

Hole states in the tin isotopes observed by the (p, t) reaction

G. M. Crawley, W. Benenson, G. Bertsch, S. Gales,* D. Weber,[†] and B. Zwieglinski[‡]

Physics Department and Cyclotron Laboratory, Michigan State University, East Lansing, Michigan 48824

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A broad structure has been observed in (p, t) reactions on the tin isotopes ^{112}Sn , ^{116}Sn , ^{118}Sn , ^{120}Sn , ^{122}Sn , and ^{124}Sn between 7 and 9 MeV of excitation energy. The width of the peak has a minimum of around 1.9 MeV for the reaction $^{116}\text{Sn}(p, t)$ and increases for both the $^{112}\text{Sn}(p, t)$ reaction and for the heavier tin targets, but the excitation energy of the structure increases with mass number. The measured angular distributions of the bump in ^{110}Sn , ^{114}Sn , and ^{120}Sn agree reasonably well with single-step, distorted-wave Born approximation calculations. The widths of the peaks observed in two neutron transfer show the same trends with mass number as the widths of the single hole states observed in one neutron transfer reactions. It appears that the peak contains components which arise from the pickup of one particle from a deep orbit and one particle from a valence orbit, plus possibly some contribution from pickup of two particles from deep orbits.

NUCLEAR REACTIONS $^{112,116,118,120,122,124}\text{Sn}(p, t)$, $^{117,119}\text{Sn}(p, d)$; $E=42$ MeV; $^{110,114,116,118,120,122}\text{Sn}$ measured E_x, Γ ; $^{110,114,120}\text{Sn}$ measured $\sigma(\theta)$. DWBA analysis; enriched targets, resolution 40 keV.

I. INTRODUCTION

Deep hole states have been observed in a variety of single neutron transfer reactions on the tin isotopes.¹⁻¹¹ The feature observed consists of one or more broad bumps which are characterized by angular momentum transfers of 4 or 1 and therefore have been identified as holes in the $1g_{9/2}$ and $2p$ orbits. In the present experiment, broad structures were also observed in the triton spectra following proton bombardment of the tin isotopes. Because the two neutrons picked up can couple to various angular momenta, it is not possible to make a unique assignment of orbital angular momentum transfer and so determine from which shell model orbits the neutrons are being picked up. In the present paper a systematic study of the (p, t) reaction on the even even tin isotopes ^{112}Sn , ^{116}Sn , ^{118}Sn , ^{120}Sn , ^{122}Sn , and ^{124}Sn is described. The broad structure is observed in all six isotopes studied, but its width and excitation energy vary from isotope to isotope. Angular distributions have been measured on ^{112}Sn , ^{116}Sn , and ^{122}Sn . A preliminary report of this work has been published.¹²

In order to investigate the systematics of the comparison of the two hole states with single hole states, a few measurements of the (p, d) reaction were made on two odd tin isotopes, ^{117}Sn and ^{119}Sn . Broad features similar to those observed on even-even targets were also observed in these cases.

II. EXPERIMENTAL METHOD

The experiment was carried out using the 42 MeV proton beam from the Michigan State Univer-

sity Isochronous Cyclotron. In the initial work, the tritons were detected using a solid state detector telescope. Because of the limitation in count rate from the elastic peak in the detectors and the presence of dead layers on the solid state detectors, the experiment was transferred to the Enge split-pole spectrograph. The same features were observed with both detection methods. Two different wire counters were used in the focal plane. One was a resistive wire counter, and the other used a delay line readout to fix position.¹³ Both counters were backed by a plastic scintillator, and deuterons and tritons were unambiguously identified by their time of flight through the spectrograph and their energy loss in the wire counter. A NaI monitor detector at a fixed laboratory angle of 90° was used to normalize different runs. The absolute cross section was obtained by measuring the elastic scattering at 8° , 10° , and 12° and comparing with an optical model calculation. The calculated elastic cross section is quite insensitive to the particular choice of optical parameters at these forward angles, and thus the absolute cross sections are accurate to better than 10%. One of the largest uncertainties in extracting the cross section for the broad feature arises from the background subtraction. An estimate of this error was made by repeating the extraction of the peak a number of times with different backgrounds. The spread of values obtained for the peak was about 10%.

