

Coulomb Displacement Energies of the $A = 4n + 3$, $T = 1/2$

Mirror Nuclei in the $1f_{7/2}$ Shell*

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Abstract

The Q-values of the ($^3\text{He}, ^6\text{He}$) reaction on ^{46}Ti , ^{50}Cr , ^{54}Fe , ^{58}Ni have been measured, and the excitation energy of levels up to 7 MeV has been determined. Angular distributions from the more strongly populated states have been taken from 4.5 to 27.0 degrees in the laboratory. These angular distributions together with comparison their $T_z = +1/2$ mirrors have been used as empirical guides to determine the spin and parity of several of the states. The Coulomb displacement energies of particle-hole states in the $T = 1/2$ mirrors have been compared to a simple model.

NUCLEAR REACTIONS ^{25}Mg , ^{27}Al , ^{46}Ti , ^{50}Cr , ^{54}Fe , ^{58}Ni ($^3\text{He}, ^6\text{He}$),

$E_{^3\text{He}} = 70$ MeV, measured $Q, \sigma(\theta)$, excitation energies, obtained mass excess of ^{43}Ti , ^{47}Cr , ^{51}Fe , ^{55}Ni , deduced Coulomb displacement energies.

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INTRODUCTION

Coulomb displacement energies have been employed in the past to deduce charge radii of nuclei. However, more direct measurements of nuclear charge radii have been provided in recent years by electron scattering and muonic X-ray data. In their review article¹, Nolen and Schiffer pointed out that the nuclear charge radii extracted from Coulomb displacement energies were too small when compared to other more accurate data. Equivalently, if nuclear charge radii which were determined by electron scattering or muonic X-rays are employed to calculate Coulomb displacement energies, the calculated values are too small by 5-10% throughout the periodic table. Nolen and Schiffer pointed out that this discrepancy persists even when both the exchange and electro-magnetic spin-orbit terms are included. Several theoretical papers have investigated other correction terms including vacuum polarization, higher order magnetic terms, the finite size of the proton, the proton-neutron mass difference, isospin mixing in the core and charge symmetry breaking of the nuclear force.²⁻¹¹ Despite the refinements of the theoretical model, a solution to the problem has not been found.

The experimental results presented in this paper are of interest because they allow accurate determination of Coulomb displacement energies for the ground and a few excited states of the $T = 1/2$ mirror nuclei throughout the $1f_{7/2}$ nuclear sub shell. Although the displacement energies between isobaric analogue states in heavier nuclei are known, the present results include the heaviest known mirror nuclei. The displacement energy of a mirror pair is expected

to depend only on the Coulomb interaction and possibly a charge-symmetry-breaking nuclear force, i.e., a difference in the nuclear part of the p-p and n-n interactions. The displacement energy of a non-mirror isobaric analogue pair may depend, in addition, upon a charge dependent nuclear force, i.e., a difference in the T=1, p-n and n-n interactions. As Sherr and Talmi have shown¹¹, it may be possible to extract the p-n and n-n difference by comparing the displacement energies of T>1/2 analogue pairs to those of the T=1/2 mirror pairs.

The results of the present experiment provide more accurate measurements of the ground state masses of ⁴³Ti, ⁴⁷Cr, ⁵¹Fe and ⁵⁵Ni than was obtained previously.^{12,13} The accurate determination of the excitation energy of several levels in these nuclei provides the necessary data to extract Coulomb displacement energies of the $J^\pi = 7/2^-, 3/2^+$ and $1/2^+$ levels for the T = 1/2 mirror pairs. The angular distributions of these low cross-section reactions and comparison to the $T_z = +1/2$ mirror nuclei provided evidence for the spin and parity assignments. The displacement energies of the $1/2^+$ and $3/2^+$ particle-hole states are compared to a model of Sherr and Bertsch¹⁴, and excellent agreement is found. However, the data show the same phenomenon which Nolen and Schiffer found in a wide range of nuclei.

