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$\alpha + \alpha$ Phase Shifts and the $p + {}^7Li$ and $n + {}^7Be$ Channels*

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Abstract

Calculations reported recently in this journal indicate a discrepancy between measurements of $\alpha + \alpha \rightarrow n + {}^7Be$ cross sections and phase shifts deduced from $\alpha + \alpha$ elastic scattering. It is demonstrated that these calculations are erroneous.

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Spallation reactions between cosmic rays and interstellar matter have apparently been important in the nucleosynthesis of the light elements. For ^{6}Li and the Be and B isotopes, in fact, spallation appears to have been the dominant production mechanism[1]. In the production of ^{7}Li, two important spallation reactions are $\alpha + \alpha \rightarrow p + {}^7$Li and $\alpha + \alpha \rightarrow n + {}^7$Be (with the ^{7}Be subsequently decaying to ^{7}Li by electron capture). From our measurements of the cross sections for these reactions[2], we concluded that existing spallation models[1] were unlikely to yield sufficient ^{7}Li production to explain the observed cosmic abundance of ^{7}Li. Thus, other ^{7}Li production mechanisms might be required.

Recently, Slobodrian[3] has made an interesting comparison between the cross sections we measured and the results of $\alpha + \alpha$ elastic-scattering phase-shift analyses. Since the $p + {}^7$Li and $n + {}^7$Be channels are the only open reaction channels in $\alpha + \alpha$ scattering below 39.6 MeV, the sum of the cross sections for these channels is the total reaction cross section in this energy region. Slobodrian calculated the total reaction cross section (incorrectly, as we shall show) using phase shifts determined in two analyses[4,5] of $\alpha + \alpha$ elastic-scattering data. In both cases, the calculated values of the total reaction cross section σ_R significantly exceeded that implied by our measurements. In his paper, Slobodrian presents previously unpublished data from Berkeley[6] which are generally in excellent agreement with our measurement of $\alpha + \alpha \rightarrow p + {}^7$Li. Hence, he suggests the apparent discrepancy must lie in the measurement of the $\alpha + \alpha \rightarrow n + {}^7$Be cross sections.

The purpose of this note is to indicate that Slobodrian's calculations are apparently erroneous. When the presence of the identical α-particles is properly treated, the expression for the total reaction cross section is

$$\sigma_R = 2\pi\lambda\!\!\!^{-2} \sum_{\substack{\ell = 0 \\ \text{even}}}^{\infty} (2\ell + 1)(1 - |e^{2i\delta_\ell}|^2) \quad (1)$$

Here the δ_ℓ are the complex elastic-scattering phase shifts. We have calculated σ_R using expression (1) and the phase-shift set of Darriulat[4] (published in a paper by Kumar and Barker[7]), which is one of the sets used by Slobodrian, and have obtained values smaller than those obtained by Slobodrian by a factor very close to 2. It therefore appears likely that Slobodrian has used an expression for σ_R which is twice that of Equation (1). Since such an incorrect expression for σ_R has appeared elsewhere[8] in the literature, it seems worthwhile to present here a short derivation of the correct result.

The properly symmetrized scattering amplitude for identical bosons is well-known[9] to be

$$f^{IB}(\theta) = f(\theta) + f(\pi - \theta) \quad (2a),$$

where $f(\theta)$ is the usual scattering amplitude for distinguishable particles:

$$f(\theta) = \frac{\lambda\!\!\!^{-}}{2i} \sum_{\ell = 0}^{\infty} (2\ell + 1)P_\ell(\cos\theta)(e^{2i\delta_\ell} - 1) \quad (2b).$$

The total cross section σ_T can then be determined from the scattering amplitude by means of the optical theorem. Using Equations (2), one finds

$$\sigma_T = 4\pi \lambda\!\!\!^{-2} \sum_{\substack{\ell = 0 \\ \text{even}}}^{\infty} (2\ell + 1)[1 - \mathrm{Re}(e^{2i\delta_\ell})].$$

The total elastic scattering cross section is the angular integral of the elastic-scattering differential cross section <u>divided by 2</u> (in order to avoid double counting of particles), and thus is

$$\sigma_E = 2\pi \lambda\!\!\!^{-2} \sum_{\substack{\ell = 0 \\ \text{even}}}^{\infty} (2\ell + 1)|1 - e^{2i\delta_\ell}|^2.$$

Since $\sigma_R = \sigma_T - \sigma_E$, one obtains Equation (1).

