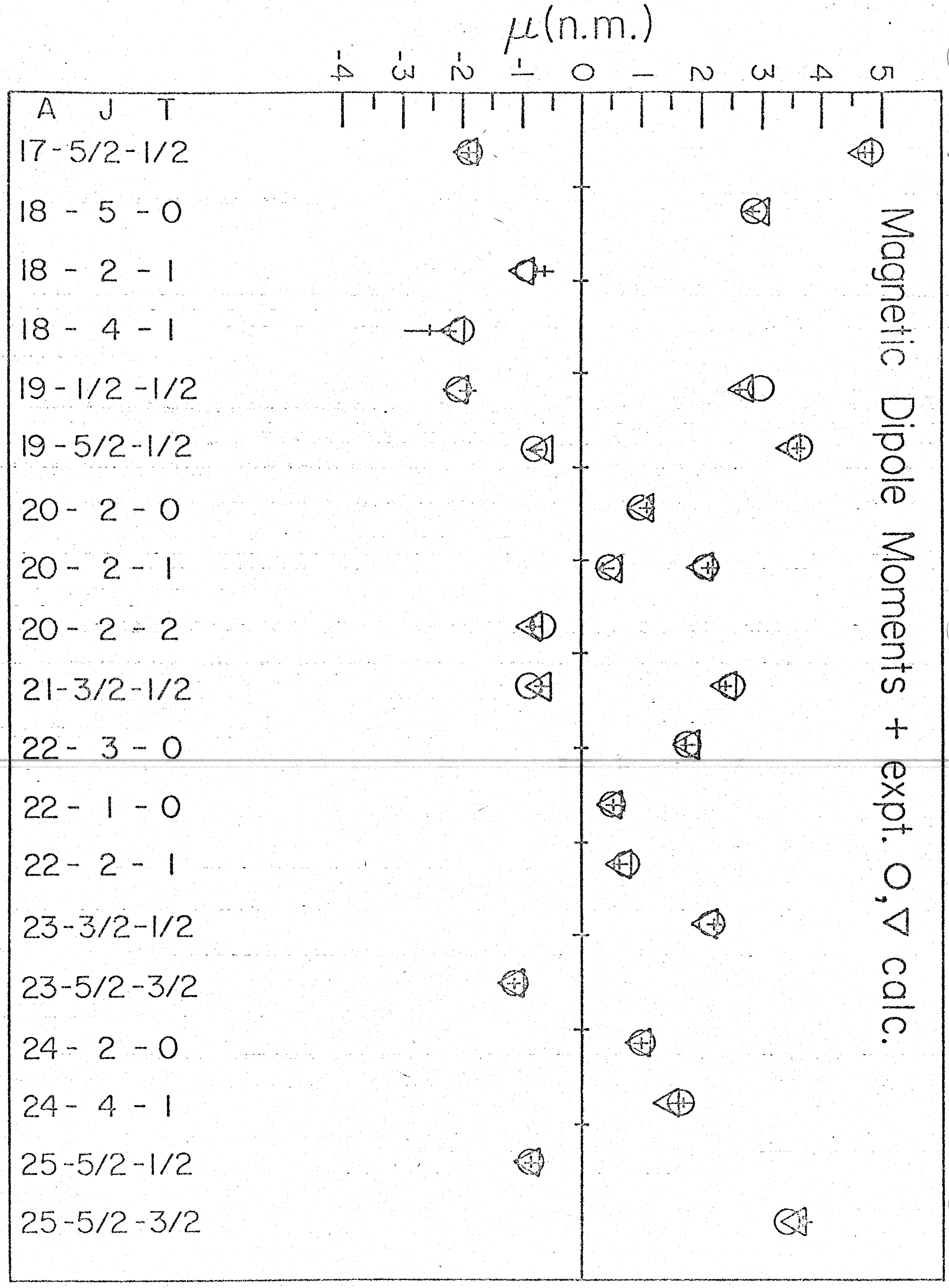
Magnetic Dipole Moments + expt. O,∇ calc.
μ(n.m.)
-4
-3
-2
-1
0
1
2
3
4
5
A J T
17-5/2-1/2
18 - 5 - 0
18 - 2 - 1
18 - 4 - 1
19-1/2-1/2
19-5/2-1/2
20 - 2 - 0
20 - 2 - 1
20 - 2 - 2
21-3/2-1/2
22 - 3 - 0
22 - 1 - 0
22 - 2 - 1
23-3/2-1/2
23-5/2-3/2
24 - 2 - 0
24 - 4 - 1
25-5/2-1/2
25-5/2-3/2