The tin spectra were calibrated using the known low lying states of the lighter tin isotopes, the oxygen and carbon impurities in the tin targets, and also the $^{58}\text{Ni}(p, t)$ and $^{64}\text{Ni}(p, t)$ reactions. This gave a consistent calibration which was

accurate to about 10 keV. The targets were self-supporting foils of various thicknesses ranging from about 5.3 to 0.3 mg/cm² with the following isotopic abundances: ¹²⁴Sn (94.7%), ¹²²Sn (90.8%), ¹¹⁸Sn (96.6%), ¹¹⁶Sn (95.6%), ¹¹²Sn (80.0%). The energy resolution, in general, was limited by the target thickness. However, a few runs were made with a thin ¹¹⁸Sn target to search for any fine structure in the region of the bump, and in this case an overall energy resolution of about 40 keV full width at half maximum was obtained.

III. RESULTS

Spectra from the (*p, t*) reaction on the isotopes ¹²⁴Sn, ¹²²Sn, ¹²⁰Sn, ¹¹⁸Sn, and ¹¹⁶Sn were presented in a previous publication.¹² However, the ¹¹²Sn (*p, t*) reaction has also been measured, and the spectrum at a laboratory angle of 16° is shown in Fig. 1. As for the other isotopes, a broad feature is observed above the low lying states, although at a somewhat lower excitation energy than for the other tin targets. The width of the peak in this case is larger than for the ¹¹⁶Sn target. A summary of the excitation energies, and the widths of the peaks for the broad peak in the even-even tin isotopes is given in Table I.

The angular distribution for the broad feature in ¹¹⁰Sn is shown in Fig. 2. This angular distribution is rather similar to those for ¹¹⁴Sn and ¹²⁰Sn, which were reported previously.¹² The angular distribution shows no structure and falls off slowly with increasing angle. In contrast, the angular distributions for the low lying states are quite structured and show a characteristic shape for a particular *l* transfer. Some examples of the angular distributions for a few low lying states in ¹¹⁰Sn are shown in Fig. 3. One must conclude that the broad structure does not consist primarily of states with the same *J* but contains states with many different *J*, and therefore different angular momentum transfers.

A plot of the absolute cross section at 16° vs *A* is shown in Fig. 4. All of the cross sections

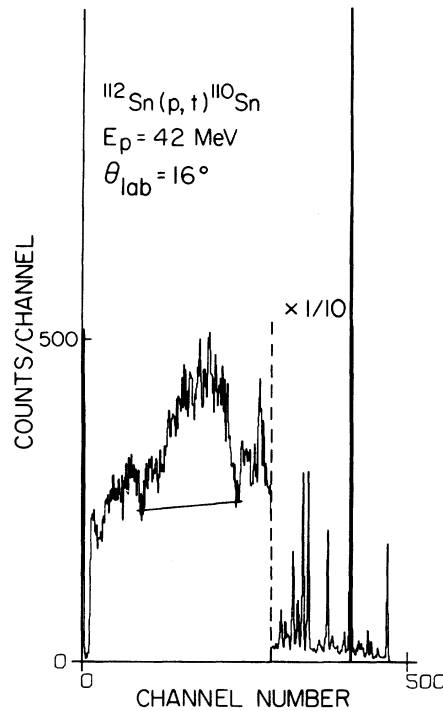


FIG. 1. Spectrum of tritons from ¹¹²Sn(*p, t*)¹¹⁰Sn. The solid line under the broad bump shows the background subtracted for cross section determination.

have about the same magnitude although the value for the ¹¹⁸Sn target is slightly higher than for the other isotopes.

IV. DWBA CALCULATIONS

As a help in investigating the possible configurations contributing to the bump in the (*p, t*) spectra, distorted-wave Born approximation calculations were carried out using the code DWUCK.¹⁴ Since such calculations were intended to serve only as a guide, no attempt was made to obtain a detailed fit to the data, but standard parameters chosen from the literature were used without variation. Becchetti-Greenless optical param-

TABLE I. Excitation energies and widths of the broad structure observed in (*p, t*) reactions on the tin isotopes.

