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INTERPRETATION OF THE ANOMALOUS ε/β^+ RATIO IN ^{22}Na*

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Abstract:

The impact of second-forbidden corrections is studied in order to relate the ε/β^+ ratio, the spectral shape factor, and the β-γ directional correlation measurements in ^{22}Na decay.

I. INTRODUCTION

The electron-capture to positron decay branching ratio (hereafter ε/β^+) for ^{22}Na was measured originally in order to test for Fierz interference.[1] The allowed theory of β decay had been well established, and theoretical ε/β^+ ratios could be readily calculated. Early experiments indicated that experiment and theory agreed to within several percent,[2] and this was used as evidence against the existence of Fierz terms.

The ^{22}Na ε/β^+ ratio has since been remeasured by several groups, leading to four especially precise results,[3-6] which are shown in Table I. The first three results agree quite closely, and, although the fourth result differs from the others, it too is in significant disagreement with theory. We outline the theoretical calculation below. The experimental differences are not explained at this time; however, they are not significant to the conclusions of this paper.

Experimental data also exist on the ^{22}Na β spectral shape and β-γ directional correlation, which are also sensitive to Fierz and/or second-forbidden effects. Wenninger, Stiewe, and Leutz measured the ^{22}Na spectral shape using both a ^{22}Na-doped NaI(Tl) crystal and a magnetic spectrometer.[8] They analyzed their data to show no Fierz term; however, these experiments did not include radiative,[9,10] annihilation in flight,[36] and detector efficiency corrections nor did their analysis contain second-forbidden form factors which, in view of the very hindered nature of this decay, could be rather significant.[11] Finally, Steffen[12] has measured a ^{22}Na β-γ directional correlation coefficient $A_{22} = 1.8(3)\times10^{-3}$ at $E = 850$ keV. Strict allowed theory predicts $A_{22} = 0$.

By the time the ^{22}Na ε/β^+-ratio anomaly had been firmly established experimentally, Fierz interference was assumed to be nonexistent, so that alternative causes were suggested. One argument[5] employed an extrapolation of Bahcall's papers on orbital electron exchange and overlap effects.[13] Such arguments were rejected by Williams[14] because an exact calculation of such effects should reveal a change only in the relative subshell capture rates, not in the total rate. Thus, capture of a K-orbital electron of the parent nucleus, for example, can result in an L-shell vacancy in the final system due to imperfect orbital overlap. The effect of the electron configurations on the total nuclear capture rate is quite small except for the case of extremely low-energy transitions. Later arguments were put forth by Firestone et al.[15,16] to the effect that the anomaly is most likely the result of the exclusion, in the simple allowed calculation, of higher-order forbidden terms, which can make significant contributions to hindered allowed decays. The implications of such higher-order forbidden terms are discussed in detail in this paper.

II. THEORETICAL CALCULATIONS

The two principal formalisms used to calculate β decay are thoroughly discussed by Holstein[17] (based on "elementary-particle" amplitudes) and by Behrens and Jänecke[18] (an extension of "standard" nuclear β-decay multipole matrix elements). The two formalisms are completely equivalent. The actual calculated values presented in this paper were generated using the Behrens and Jänecke approach; however, in the main text the discussion is given in terms of the somewhat more transparent elementary particle approach. A translation dictionary between these terminologies is given in the Appendix.

We assume the canonical V-A form for the weak interaction. Thus, for β^+ decay,

$$T_{wk} = \frac{G}{\sqrt{2}} \cos\theta_c \langle \beta_{p_2} | V_\lambda + A_\lambda | \alpha_{p_1} \rangle \bar{u}_\nu(k) \gamma^\lambda (1+\gamma_5) v_e(p) \quad (1)$$

where p_1, p_2, p, and k represent the respective four-momenta of the parent nucleus α, daughter nucleus β, positron, and neutrino; G ($\simeq 10^{-5}\, m_p^{-2}$) is the weak decay constant; and θ_c ($\simeq 15°$) is the Cabibbo angle. Letting M_1 and M_2 be the respective parent and daughter masses, we also define

$$P = p_1 + p_2 \ ;$$

$$q = p_1 - p_2 = p + k \ ;$$

$$M = \frac{1}{2}(M_1 + M_2) \ ;$$

and

$$\Delta = M_1 - M_2 \ . \quad (2)$$

Then, to first order in recoil, the decay spectrum becomes

$$d\lambda_{\beta^+} = \frac{|T|^2}{(2\pi)^5} \left(1 + \frac{3E - E_0 - 3\underset{\sim}{p}\cdot\hat{k}}{M}\right)(E_0 - E)^2 pE\, d\Omega_e\, d\Omega_\nu \ , \quad (3)$$

where $E(\underset{\sim}{p})$ is the positron energy (momentum), $\hat{k}$ is a unit vector in the direction of neutrino momentum, and E_0 is the maximum positron energy,

$$E_0 = \Delta \left(\frac{1 + \frac{m_e^2}{2M\Delta}}{1 + \frac{\Delta}{2M}} \right) . \quad (4)$$

We define for an arbitrary Gamow-Teller transition,[17]

$$\langle \beta_{p_2} | V_\lambda + A_\lambda | \alpha_{p_1} \rangle \ell^\lambda =$$

$$-\frac{1}{4M} C^{M'k;M}_{J'1;J} \epsilon_{ijk} [2b\ell_i q_j + i\epsilon_{ij\lambda n} \ell^\lambda (cP^n + dq^n)$$

$$+ i\epsilon_{ij\lambda n} q^\lambda p^n q\cdot\ell \frac{h}{(2M)^2}] + \ldots \ , \quad (5)$$

where J and J' are the spins of the parent and daughter nucleus, respectively, and M and M' represent the initial and final components of nuclear spin along some axis of quantization. Here c represents the usual Gamow-Teller matrix element, b is the so-called weak magnetism contribution, h is the induced pseudoscalar, while d, often called the induced tensor, is uniquely correlated with the existence of a second-class axial current if α and β are isobaric analog states.[19,20]