II. Experimental Method

The measurement of the reaction Q-values was made by comparing the magnetic rigidity of the ⁶He particles from the reactions of interest to those from the ²⁷Al(³He, ⁶He)²⁴Al (g.s.) and ²⁵Mg(³He, ⁶He)²²Mg (3.3082) reactions in a magnetic spectrograph. The Q-values of the calibration reactions -19.812 (3) MeV^{15,16} and -18.7656 (40) MeV^{16,17,18,19} respectively are very close to those

being measured, and therefore the scaling of the field in the spectrograph contributes very little (~1.0 keV) to the uncertainty in the measured Q-values. A beam of 70 MeV ³He particles from the Michigan State University Cyclotron was employed to induce the reactions on thin, isotopically-enriched, carbon-backed, metal foils. The position of the ⁶He particles in the focal plane of the spectrograph was measured by a resistive-wire gas-proportional counter. The ⁶He particles were identified by their energy loss in two proportional counters, time-of-flight through the spectrograph, and light output from the plastic scintillator which followed the two proportional counters. The method is similar to that described by Kashy et al.²⁰ except for the use of a second proportional counter and event recording the data on magnetic tape in the present experimental arrangement. The target thicknesses were measured by means of the energy loss of 56 MeV ⁶Li ions from the ¹²C(³He, ⁶Li) reaction induced on the carbon backings. The targets were rotated so that the carbon backing faced either towards or away from the spectrograph aperture. The difference between ⁶Li energies in the two target orientations gives the ⁶Li energy loss. Table 1 gives the results of the target thickness measurements for these thin, carbon-backed targets as well as the thicknesses of the backings, which were measured by comparison of the yields to that from a 217 $\mu\text{g}/\text{cm}^2$ carbon foil. The 10% uncertainty in the target thickness implies only about a 1 keV uncertainty in the Q-value measurements. The Q-value measurements were made at lab angles of 6° and 10° with a solid angle of 1.2×10^{-3} sr.

Results

The results of the Q-value measurements and the deduced mass-

excesses are compared to previous measurements in Table 2. The primary differences between the present and previous measurements at M.S.U. are that thinner targets and a second calibration were employed in obtaining the present results while a greater number of repeated measurements were used previously^{12,22}. The only substantial difference between the two results is in the ⁴⁷Cr ground state mass. This difference is a result of the better resolution obtained which allowed a more accurate determination of the centroid of the previously unresolved ground state. Figures 1 and 2 show the high resolution spectra obtained at 10° in the laboratory.

Table 3 lists the excitation energies of levels in the final nuclei which have been observed in thin target runs with a cross-section greater than about 50 nb/sr at either 6° or 10° in the laboratory or which are observed as isolated peaks in the thick target spectra. The spin and parity assignments have been made for some levels by comparison of the $T_z = -1/2$ to the $T_z = 1/2$ levels. These assignments are further supported by the comparison of the angular distribution of the states of interest to those from the ⁴²Ca(³He, ⁶He)³⁹Ca reaction, in which the spins and parities of the final states are well known. Figure 3 illustrates the results of this empirical comparison. The solid lines shown in figure 3 are drawn to facilitate comparison of the angular distributions from the various targets. The angular distributions leading to either the 7/2⁻ or 3/2⁺ state are quite similar throughout this set. Only the L = 0 transfers seem to vary from one case to another. Despite the recent success of Delic and Kurath²³ in describing the qualitative features of the ¹³C(³He, ⁶He)¹⁰C reaction by a finite-range distorted-wave Born-approximation, the (³He, ⁶He)

reaction mechanism is not yet well understood. So, no attempt to perform a DWBA analysis of these results has been made. The angular distributions are presented in the hope that they may be of some use in the future as the theory of 3-nucleon transfer-reaction theory improves.