Fig. 1

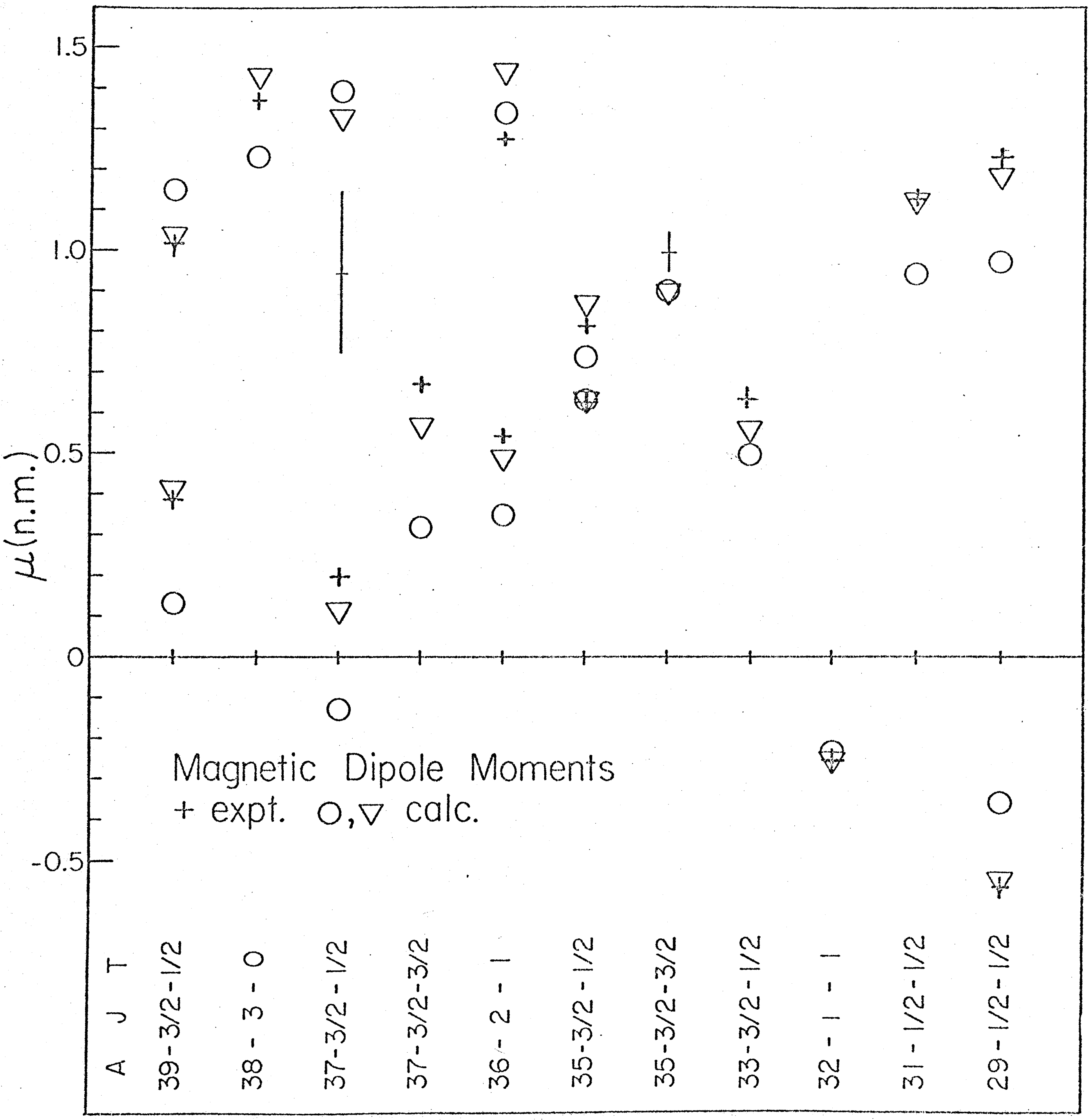
1.5
1.0
0.5
0
-0.5
μ(n.m.)
Magnetic Dipole Moments
+ expt. ○,▽ calc.
A J T
39-3/2-1/2
38 - 3 - 0
37-3/2 - 1/2
37-3/2-3/2
36 - 2 - 1
35-3/2 - 1/2
35-3/2-3/2
33-3/2 - 1/2
32 - 1 - 1
31 - 1/2 - 1/2
29 - 1/2 - 1/2

Fig. 2