Each form factor (b, c, d and h) is a function of the four-momentum transfer q^2. However, for present purposes it is sufficient to include this feature only for the Gamow-Teller term,

$$c(q^2) \equiv c_1+c_2q^2+\dots \quad (6)$$

In terms of this notation, ε/β^+ has been calculated in a previous communication.[21] If $(\varepsilon/\beta^+)_0$ is the theoretical electron-capture to positron ratio for a strictly allowed decay, we define,

$$\mathrm{V} = (\varepsilon/\beta^+)_{exp}/(\varepsilon/\beta^+)_0 \quad (7)$$

where V is hereafter referred to as the skew ratio. Then, neglecting the electron binding energy with respect to m_e, we have[19]

$$\mathrm{V} = 1+\frac{1}{2M}(m_e-\frac{20}{3}\langle E\rangle+\frac{4}{3}m_e^2\left\langle\frac{1}{E}\right\rangle+\frac{7}{8}\Delta_0)$$

$$-\frac{c_2}{c_1}[\frac{40}{9}\Delta(m_e+\langle E\rangle)+\frac{40}{9}(m_e^2-\langle E^2\rangle)-\frac{4}{9}m_e^2\Delta(\frac{1}{m_e}+\left\langle\frac{1}{E}\right\rangle)$$

$$-\frac{2}{3}\alpha E_0R-\frac{9}{2}\alpha^2(2Z'-1)+\frac{20}{3}\frac{\alpha Z'}{R}(m_e+\eta\langle E\rangle)]$$

$$-\frac{\alpha}{2MR}\frac{c_1-d-2b}{c_1}+\frac{4}{3M}\frac{b}{c_1}(m_e+\langle E\rangle)$$

$$-\frac{m_e^2}{3M}(\frac{d+2b+h\frac{E_0}{2M}}{c_1})(\frac{1}{m_e}+\left\langle\frac{1}{E}\right\rangle)+\frac{h}{c_1}\frac{1}{2MR}(\alpha\frac{E_0}{2M}-\frac{3}{2}\alpha^2\frac{2Z'-1}{2MR}), \quad (8)$$

where Z' = 11 is the charge of the parent ^{22}Na nucleus,

$$\eta = \frac{Z'-1}{Z}, \quad (9)$$

and

$$\langle E^n\rangle = \frac{\int_{m_e}^{E_0} dEpE^{n+1}(E_0-E)^2F_+(Z,E)}{\int_{m_e}^{E_0} dEpE(E_0-E)^2F_+(Z,E)} \quad (10)$$

is the n^{th} moment of the positron energy for the β^+ transition. There does exist an important omission in Eq. 8 - the radiative correction, which accounts for real photon emission and other non-Coulombic electromagnetic effects. This has been calculated for the β^+ decay and reduces $(\varepsilon/\beta^+)_0$ by about 1.6%.[22] There exists in addition a radiative correction to account for similar effects in the ε process. This should tend to reduce the 1.6% number somewhat. However, calculation of the ε radiative correction has not yet been made, so that in the following discussion we discard the β^+ radiative correction.

We find, then, for ^{22}Na, using E_0 = 2.070 m_e,[21] that

$$(\varepsilon/\beta^+)_0 = 0.1152\pm0.0003 \quad (11)$$

and

$$V-1 = -[18.0\frac{c_2}{c_1R^2} - 1.72\frac{b}{Ac_1} + 0.71\frac{d}{Ac_1} + 0.0017\frac{h}{A^2c_1}]\times 10^{-3}. \qquad (12)$$

In Fig. 1 we show the dependence of V on b, c_2, d and h separately for reasonable values of these parameters. The slight deviation of the graph from the approximate expression given in Eq. 12 results from the fact that the figures were generated using a more complete form, including higher-order quadratic and Coulomb effects. At large values of d quadratic effects dominate the skew ratio, allowing little net effect.

The shape factor $f_1(E)$ is defined for this β^+ decay by

$$d\lambda_{\beta^+} = 2F_+(Z,E)\frac{G^2\cos^2\theta_c}{(2\pi)^4}(E_0-E)^2pEf_1(E)dE\ , \qquad (13)$$

where $F_+(Z,E)$ is the Behrens-Jänecke Fermi function[18] with $Z = 10$ (for the daughter ^{22}Ne nuclear charge) and

$$f_1(E) = c_1^2-\frac{2}{3}\frac{E_0}{M}c_1(c_1-d-b)+\frac{2}{3}\frac{E}{M}c_1(5c_1-2b)$$

$$-\frac{m_e^2}{3ME}c_1(2c_1-d-2b)+2c_1c_2(\frac{11}{9}m_e^2+\frac{20}{9}EE_0-\frac{20}{9}E^2$$

$$-\frac{2}{9}m_e^2\frac{E_0}{E}-\frac{1}{3}\frac{\alpha ZE_0}{R}+\frac{10}{3}\frac{\alpha ZE}{R}-\frac{9}{4}(\frac{\alpha Z}{R})^2)$$

$$-\frac{\alpha Z}{2MR}c_1(c_1-2b-d)+\frac{c_1h}{(2M)^2}(\frac{\alpha Z}{R}E_0-\frac{3}{2}(\frac{\alpha Z}{R})^2$$

$$-\frac{2}{3}m_e^2+\frac{2}{3}m_e^2\frac{E_0}{E}). \qquad (14)$$

An important feature of the shape factor is its energy dependence. Most of the experimental data points are taken between $E_1 = 600$ keV and $E_2 = 900$ keV.[8] Thus, we define the average slope parameter S by

$$S = \frac{1}{E_2-E_1}\frac{f_1(E_2)-f_1(E_1)}{f_1(E_1)} = -[1.79\frac{b}{Ac_1} - 7.78\frac{c_2}{c_1}R^2$$

$$+0.178\frac{d}{Ac_1} + 0.00010\frac{h}{A^2c_1}]\times 10^{-3} \qquad (15)$$

Shown in Fig. 2 is the variation of S with respect to b, c_2, d and h. At large values of h, quadratic effects again dominate the slope to give little net effect.

Finally, for the β-γ directional correlation, we define

$$d\lambda_{\beta^+,\gamma} = \frac{1}{2}F_+(Z,E)\frac{G^2\cos^2\theta_c}{(2\pi)^5}(E_0-E)^2pEdEd\Omega_e d\Omega_\gamma$$

$$\times[f_1(E)+f_3(E)((\frac{\vec{p}\cdot\hat{s}}{E})^2-\frac{1}{3}\frac{p^2}{E^2})]\ , \qquad (16)$$

where $\hat{s}$ is a unit vector in the photon direction. Here $f_1(E)$ is the shape factor as given in Eq. 14, while the β-γ directional correlation parameter A_{22} is given by

$$A_{22} \equiv \frac{2}{3}\frac{f_3(E)}{f_1(E)} = -\frac{E}{21M}\frac{c_1-b+d+\frac{8}{3}c_2M(E_0-E)-\frac{h}{2M}\frac{3}{2}\frac{\alpha Z}{R}}{c_1}$$

$$\simeq (\text{at } E=850\text{ keV})[4.4\frac{b}{Ac_1} - 4.4\frac{d}{Ac_1} + 0.015\frac{h}{A^2c_1}$$

$$-0.60\frac{c_2}{c_1R^2}]\times 10^{-5}\ . \qquad (17)$$

In Fig. 3 the variation of A_{22} with respect to b, c_2, d and h is shown.