Discussion

The results of the mass measurements and the determination of excitation energies allow the extraction of Coulomb displacement energies listed in Table 4 for the $J^\pi = 7/2^-, 3/2^+$, and $1/2^+$ levels in the A = 4n + 3, T = 1/2 mirror nuclei. Recently Sherr and Bertsch¹⁴ employed the Bansal-French-Zamick model to calculate the Coulomb displacement energies of excited particle-hole states in light nuclei and reported that the level shifts are reproduced to within 50 keV. Using this model, the Coulomb displacement energy of the lowest $J^\pi = 3/2^+$ state in ⁴³Ti and ⁴³Sc is given by

$$\Delta E_C(43, 3/2^+) = \Delta E_C(39, 3/2^+) + 2c(3/2^+, 7/2^-)$$

where $E_C(39, 3/2^+)$ is the Coulomb energy difference of the 3/2⁺ levels in ³⁹Ca and ³⁹K, and $c(3/2^+, 7/2^-)$ is the Coulomb interaction of a d_{3/2} proton with an f_{7/2} proton. The factor of two in the second term reflects the greater number of d_{3/2} - f_{7/2} proton interactions in ⁴³Ti as compared to ⁴³Sc. The results of predictions using this model for both the 1/2⁺ and 3/2⁺ states in the A = 4n + 3, T = 1/2 mirror nuclei are indicated by the solid lines in figure 4. The values $c(3/2^+, 7/2^-) = 296$ keV and $c(1/2^+, 7/2^-) = 302$ keV, were obtained by performing a least squares fit to the data which are indicated by the points. These values of c are for many-particle-

one-hole states, but they can be compared with the 289 and 286 keV values which were obtained in Ref. 14 for one-particle-many-hole states. Recently, it was pointed out that the $^{39}\text{Ca} - ^{39}\text{K}$ pair has an anomalously large Coulomb displacement energy which results from the large binding energy of these nuclei.²² Hence, it is reasonable to make a fit to the data excluding the $A = 39$ values. The results are indicated by the dashed lines in figure 4. It is then found that $c(3/2^+, 7/2^-) [c(1/2^+, 7/2^-)] = 316$ keV [321 keV], and that the predicted Coulomb displacement energy of the $3/2^+ [1/2^+]$ states in $^{39}\text{Ca} - ^{39}\text{K}$ is 126 keV [117 keV] below the experimental values. This is consistent with results obtained from the systematics of the series of analogue states in the odd mass Ca-K isotopes.²²

Calculations of the $d_{3/2} - f_{7/2}$ and $s_{1/2} - f_{7/2}$ proton-proton interactions were performed using both harmonic oscillator and Woods-Saxon wave functions. An oscillator parameter of 0.258 fm^{-2} was used to obtain the harmonic oscillator wave functions. A Woods-Saxon well depth which reproduced the one-neutron separation energy of ^{43}Sc and a radius consistent with the charge radius obtained in electron scattering experiments²³ were used to obtain the Woods-Saxon wave functions. Table 5 shows the surprisingly good agreement between the results obtained in the calculations and the experimentally determined values.

The Coulomb displacement energies of the $A = 4n + 3$, $T = 1/2$ mirror nuclei in the $1f_{7/2}$ shell were calculated using the method outlined by Nolen and Schiffer¹ for the direct term. The depth of a Woods-Saxon well was varied to fit the experimental neutron

separation energy of the $T = +1/2$ nucleus. Then the separation energy of a proton from the $T_z = -1/2$ nucleus was calculated using the same well depth and assuming a uniformly charged spherical core for the nucleus. The radius of the core charge distribution for these nuclei is not experimentally known, however addition of $f_{7/2}$ shell neutrons to a nucleus has been shown to have a small effect on the nuclear charge radius. For example in the Ti or Ca isotopes, in which comparison of nuclei having different numbers of $f_{7/2}$ neutrons is possible, the charge radius varies only about one percent between extremes.²³ Using a charge radius consistent with electron scattering in the above model, we find the calculation of the Coulomb displacement energy to be 450 keV to 650 keV less than the experimental values found in table 4. The calculations for the $3/2^+$ levels agree with the data better than do either the $7/2^-$ or $1/2^+$ levels by 100 to 150 keV. Therefore, although the Nolen, Schiffer anomaly persists for these mirror levels, it is interesting that the values for the $f_{7/2} - d_{3/2}$ and $f_{7/2} - s_{1/2}$ Coulomb interactions agree so well with the values extracted from Sherr and Bertsch's model.