Reaction	E_x (of maximum enhancement) (MeV)	Full width at half maximum (MeV)
¹¹² Sn(<i>p, t</i>) ¹¹⁰ Sn	7.15 ± 0.10	2.58 ± 0.10
¹¹⁶ Sn(<i>p, t</i>) ¹¹⁴ Sn	8.00 ± 0.04	1.93 ± 0.07
¹¹⁸ Sn(<i>p, t</i>) ¹¹⁶ Sn	8.40 ± 0.04	2.12 ± 0.07
¹²⁰ Sn(<i>p, t</i>) ¹¹⁸ Sn	8.51 ± 0.04	2.16 ± 0.07
¹²² Sn(<i>p, t</i>) ¹²⁰ Sn	8.53 ± 0.08	2.58 ± 0.10
¹²⁴ Sn(<i>p, t</i>) ¹²² Sn	8.65 ± 0.08	2.72 ± 0.10

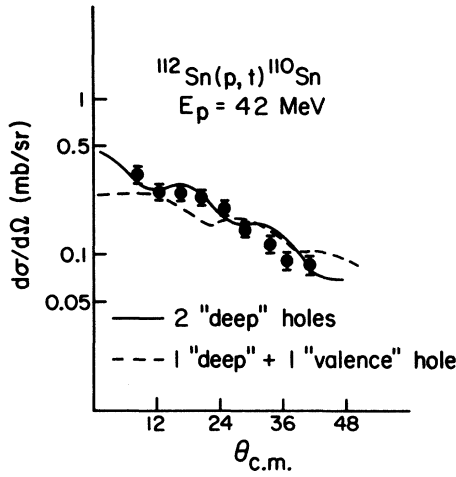


FIG. 2. Angular distribution of the broad feature observed in ^{110}Sn . The solid and dashed lines are DWBA calculations for the "two deep hole" and "one valence-one deep hole" assumptions as described in the text.

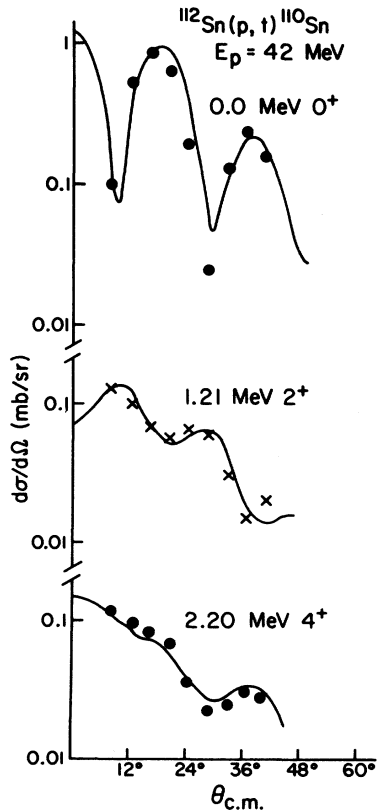


FIG. 3. Angular distributions of some low lying states in ^{110}Sn . The solid lines are DWBA calculations as described in the text.

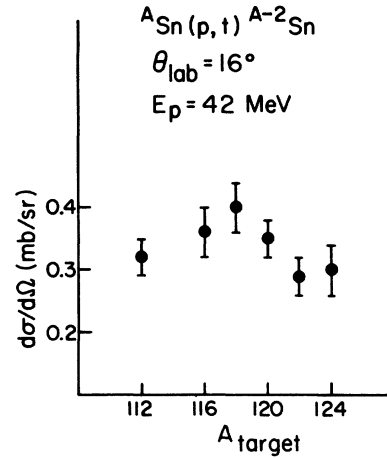


FIG. 4. Differential cross sections for the broad feature observed in the (p,t) reaction on the tin isotopes at a laboratory angle of 16° . The main uncertainties reflected in the error bars arise from the background subtraction and the elastic scattering normalization.

eters¹⁵ were used for the proton channel, and the triton parameters were taken from a $^{90}\text{Zr}(p,t)$ experiment with 38 MeV protons.¹⁶ These parameters are listed in Table II.

The shell model orbits used in the calculations, viz., the $1g_{9/2}$, $2p_{3/2}$, or $2p_{1/2}$ orbits, were always assumed to be full so that the maximum pickup strength was calculated. The value of D_0^2 was set equal to the usual value¹⁷ of $22.0 \times 10^4 \text{ MeV}^2 \text{ fm}^3$ in all cases. The calculations were tested on a few of the low lying states for which it was assumed, for example, that two $d_{5/2}$ neutrons were picked up coupled to 0^+ , 2^+ , or 4^+ . These calculations (solid lines) are compared with the experimental angular distributions for some of the relevant low lying states in ^{110}Sn in Fig. 3. The agreement is quite satisfactory, although the calculation does not reproduce the deep minimum at about 28° observed in the 0^+ angular distribution, and is very slightly out of phase in all

TABLE II. Optical parameters used in the DWBA calculations.