III. DISCUSSION OF EXPERIMENTAL DATA

According to the error bars associated with the measured ϵ/β^+ ratios (cf. Table I), all of these results are quite precise. The deviation of the most recent experiment is therefore disturbing; however, even this result differs by 6.0% from the strictly allowed theoretical value. The errors in the allowed theory are quite small because the decay energy is precisely known. Radiative corrections can account for at most $\simeq$1.6% of this deviation, so at least a 4.4% effect remains unaccounted for.

From Fig. 1 we see that the induced tensor form factor d cannot produce even a 1% change in the skew ratio for any value. Somewhat large values of weak magnetism, $b \text{-} b/Ac_1 \simeq -35$; the Gamow-Teller q dependence, $c_2 \text{-} c_2/c_1R^2 \simeq +3.5$; and the induced pseudoscalar, $h \text{-} h/A^2c_1 \simeq 3.5\times10^4$, can result in a 6% deviation. We discuss the implications of such values of these parameters in detail below with regard to the remaining body of ^{22}Na data.

The value of c_1 can be determined from the experimental $\log ft = 7.42$:

$$|c_1| \simeq [2ft^{\mathrm{Fermi}}/ft(^{22}\mathrm{Na})]^{1/2} \simeq 0.016, \qquad (18)$$

where $ft^{\mathrm{Fermi}} = 3085$ sec is the ft value for pure Fermi transitions.[24] This value for c_1 is strictly correct only in the purely allowed sense; however, the introduction of second-forbidden corrections changes c_1 by no more than a few percent.

The weak magnetism term b can be obtained by measurement of the analog $M1$ γ-ray transition in ^{22}Na from the T=1 analog of the $J^{\pi} = 2^{+}$ 1.274-MeV state in ^{22}Ne. The $M1$ component of this transition is directly related to the weak magnetism beta decay form factor by CVC theory.[25] The T=1 analog in ^{22}Na is well known to occur at 1.952 MeV and is observed to decay only to the T=1 0.583 MeV state, as indicated in Fig. 4. The half-life of the T=1 state was measured to be 9±5 fsec,[26] and an upper bound of 0.25% was recently set for the branching ratio to the ground state.[27] This yields, using CVC theory, an upper limit of $|b/Ac_1|<14$ and a deviation of the ε/β^{+} ratio of less than 2.1%. It is therefore not possible for weak magnetism alone to account for the skew ratio. It should be emphasized that this upper limit assumes a pure $M1$ transition. Any $E2$ competition lowers the weak magnetism prediction even further!

There exist no simple analog experiments to determine c_2, d, or h, as these derive from the non-conserved axial current. We noted above, however, that the experimental skew ratio can be understood if $c_2/c_1R^2 \simeq +3.5$ or $h/A^2c_1 \simeq 3.5\times10^4$. In order to decide if such values are "reasonable" we have calculated all the relevant form factors in the impulse approximation using the extensive s-d shell-model wavefunctions generated by Chung and Wildenthal.[28] The results[29] are:

$$c_1 \simeq g_A M_{GT} = +0.00266 ;$$

$$\frac{b}{Ac_1} \simeq \frac{g_M}{g_A} + \frac{g_V}{g_A}\frac{M_L}{M_{GT}} = -117 ;$$

$$\frac{d}{Ac_1} = \frac{M_{\sigma L} + m_A E_0/5(M_{\sigma r^2} + 2M_{\sigma\cdot rr})}{M_{GT}} \simeq +19 ;$$

$$\frac{c_2}{c_1 R^2} = \frac{1}{10R^2}\,\frac{2M_{\sigma r^2} - M_{\sigma\cdot rr}}{M_{GT}} \simeq -2.25 ;$$

and

$$\frac{h}{A^2c_1} \simeq \frac{3}{10}(2m_A)^2\,\frac{M_{\sigma\cdot rr} - \frac{1}{3}M_{\sigma r^2}}{M_{GT}} \simeq +7.3\times10^3 , \qquad (19)$$

where m_A is the atomic mass unit, and the M's represent the reduced matrix elements

$$M_{GT} = \langle || \sum_j \tau_j \sigma_j || \rangle ;$$

$$M_{\sigma L} = \langle || i\sum_j \tau_j \sigma_j \times L_j || \rangle ;$$

$$M_L = \langle || \sum_j \tau_j L_j || \rangle ;$$

$$M_{\sigma r^2} = \langle || \sum_j \tau_j \sigma_j r_j^2 || \rangle ;$$

and

$$M_{\sigma\cdot rr} = \langle || \sum_j \tau_j \sigma_j \cdot r_j r_j || \rangle . \qquad (20)$$

Comparison of the calculated value of c_1 with its experimental value (from Eq. 18) indicates that the shell-model prediction is nearly an order of magnitude too small. However, the wavefunction calculation of M_{GT} involves considerable cancellation among several terms of comparable magnitude and thus could well be unreliable even as to the overall sign! Such cancellations are not such important features in the remaining reduced matrix elements, so that perhaps estimates of b, c_2, d and h may be more reliable. To the extent the impulse approximation is good, the values calculated in Eq. 19 should perhaps be considered upper bounds for the actual numbers.

Thus, using the shell-model calculation and the experimental $c_1 = 0.016$, we find

$$\frac{b}{Ac_1} \simeq -19 ;$$

$$\frac{d}{Ac_1} \simeq 3.2 ;$$

$$\frac{c_2}{c_1 R^2} \simeq -0.37 ;$$

and

$$\frac{h}{A^2 c_1} \simeq 1200 . \qquad (21)$$

Together, these results predict a $\simeq 2.8\%$ effect on the skew ratio, primarily due to the large calculated weak-magnetism contribution. The experimental limit on b is already below this calculated value, leaving

much less calculable effect. Both h and c_2 would need to be an order of magnitude larger to explain the skew ratio; however, there are good reasons to suspect that the impulse approximation is unreliable for hindered transitions in calculating c_2.[30] Similar considerations may also apply to d and h.[31] In the best possible case, assuming $b \simeq -13$, we would need values like $c_2/c_1R^2 = 1.2$ and $h/A^2c_1 = 1.2\times10^4$ simultaneously to explain the ϵ/β^+ anomaly.