We hope to be able to extend these results to the hole-states in the $A = 4n + 1$, $T_z = -1/2$ nuclei by means of the (^3He , Li) or (p, He) reaction but resolution as high as in the present experiment will be much more difficult to obtain.

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Table 2

Nucleus	Mass Excess (MeV)		
	Previous ^a	Present	Average
⁴³ Ti	-29.328±0.012	-29.305±0.014	-29.319±9.008 ^b
⁴⁷ Cr(g.s.)	-34.608±0.040 ^c	-34.553±0.015	-34.561±0.012 ^c
(7/2 ⁻)	-34.386±0.012	-34.371±0.013	-34.379±0.010
⁵¹ Fe(g.s.)	-40.219±0.017	-40.200±0.015	-40.201±0.012 ^d
(7/2 ⁻)	-39.940±0.013	-39.938±0.013	-39.939±0.010
⁵⁵ Ni	-45.337±0.011 ^e	-45.327±0.013	-45.333±0.010

Reaction	Q-Value (MeV)	
	Previous ^a	Present ^f
⁴⁶ Ti(³ He, ⁶ He) ⁴³ Ti	-17.463±0.012	-17.486±0.014
⁵⁰ Cr(³ He, ⁶ He) ⁴⁷ Cr(g.s.)	-18.313±0.040	-18.368±0.014
(7/2 ⁻)	-18.535±0.012	-18.550±0.013
⁵⁴ Fe(³ He, ⁶ He) ⁵¹ Fe(g.s.)	-18.698±0.017	-18.697±0.015
(7/2 ⁻)	-18.969±0.013	-18.971±0.013
⁵⁸ Ni(³ He, ⁶ He) ⁵⁵ Ni	-17.555±0.011	-17.565±0.013

- a) Ref. 13
- b) The average mass excess of ⁴³Ti included the measurement of Ref. 24 (-29.321±0.010 MeV).
- c) In the previous measurement, the ⁴⁷Cr g.s. was not resolved. The separation of the 7/2⁻ and the ground state was taken from the present measurements.
- d) The relative excitation of the 7/2⁻ and 5/2⁻ ground state from the present measurement was employed to deduce the ⁵¹Fe and ⁴⁷Cr ground state masses.
- e) This value represents an 8.2 keV increase in the mass of ⁵⁵Ni from the previous value¹ due to the measurement of the ⁵⁸Ni mass of Jolivet et al.²
- f) Q-values measured relative to the ²⁷Al(³He, ⁶He)²⁴Al(g.s.) and ²⁵Mg(³He, ⁶He)²⁴Mg(3.3082) reaction Q-values^{3,4} -19.812±0.003 MeV^{15,16} and -18.7655±0.004 MeV¹⁶⁻¹⁹ respectively.

Table 1. Target Thickness

Frame Number	Isotope	Thickness (μg/cm ²) ^a		Energy Loss (keV) ^b		Enrichment
		Target	Backing ^c	⁶ He	³ He	
426	⁵⁰ Cr	82	47	5.5	6.3	96.80(5)
227	²⁵ Mg	61	30	4.8	4.6	99.21(5)
207	⁵⁸ Ni	89	37	6.0	5.5	99.890
632	⁴⁶ Ti	40	27	2.9	3.5	81.2
634	⁵⁴ Fe	99	31	6.6	5.5	96.81(5)
627	²⁷ Al	62	27	4.8	4.4	100

a) The uncertainty in the thicknesses are 10% for the target and 15% for the backing.

b) The energy loss for ⁶He is that of 52 MeV ⁶He particles in one half the target thickness, while for ³He, the energy loss is that for 70 MeV ³He particles in one half the target thickness plus that in the backing.

c) The measurement of thickness of the backings employed the relative yields from the ¹²C(³He, ⁶Li) reaction, as well as the ⁶Li energy loss.