	Protons	Tritons
V (MeV)	47.28	170.1
r_0 (fm)	1.17	1.15
a (fm)	0.75	0.739
W (MeV)	6.54	19.0
W_D (MeV)	2.59 ^a	0.0
r_I (fm)	1.32	1.515
a_I (fm)	0.585	0.758
V_{s0} (MeV)	6.2 ^a	0.0
r_{s0} (MeV)	1.01	0.0
a_{s0} (fm)	0.75	0.0

^aDWUCK uses 4 times this value.

cases. However, such calculations are clearly satisfactory as a guide to the angular distributions for states of unknown J at higher excitation energy.

Two different assumptions were made in the calculations. In one case both particles were assumed to be picked up from one of the deep hole orbits $g_{9/2}$, $p_{1/2}$, or $p_{3/2}$. Earlier calculations¹² included the $f_{5/2}$ orbit for ^{118}Sn and ^{122}Sn , but this was shown to make only a small contribution to the total cross section, and therefore this orbit was excluded from the present calculations. In the other case, one particle was picked up from either valence orbit $g_{7/2}$ or $d_{5/2}$, and the other particle was assumed to come from the $g_{9/2}$ orbit.

For the case of both particles picked up from a deep orbit, the calculated angular momentum transfers which are strongest are $l=0, 2, 3$, and 5 . The $l=4, 6$, and 8 transfers are about an order of magnitude weaker. A similar trend was observed for the $^{118}\text{Sn}(p, t)$ and $^{122}\text{Sn}(p, t)$ cases reported earlier, although the $l=4$ transfer becomes somewhat stronger for the heavier mass nuclei. No one transition dominates, and the incoherent sum is featureless, as can be seen in Fig. 2, in which the theory (solid curve) is compared with the measured cross section.

For the one valence-hole, one deep-hole case, there are no $l=0$ transitions, of course, and the remaining l transfers are comparable in magnitude. The sum is shown as a dashed curve in Fig. 2. Without accurate wave functions for the valence orbits, the magnitude of the cross section calculated including one valence particle is not meaningful. For the calculations with two particles coming from deep orbits, it is reasonable to assume that these orbits are initially full. In this case, the magnitude of the experimental cross section is about 50% of that predicted, in agreement with the results reported for ^{114}Sn and ^{120}Sn .¹²

While no definitive conclusions can be drawn from the comparison with the shapes of the theoretical angular distributions, it appears that the case for which both particles are picked up from a deep orbit gives slightly better agreement with the data at angles forward of 24° .

A few calculations have also been carried out for the two step sequential $(p, d)(d, t)$ process using the program CHUCK.¹⁸ These calculations predict angular distributions which differ in shape from the one step calculations only at forward angles and do not fit the shape of the experimental angular distributions for the low lying states observed in the $^{112}\text{Sn}(p, t)$ reaction very well. In addition, the predicted magnitude of this sequential process is weaker than the single step process by

a factor of 5 to 10 in the cases considered. Therefore these two step processes will not be considered further in this paper.

V. SINGLE NEUTRON TRANSFER ON ^{117}Sn AND ^{119}Sn

In order to investigate further the two-hole phenomenon in ^{116}Sn and ^{118}Sn , the (p, d) reaction on the odd tin isotopes ^{117}Sn and ^{119}Sn was studied. The experimental arrangement was similar to that for the (p, t) reaction, with the deuterons being detected in a position sensitive proportional counter with delay line readout placed in the focal plane of the Enge spectrograph. The energy spectra from the $^{117,119}\text{Sn}(p, d)$ $^{116,118}\text{Sn}$ reactions are shown in Fig. 5 together with the ones obtained from the (p, t) study on $^{118,120}\text{Sn}$ isotopes. The deuteron spectra obtained for both odd Sn isotopes show similar features, namely a cluster of mainly resolvable states between 2 and 4 MeV of excitation, and a broad structure at higher excitation. There does appear to be a double cluster of low lying states in ^{118}Sn , one cluster centered near 2.7 MeV and the other near 3.8 MeV. In ^{116}Sn a single broad cluster of states is observed.