Now finally, we apply these considerations to the interpretation of the remaining pieces of ^{22}Na-decay data. The best available shape-factor measurements were performed by Wenninger, Stiewe, and Leutz.[8] They made two independent measurements of the spectral shape factor – with a ^{22}Na doped NaI(Tl) crystal and with a magnetic spectrometer. We have reanalyzed their published data in the range $100<E_\beta<400$ keV, including corrections of the NaI(Tl) data for photon emission,[9,10] annihilation in flight,[36] and escape of the positron from the detector. These data are displayed in Fig. 5. The magnetic spectrometer data were corrected only for photon emission, although annihilation in flight at the detector and slit scattering will tend to eliminate higher energy positrons, preferentially while backscatter at the detector eliminates mainly those at lower energy. The best linear slope to these data is presented in Table II for both the corrected and uncorrected data.[32] The NaI(Tl) data yield a positive slope despite the large correction, and it is possible that appropriate corrections to the spectrometer data will also make its measured slope positive.

In Fig. 2 we showed that only an extraordinarily large value of d can give a large negative slope, h gives almost no slope, and b or c_2 give positive slopes for values consistent with the measured skew ratio. A negative slope is therefore very unlikely because d cannot explain the skew ratio; thus, the apparent positive slope from the NaI(Tl) data is probably correct. Nevertheless, the measured slope discussed here is too uncertain to be trusted completely.

The best measurement of the β-γ directional correlation A_{22}, performed by Steffen,[12] is

$$A_{22}^{\mathrm{exp}} = -(1.8 \pm 0.3)\times 10^{-3} \quad (22)$$

measured at E_β = 350 keV. Here, unfortunately, we cannot draw any firm conclusions because there exist two additional axial form factors of rank 2 and 3 plus two polar form factors of rank 2 which can in principle also affect the resulting A_{22}, although not the skew ratio or slope. In Fig. 3 we showed the calculated A_{22} values for b, c_2, d, and h. Only b and d can give a significant negative A_{22}, and values of h sufficient to explain the skew ratio would require very large offsetting second rank tensor contributions.

IV. CONCLUSIONS

The explanation of the ^{22}Na ϵ/β^+ anomalous ratio offers several avenues. The most likely possibility seems to us to be that the second-forbidden terms are larger than expected. Thus for example if $b/Ac_1 = h/A^2c_1 = 0$, $c_2/c_1R^2 = +4$, and $d/Ac_1 = +60$, we can obtain $\mathbb{V} = 0.93$, $S = +2\%$/MeV, and $A_{22} = -1.7\times 10^{-3}$, which are consistent with all the existing data for ^{22}Na. Given the experimental uncertainties this solution is not unique, but it must be emphasized that any solution consistent with the skew ratio requires large values for the second-forbidden terms. Additionally, we mention for completeness that a Fierz term of size $b_F \simeq -0.05$ will also explain the skew ratio; however, it will require a significant negative slope for the shape factor. The ^{22}Na slope data do not support this conclusion and in view of the present limits on the absence of Fierz interference[34] we consider this possibility a remote one.

In order to clear up this situation further we suggest two courses. First is a very careful remeasurement of the shape factor in order to confirm the size and sign of the slope S. Second, it is important to have an additional *independent* experiment on this system. One possibility is a precise measurement of the positron longitudinal polarization - P_L - to a precision of about 1 part in 10^3. This may be quite feasible using a newly designed polarimeter.[35] We find

$$\frac{E}{P} P_L \simeq 1 + 10^{-4}\Big(3.0\frac{b}{Ac_1} - 1.07\frac{d}{Ac_1} + 0.00036\frac{h}{A^2c_1} + 0.00002\frac{c_2}{c_1R^2}\Big) \quad (23)$$

at E = 760 keV. This is displayed graphically in Fig. 6. Large values of b or d, as discussed previously, will yield larger deviations from unity in opposite directions, h yields only a small deviation; and c_2 can yield none. Thus, both a new spectral shape remeasurement and a measurement of F_L would be most welcome in order to resolve the ^{22}Na question entirely.

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Appendix:

We give here the relationships between the "elementary-particle" amplitudes $c, d, \ldots$, used in the text[17] and the multipole matrix elements ${}^{A}F^{(0)}_{101}$, ${}^{V}F^{(0)}_{111}, \ldots$, employed by Behrens and Jänecke.[18] The symbols are defined in the text for the "elementary-particle" amplitudes and in Ref. 18 for the multipole matrix elements.

$$
\begin{aligned}
{}^{A}F^{(0)}_{101} &= c_1 + d\frac{E_0}{2M} + E_0^2 c_2 \\
{}^{A}F^{(1)}_{101} &= \frac{1}{R^2}\left(6c_2 + 2\frac{h}{4M^2}\right) \\
{}^{A}F^{(0)}_{110} &= \frac{\sqrt{3}}{2MR}\left(c_1 - d + h\frac{E_0}{2M}\right) \\
{}^{A}F^{(0)}_{121} &= \frac{5\sqrt{2}}{(2MR)^2}h \\
{}^{V}F^{(0)}_{111} &= -\sqrt{\frac{3}{2}}\,\frac{1}{MR}b
\end{aligned}
\qquad \text{(A1)}
$$

Equivalently:

$$
c_1 = \frac{1}{\left(1+\frac{E_0}{2M}\right)}\left[{}^{A}F^{(0)}_{101} - \frac{1}{6}(E_0R)^2\,{}^{A}F^{(1)}_{101} + \frac{E_0R}{\sqrt{3}}\,{}^{A}F^{(0)}_{110} - \frac{\sqrt{2}}{15}(E_0R)^2\,{}^{A}F^{(0)}_{121}\right]
$$

$$
\begin{aligned}
\frac{1}{R^2}c_2 &= \frac{1}{6}\,{}^{A}F^{(1)}_{101} - \frac{1}{15\sqrt{2}}\,{}^{A}F^{(0)}_{121} \\
d &= \frac{1}{\left(1+\frac{E_0}{2M}\right)}\left[{}^{A}F^{(0)}_{101} - \frac{1}{6}(E_0R)^2\,{}^{A}F^{(1)}_{101} - \frac{2MR}{\sqrt{3}}\,{}^{A}F^{(0)}_{110} + \frac{2ME_0R^2}{5\sqrt{2}}\left(1+\frac{1}{3}\frac{E_0}{2M}\right){}^{A}F^{(0)}_{121}\right] \\
h &= \frac{(2MR)^2}{5\sqrt{2}}\,{}^{A}F^{(0)}_{121} \\
b &= -\sqrt{\frac{2}{3}}MR\,{}^{V}F^{(0)}_{111}
\end{aligned}
\qquad \text{(A2)}
$$

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Table I. ^{22}Na Experimental ϵ/β^+ Decay Branching Ratios

ϵ/β^+	V, Skew Ratio[a]	Ref.
0.1048±0.0007	0.910±0.008	Leutz and Wenninger (1967), Ref. 4
0.1042±0.0010	0.905±0.011	Vatai, Varga, and Uchrin (1968), Ref. 5
0.1041±0.0010	0.904±0.011	Williams (1964), Ref. 3
0.1077±0.0006	0.935±0.008	MacMahon and Baerg (1976), Ref. 6
0.1152±0.0003	—	Theory - This Paper

[a] A recent measurement by Bosch et al. is omitted from this Table because of experimental uncertainties discussed in Ref. 7.