It should be mentioned that the requirement of dividing total integrated differential cross sections by 2 to avoid double counting where there are identical particles in the final state yields an extra factor of 2 in the usual detailed balance expression. When this is included, we find that our cross sections for the ^{7}Li ground-state channel are approximately one-half of that predicted by detailed balance from the inverse reaction ^{7}Li(p,α) cross sections measured by Mani et al.[10]. This point also seems to have been overlooked by Slobodrián.

Figure 1 shows our calculations of σ_R from the phase-shifts of Darriulat compared with the sum of our measured cross sections for the p + ^{7}Li and n + ^{7}Be channels. This sum should correspond to σ_R up to 39.6 MeV, where the p + t + α channel has its threshold. As can be seen, below this energy the two curves agree remarkably well, and above it the phase-shift predictions for σ_R are larger than the sum of our measurements, as is to be expected. (The phase shifts of Bacher et al.[11] also yield values of σ_R in agreement with our measurements up to the $\alpha + \alpha \rightarrow p + t + \alpha$

threshold, but the Bacher values for σ_R are considerably larger than the Darriulat values above this threshold.)

Slobodrian seems to find it surprising that our measured n + ^{7}Be cross sections are smaller than those for p + ^{7}Li in the energy region below 42 MeV. However, he neglects the essential point that the n + ^{7}Be threshold is higher than that for p + ^{7}Li. This makes the neutron penetrabilities smaller than the proton penetrabilities. In a compound-nucleus reaction model, charge symmetry would then imply that the neutron cross sections are reduced from the proton cross sections by the ratio of the penetrabilities. This, in fact, gives a good qualitative reproduction of our neutron cross section measurements, as is also shown in figure 1.

We conclude, then, that there is no discrepancy between our measurements and $\alpha + \alpha$ elastic-scattering phase-shifts. In fact, these phase shifts tend to confirm our measurements. Hence, there is no reason to reject our original conclusion concerning the need to find new astrophysical production mechanisms for ^{7}Li.