The broad feature, on closer examination, has features similar to those observed in (d, t) and $(^3\text{He}, \alpha)$ experiments on even-even tin isotopes. In both ^{116}Sn and ^{118}Sn , the broad feature appears to have two components, a narrow component at lower excitation energy, and a broader component not clearly resolved at a higher excitation energy. The angular distributions suggest that the lower excitation energy component is mainly excited by $l=4$ transfer and the upper component by $l=1$ transfer. The peak of the complete feature occurs at an excitation energy of 8.14 ± 0.08 MeV in ^{118}Sn and at 7.79 ± 0.08 MeV in ^{116}Sn . From the spectra displayed in Fig. 5, striking similarities are observed between the broad features populated via the (p, d) reaction on the odd Sn isotopes and the ones excited in the (p, t) reactions on the even-even Sn isotopes. These results will be discussed in the following section.

VI. DISCUSSION

A. Excitation energy of the peak

From the energy systematics of the peak seen in two nucleon transfer, it appears to be closely related to the corresponding peak in single-nucleon transfer reactions. The excitation energy of these peaks is compared in Fig. 6. We see that in both cases the excitation energy increases smoothly with A , and that the energy of the two-nucleon transfer peak is roughly 3 MeV higher than the

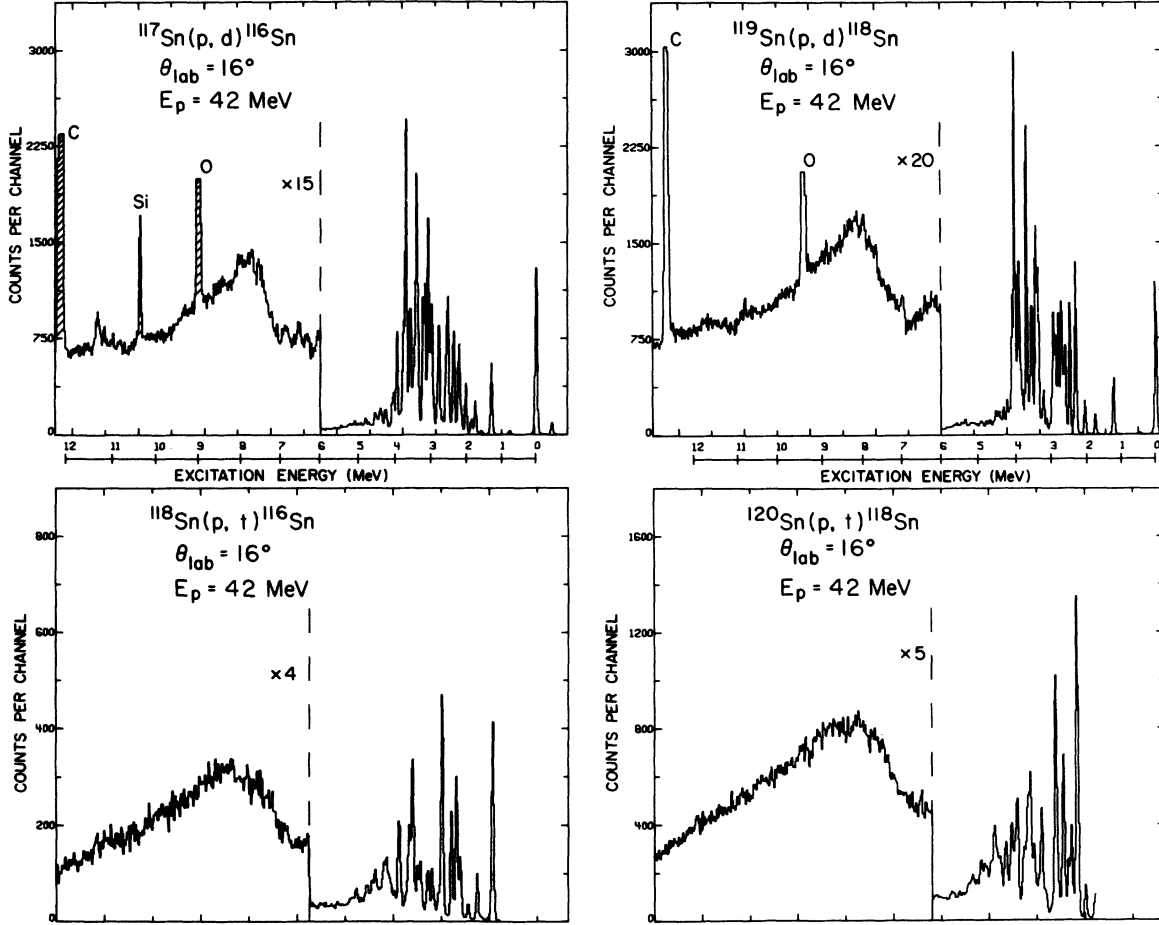


FIG. 5. Comparison of the energy spectra from the $^{117,119}\text{Sn}(p,d)$ reactions (top) with those obtained from the $^{118,120}\text{Sn}(p,t)$ reactions (bottom) leading to the same final nucleus. The energy scale applies to both deuteron and triton spectra.

energy of the single-nucleon transfer peak.