Table II. ^{22}Na Shape-Factor Experimental Slope

Experiment	Slope (%/MeV) Uncorrected	Corrected
^{22}NaI(Tl)	-11.4±6.4	+5.9±3.0[a]
Magnetic Spectrometer	- 6.8±5.8	-6.1±4.9[b]

[a]Corrected for photon emission, annihilation in flight, and positron escape.

[b]Corrected for photon emission only.

Figure Captions:

Fig. 1 Dependence of the ^{22}Na ϵ/β^+ skew ratio V on the "elementary-particle" amplitudes b (weak magnetism), c_2 (Gamow-Teller four-momentum dependent term), d (induced tensor), and h (induced pseudoscalar).

Fig. 2 Variation of the ^{22}Na β^+-decay shape factor S on the "elementary-particle" amplitudes b, c_2, d, and h.

Fig. 3 Variation of the ^{22}Na β-γ directional correlation coefficient A_{22} with the "elementary-particle" amplitudes b, c_2, d, and h.

Fig. 4 Level schemes for A=22 showing the decays of the T=1 analog states in ^{22}Na and ^{22}Ne.

Fig. 5 ^{22}Na β^+-decay shape-factor data obtained by Wenninger, Stiewe, and Leutz.[8] Curve a) was obtained with a NaI(Tl) crystal, and curve b) was obtained with a magnetic spectrometer. See text for an explanation of the analysis and corrections.

Fig. 6 Predicted variation of the ^{22}Na β^+ longitudinal polarization P_L (actually $\frac{E}{p} P_L$) with the "elementary-particle" amplitudes b, c_2, d, and h.

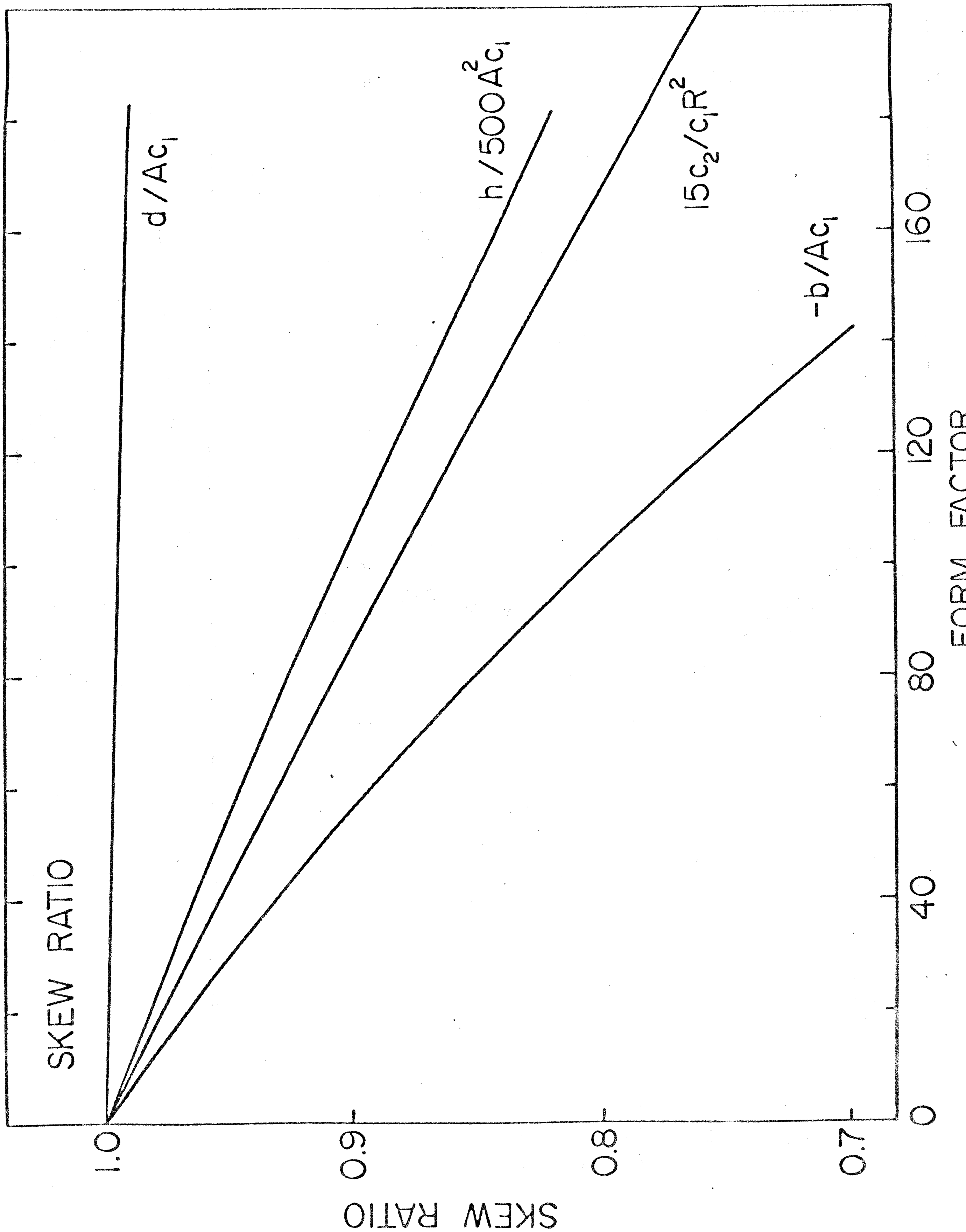
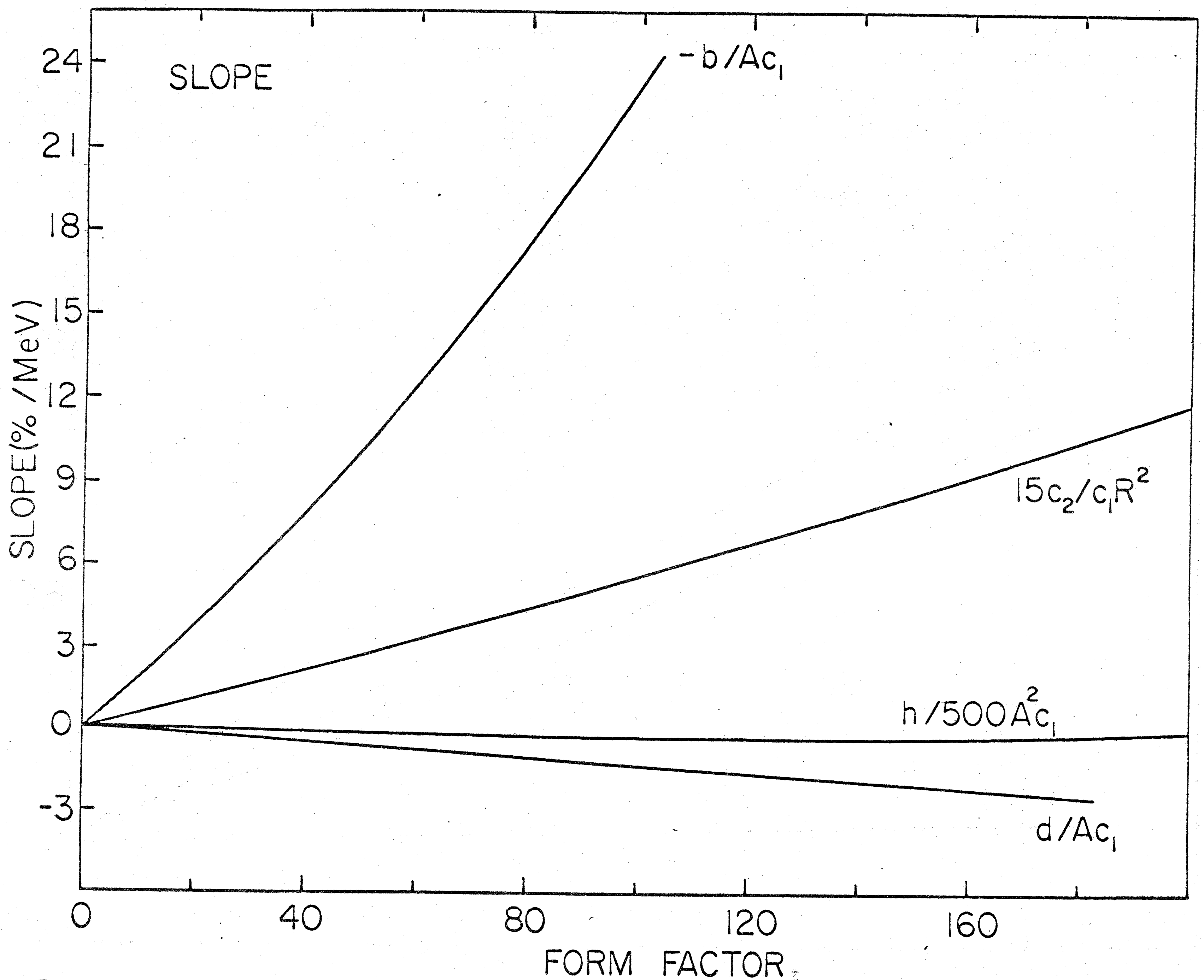