Table 3. Excitation Energies

55Ni		51Fe		47Cr		43Ti	
Ex (MeV)	J ^π	Ex (MeV)	J ^π	Ex (MeV)	J ^π	Ex (MeV)	J ^π
0.000 (0)	7/2 ⁻	0.000 (0)	5/2 ⁻	0.000 (0)	3/2 ⁻	0.000 (0)	7/2 ⁻
2.089 (6)		0.262 (6)	7/2 ⁻	0.102 (10)	5/2 ⁻	0.319 (6)	3/2 ⁺
2.462 (5)		1.218 (10)		0.182 (7)	7/2 ⁻	0.475 (10)	
2.839 (5)		1.525 (9)		0.478 (7)	3/2 ⁺	0.998 (10)	1/2 ⁺
2.888 (7)		1.866 (13)		0.890 (20)		1.160 (10)	
3.185 (6)	1/2 ⁺	2.063 (7)	3/2 ⁺	1.355 (8)		1.470 (10)	
3.502 (15)		2.489 (8)	1/2 ⁺	1.451 (9)		1.800 (15)	
3.592 (15)		3.013 (9)		1.541 (15)		2.250 (10)	
3.752 (7)	3/2 ⁺	3.127 (9)		1.831 (8)	1/2 ⁺	2.438 (9)	
3.784 (15)		3.310 (10)		2.131 (9)		2.990 (15)	
4.046 (9)		3.964 (12) (doublet)		2.406 (10)			
4.444 (10) (doublet)		4.456 (13)		2.557 (10)			
4.616 (11)				2.609 (10)			
4.743 (12)				2.661 (10)			
4.983 (11)				2.848 (10)			
5.178 (11)				3.430 (10)			
5.339 (12)				3.504 (11)			
5.876 (13)				3.747 (11)			
5.937 (13)				4.169 (12)			
6.600 (50)				4.295 (12)			
6.870 (50)				5.409 (15)			

Table 4

Displacement energies of the

T = 1/2 mirror pairs in the 1f_{7/2} shell.

A	J ^π	ΔE _C (MeV)	Reference
41	7/2 ⁻	7.278 ± 0.005	16
	3/2 ⁺	7.364 ± 0.009	16,19
	1/2 ⁺	7.323 ± 0.007	16,19
43	7/2 ⁻	7.642 ± 0.009	16, present
	3/2 ⁺	7.809 ± 0.011	16,19, present
	1/2 ⁺	7.947 ± 0.013	16,19, present
45	7/2 ⁻	7.906 ± 0.018	13,16
47	7/2 ⁻	8.262 ± 0.010	16,26, present
	3/2 ⁺	8.445 ± 0.013	16,26, present
	1/2 ⁺	8.397 ± 0.013	16,26, present
49	7/2 ⁻	8.487 ± 0.018	13,25
51	7/2 ⁻	8.846 ± 0.011	16,27, present
	3/2 ⁺	9.069 ± 0.013	16,27, present
	1/2 ⁺	9.036 ± 0.013	16,27, present
53	7/2 ⁻	9.073 ± 0.023	13,16
55	7/2 ⁻	9.477 ± 0.010	25, present
	3/2 ⁺	9.703 ± 0.012	25,28, present
	1/2 ⁺	9.743 ± 0.012	25,28, present