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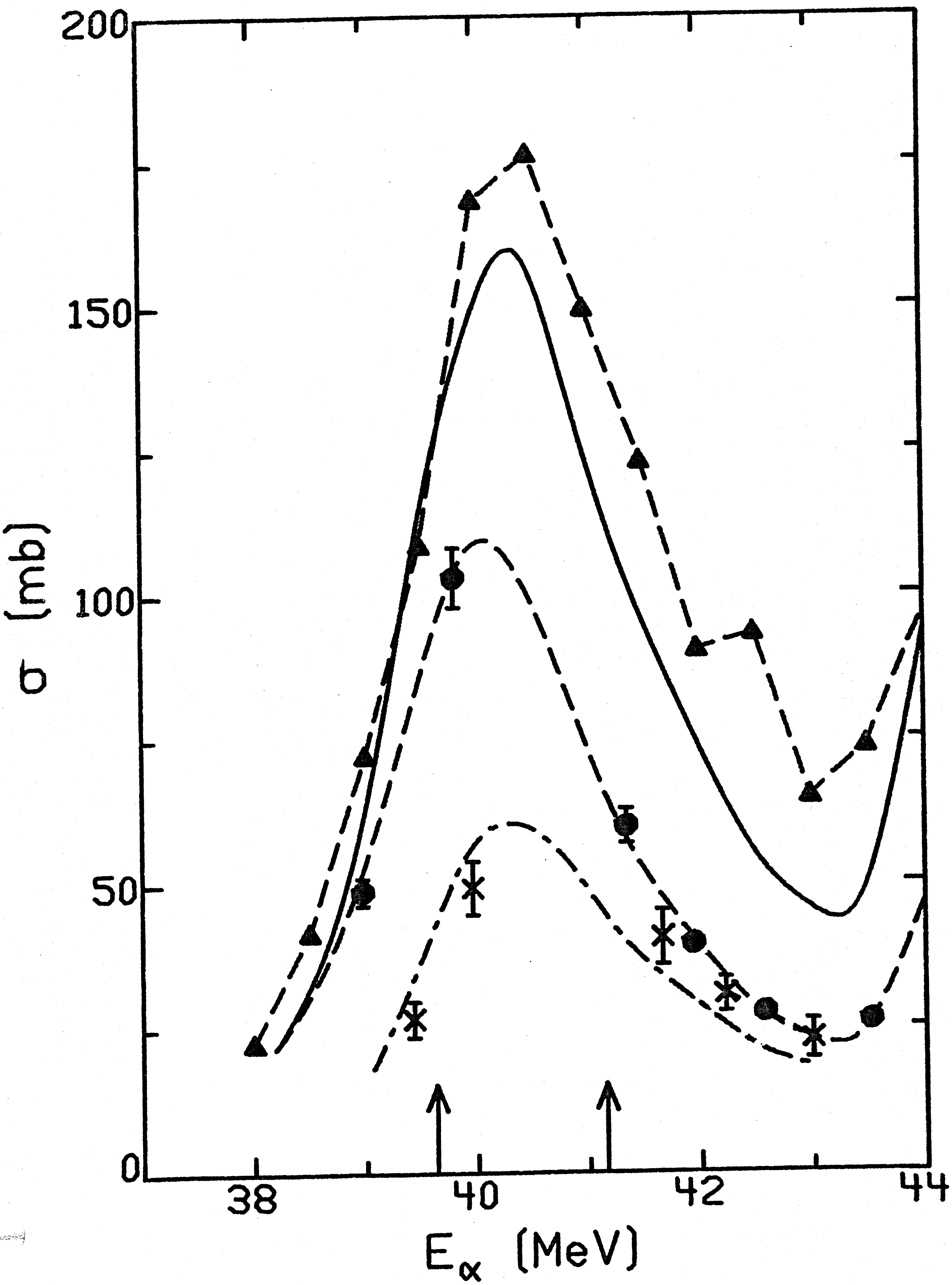
Figure Caption

Comparison between measured $\alpha + \alpha \rightarrow p + {}^7Li$ and $\alpha + \alpha \rightarrow n + {}^7Be$ total cross sections and those predicted from $\alpha + \alpha$ phase shifts. The circles are the measurements of ref. 2 for the $\alpha + \alpha \rightarrow p + {}^7Li$ (g.s. + 0.478 MeV) cross sections; and the crosses, that for the $\alpha + \alpha \rightarrow n + {}^7Be$ (g.s. + 0.429 MeV) cross sections. The solid line is the sum of these two cross sections. The triangles are the predictions of σ_R based on the $\alpha + \alpha$ phase shifts of Darriulat. The dashed curves are to guide the eye through the points. The dash-dot curve is obtained from the curve representing the $\alpha + \alpha \rightarrow p + {}^7Li$ cross sections after multiplication by P_n/P_p, the ratio of the neutron to proton penetrabilities. The arrows indicate the thresholds for the first two three body reactions: $\alpha + \alpha \rightarrow \alpha + p + t$ (39.63 MeV) and $\alpha + \alpha \rightarrow \alpha + n + {}^3He$ (41.16 MeV).



200
150
100
50
0
σ (mb)
38
40
42
44
Eα (MeV)