SKEW RATIO
d/Ac_1
h/500A^2c_1
15c_2/c_1R^2
-b/Ac_1
0
40
80
120
160
FORM FACTOR
1.0
0.9
0.8
0.7
SKEW RATIO

Figure 1


SLOPE
SLOPE(% /MeV)
24
21
18
15
12
9
6
3
0
-3
0
40
80
120
160
FORM FACTOR
$-b/Ac_1$
$15c_2/c_1R^2$
$h/500A^2c_1$
d/Ac_1

..gure 2

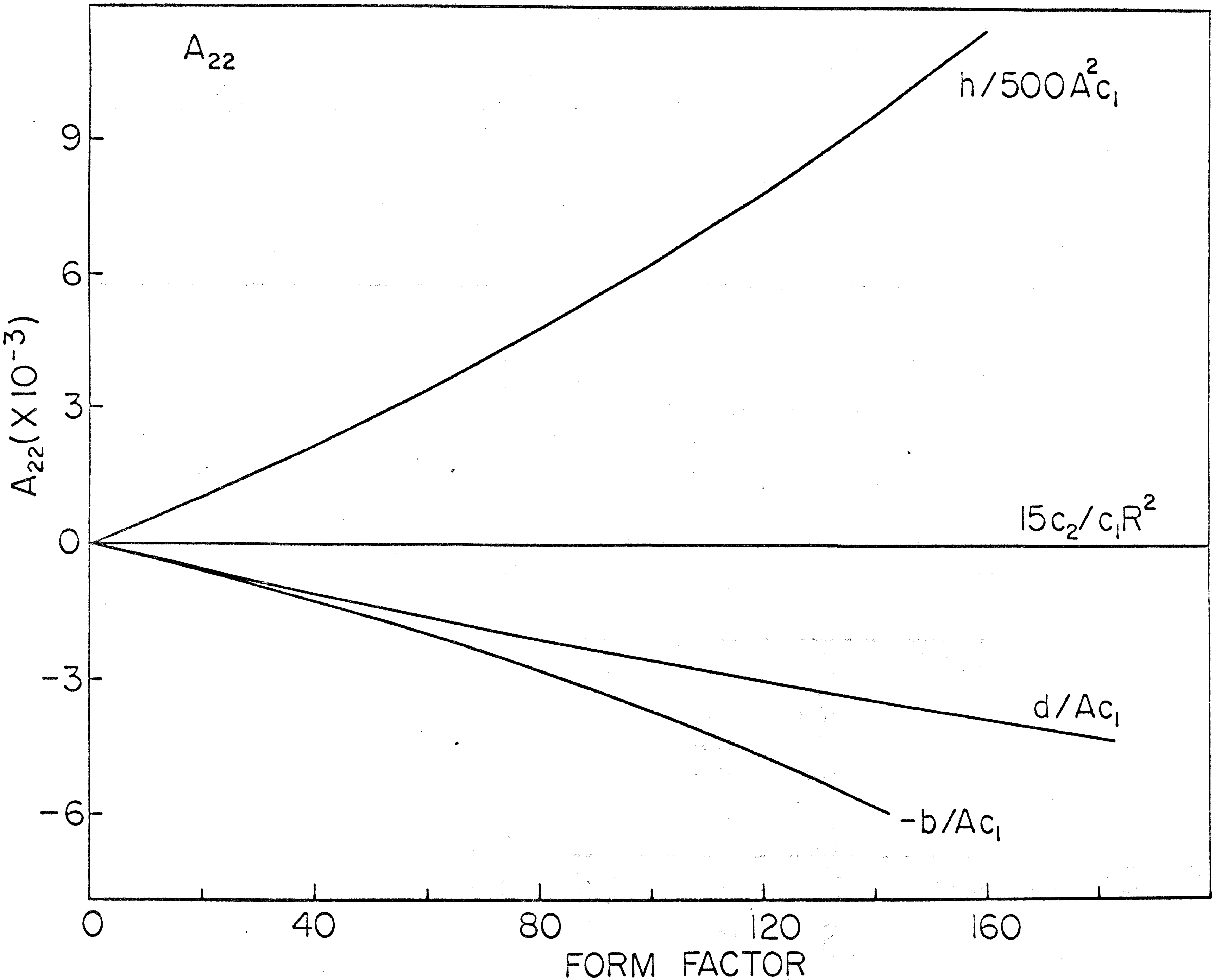

A22
h/500A2c1
15c2/c1R2
d/Ac1
-b/Ac1
9
6
3
0
-3
-6
A22(X10-3)
0
40
80
120
160
FORM FACTOR