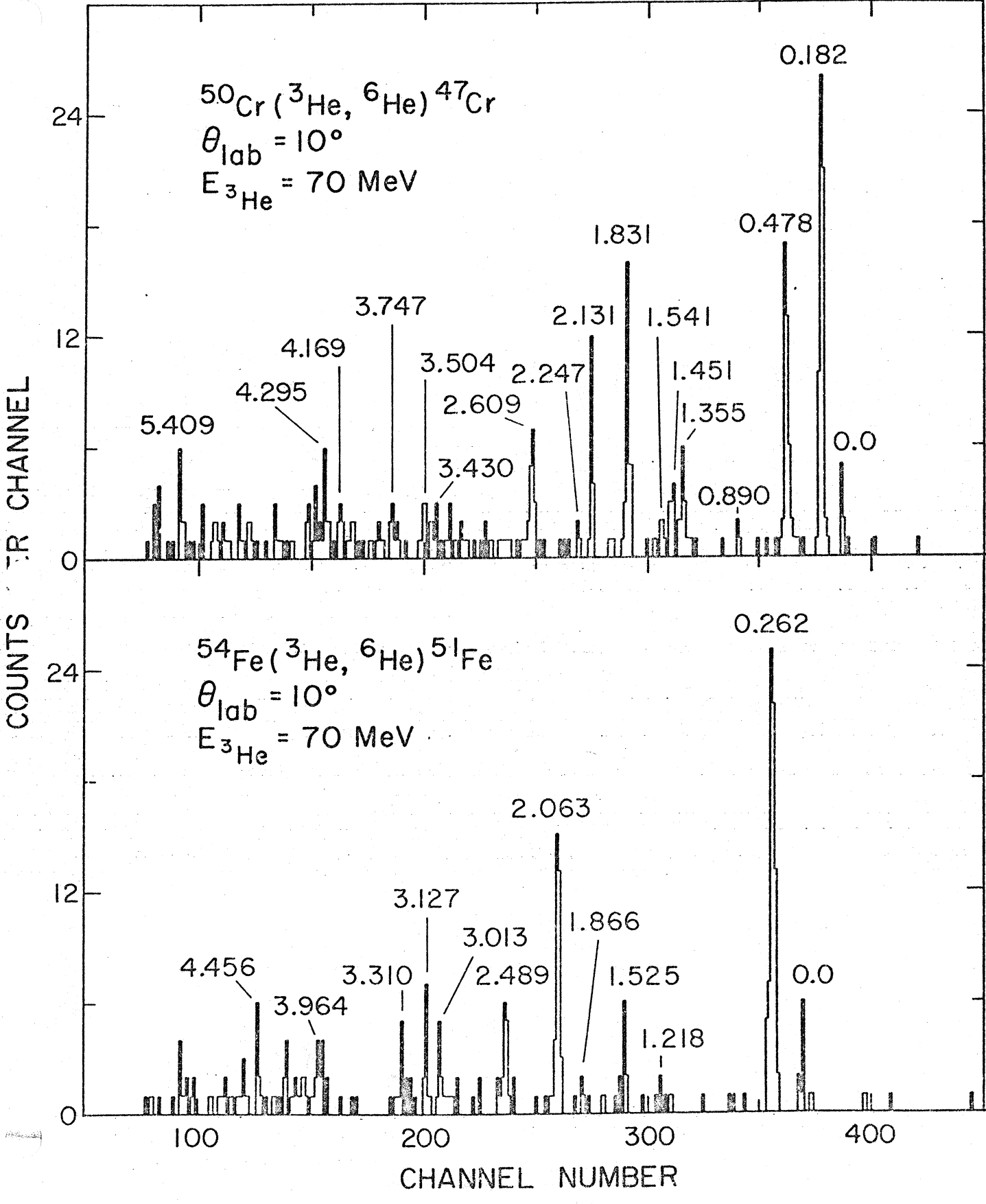
Table 5. Coulomb Interaction Between Protons in Different Shells