We can understand these systematics qualitatively within a simple pairing description of the energetics. In pairing theory, the energy of a state with n quasiparticles is

$$E_{n\text{qp}} = \sum^n e_i + \sum V_{ij}, \quad (1)$$

where V_{ij} is the residual interaction between quasiparticles and e_i is the quasiparticle energy. The quasiparticle energies are related to the Hartree-Fock energy ϵ_i and the pairing gap Δ by

$$e_i = [(\epsilon_i - \epsilon_F)^2 + \Delta^2]^{1/2}, \quad (2)$$

where ϵ_F is the Fermi energy. We can now compare the excitation energy of a one quasiparticle state (1qp), given by the single-nucleon transfer reaction, with the excitation energy of 2 quasiparticle states (2qp) populated in the (p,t) reaction. There are two main models for the 2qp state. In the original experiments,¹² it was sug-

gested that both particles were removed from deep orbits [the (2d) model]. Nomura¹⁹ has suggested that the energetics are better described by a model with only one particle removed from a deep orbit, the other coming from the Fermi surface [the $(v+d)$ model]. The energetics for the different states are summarized in Table III. The simple formulas are obtained by approximating e_i in Eq. (2) by the larger of $(\epsilon_i - \epsilon_F)$ and Δ .

From the numerical estimate, we see that the (2d) model predicts much too high an excitation energy. On the other hand, the $(v+d)$ model predicts an excitation energy $\approx 2\Delta$ higher than the (1d) state, which is consistent with the data. Thus the energetics suggest that the state is a combination of valence hole and deep hole.

In addition to the absolute value of the excitation energy, the variation of the excitation energy with the mass number A also implies that only one particle is picked up from a deep orbit. As seen in Table III, the excitation energy of the

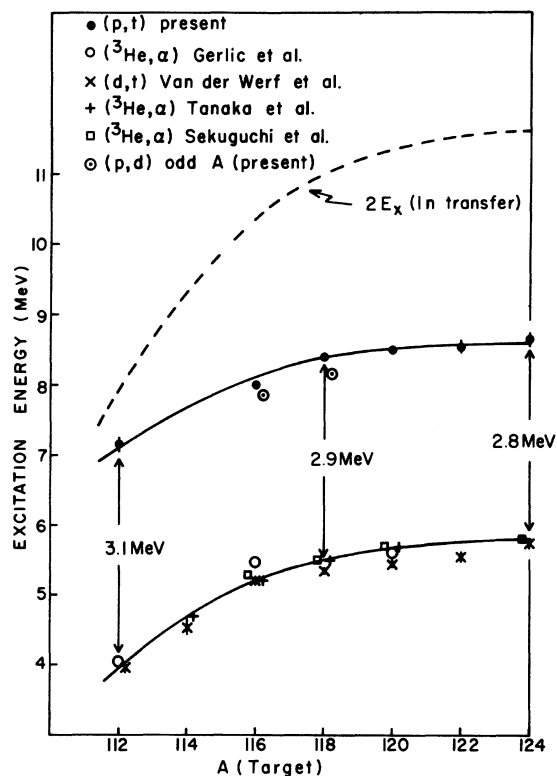


FIG. 6. Excitation energy of the broad features observed in one and two neutron pickup reactions on the even-even tin isotopes versus target mass. The points plotted as dots surrounded by open circles came from the (p, d) reaction on ^{119}Sn and ^{117}Sn . The solid lines are to guide the eye. The dashed line connects points of twice the excitation energy of the single hole state observed in one neutron transfer.

$(2d)$ state would be twice the excitation energy of the $(1d)$ state plus a constant (about 0.9 MeV for the numbers given in Table III). This dependence is shown as a dashed line in Fig. 6 and clearly does not represent the trend observed in the data. As noted, there is a reasonably constant difference between the excitation energies of the peaks observed in one and two neutron transfer, which is the behavior expected if the feature in the (p, t) reaction is produced by picking up one

particle from a deep orbit and one from a valence orbit.