Figure 3

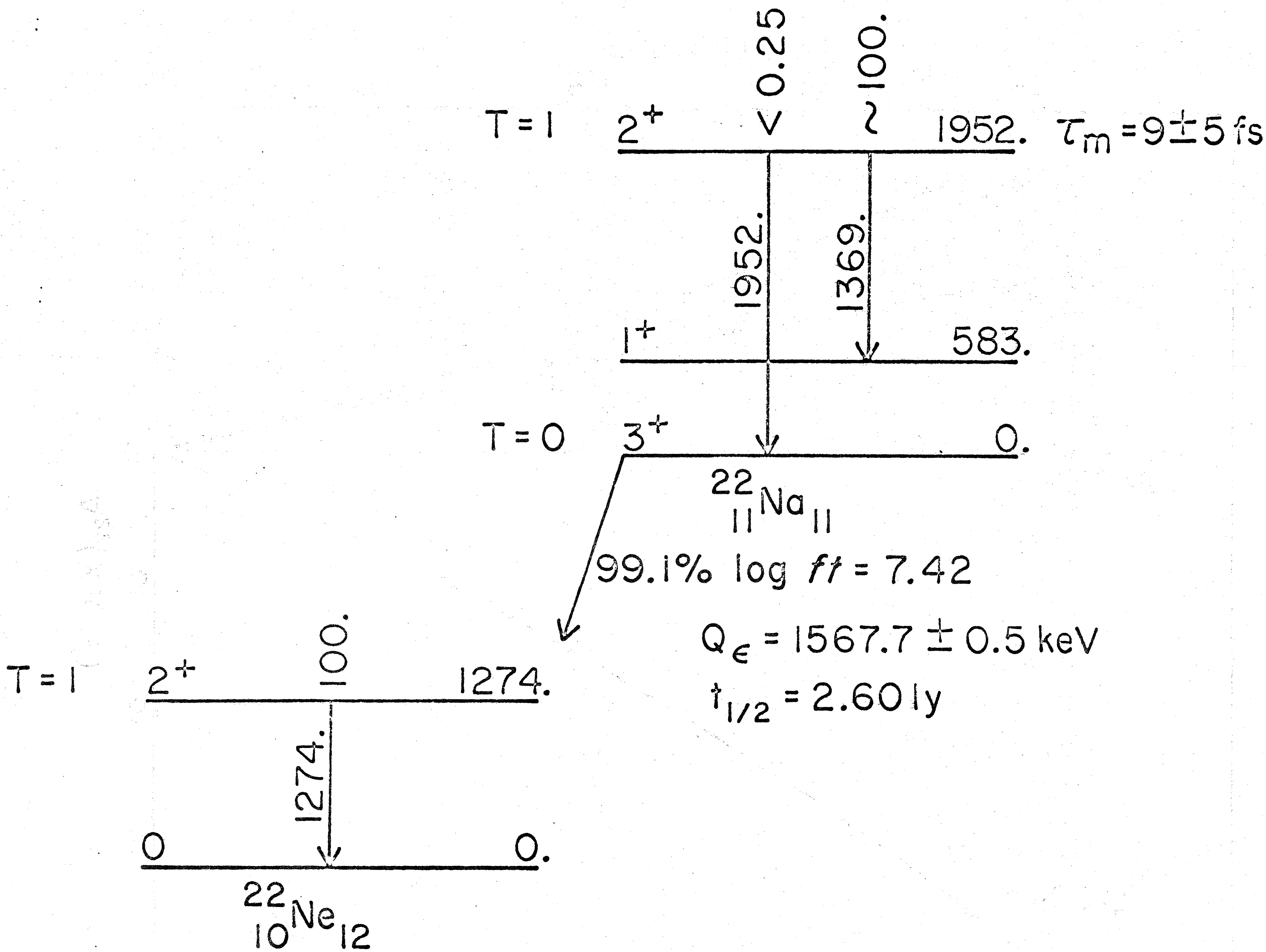
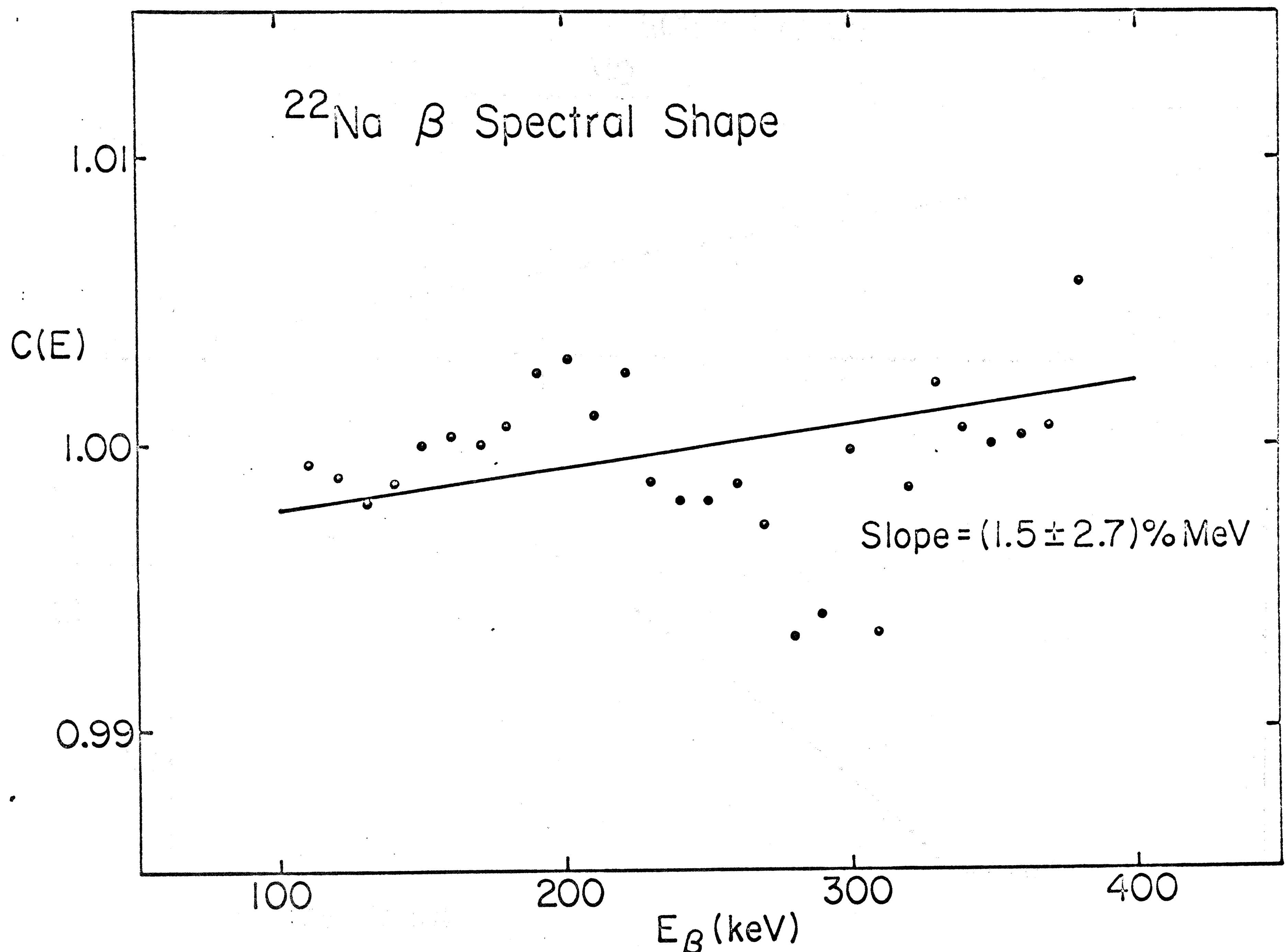