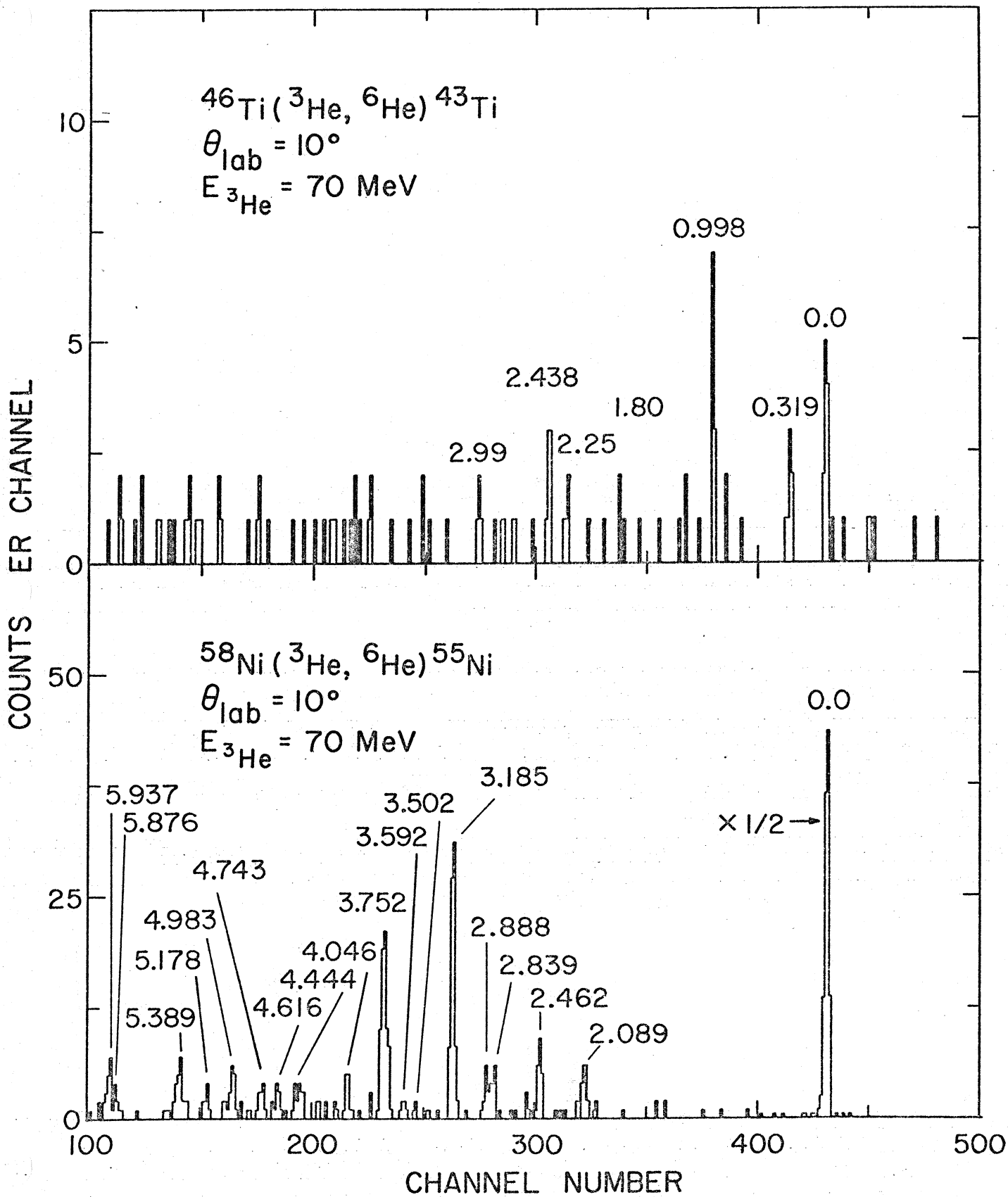
	Exp. (keV)	H.O. a) (keV)	W.S. (keV)	
			Direct	Exchange
$f_{7/2-d_{3/2}}$	316	308	338	-16
$f_{7/2-2s_{1/2}}$	321	294	328	- 6

a) Calculation made using the oscillator parameter
 $= 0.258 \text{ fm}^{-2}$.

Figure Captions

- Fig. 1) High resolution spectra of ${}^6\text{He}$ particles from the ${}^{50}\text{Cr}({}^3\text{He}, {}^6\text{He})$ and ${}^{54}\text{Fe}({}^3\text{He}, {}^6\text{He})$ reactions.
- Fig. 2) High resolution spectra of ${}^6\text{He}$ particles from the ${}^{46}\text{Ti}({}^3\text{He}, {}^6\text{He})$ and ${}^{58}\text{Ni}({}^3\text{He}, {}^6\text{He})$ reactions.
- Fig. 3) Angular distributions of the $({}^3\text{He}, {}^6\text{He})$ reaction at 70 MeV. The ${}^{42}\text{Ca}({}^3\text{He}, {}^6\text{He})$ results are from ref. 28.
- Fig. 4) Coulomb displacement energies versus mass number for $A=4n+3$, $T=1/2$ mirror nuclei in the $1f_{7/2}$ shell. The points represent the experimental data while the lines connect the predictions discussed in the text.





(³He, ⁶He)