T = 1
2+
< 0.25
~ 100.
1952.
τm = 9 ± 5 fs
1952.
1369.
1+
583.
T = 0
3+
0.
22 11 Na 11
99.1% log ft = 7.42
Qε = 1567.7 ± 0.5 keV
t1/2 = 2.601 y
T = 1
2+
100.
1274.
1274.
0
0.
22 10 Ne 12

Figure 1



22Na β Spectral Shape
C(E)
1.01
1.00
0.99
Slope = (1.5 ± 2.7)% MeV
100
200
300
400
Eβ (keV)

Figure 5

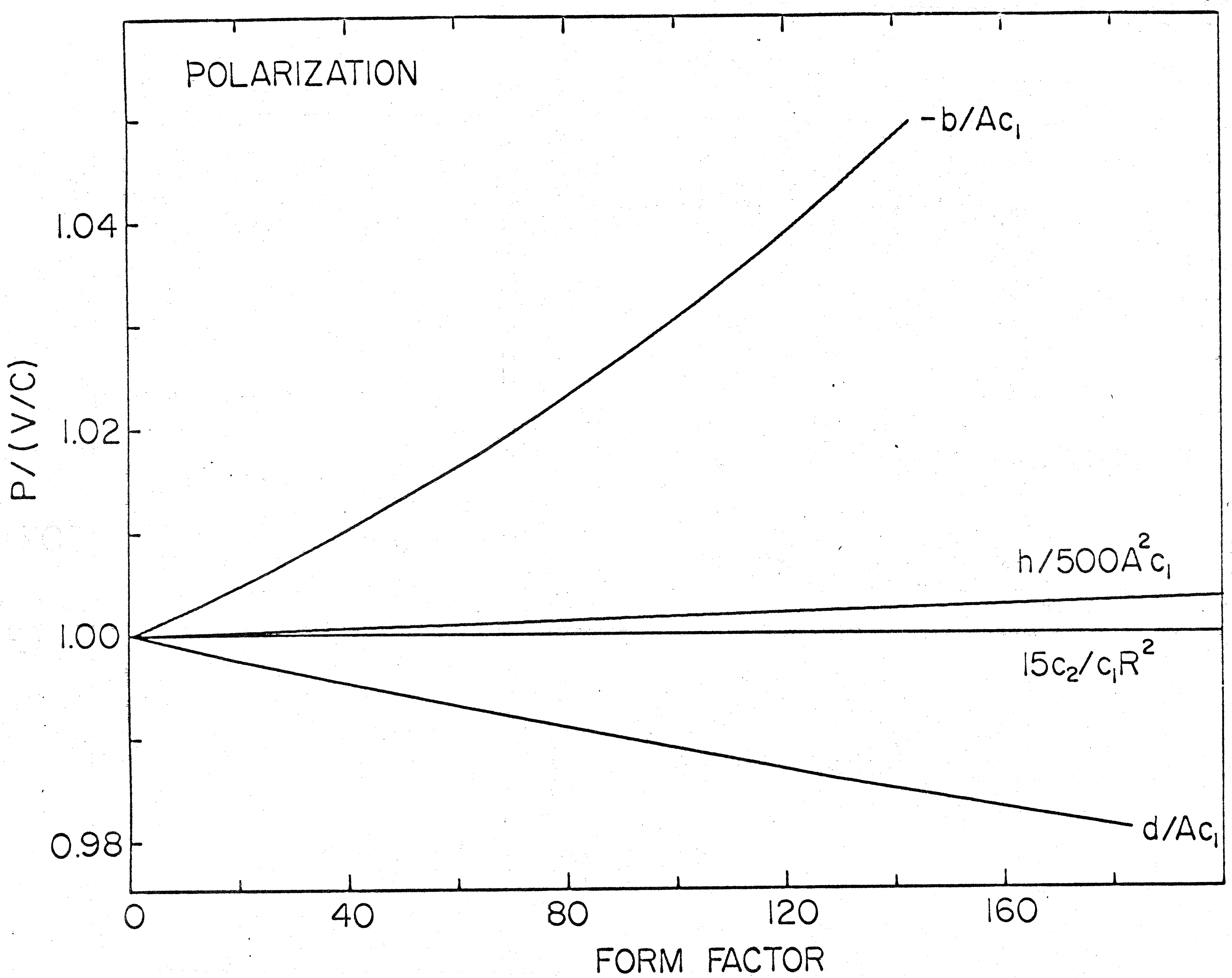

POLARIZATION
-b/Ac₁
h/500A²c₁
15c₂/c₁R²
d/Ac₁
P/(V/C)
1.04
1.02
1.00
0.98
0
40
80
120
160
FORM FACTOR

Figure 6