Further evidence for this conclusion is provided by the (p, d) reaction on the two odd isotopes ^{117}Sn and ^{119}Sn . As described above, spectra for both the residual ^{116}Sn and ^{118}Sn show a broad feature similar to that observed in the two neutron pickup. In the (p, d) direct pickup reaction, this feature can only come from removing a single particle from a deep orbit. The structure observed in ^{116}Sn and ^{118}Sn must arise from the coupling of a hole in this deep orbit with the single hole near the Fermi surface which was present in the ground states of ^{117}Sn and ^{119}Sn . In other words, the (p, d) reaction will excite the $(\nu + d)$ state but not the $(2d)$ state. However, the excitation energies of the peak in both ^{116}Sn (7.8 MeV) and ^{118}Sn (8.1 MeV) are very close to the values observed for the peaks in the (p, t) reaction to these same two final nuclei. These values are also plotted in Fig. 6. This observation supports the conclusion that the feature observed in the (p, t) reaction has a significant component which arises from pickup of a neutron from a deep orbit plus a neutron from a valence orbit.

B. Width of the peak

Empirically, the width of the $(2qp)$ peak is larger than the $(1qp)$ width, but it appears to follow the trend of the $(1qp)$ width as a function of mass number. In Fig. 7, the widths of the peaks observed in (p, t) are plotted together with the widths observed for the one-particle states. In both cases, the general trend is for the width to increase with A , at least from $A = 116$ to $A = 124$. The case of ^{112}Sn is somewhat ambiguous. In the (p, t) experiment, the observed width is certainly greater than that for $A = 116$. However, while some of the single neutron pickup experiments also report an increased width, the recent experiment of Gerlic *et al.*¹¹ suggests that the $l = 4$ strength is similar to the case of ^{116}Sn . Part of the problem probably arises from the difficulty of distinguishing $l = 4$ and $l = 1$ transfers in different reactions.

On the theoretical side, there have been several

TABLE III. Excitation energies in the pairing model.

State	Excitation energy formula	Numerical ^a
single deep hole $(1d)$	$e_d - e_\nu \approx (\epsilon_F - \epsilon_d) - \Delta$	5.4 MeV
deep hole pair $(2d)$	$2e_d + V_{dd} \approx 2(\epsilon_F - \epsilon_d) + V_{dd}$	11.9 MeV
valence hole plus deep hole $(\nu + d)$	$e_d + e_\nu + V_{\nu d} \approx (\epsilon_F - \epsilon_d) + \Delta$	8.2 MeV

^aNumerical: $\Delta = 1.4$ MeV, from average odd-even mass difference. $V_{dd} = -1.7$ MeV, from Ref. 20. $\epsilon_F - \epsilon_d = 6.8$ MeV, from empirical $(1d)$ energy.

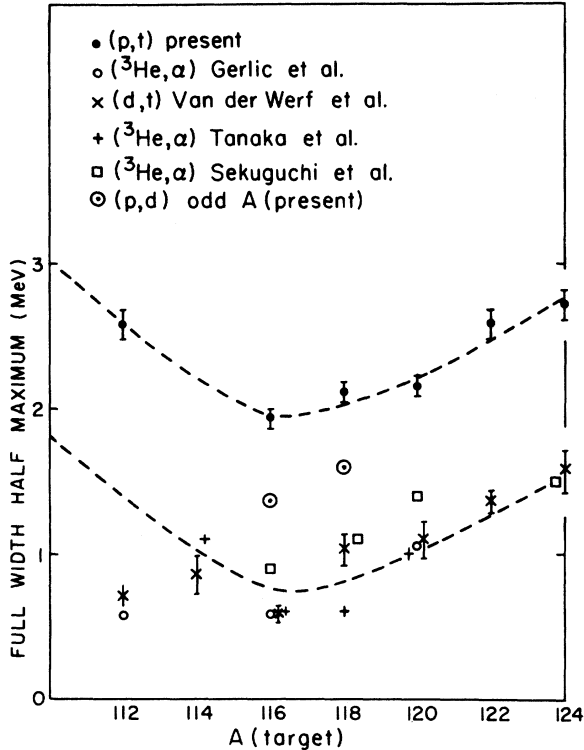


FIG. 7. The full width at half maximum of the broad peak observed in one and two neutron pickup reactions on the even-even tin isotopes plotted as a function of target mass. The dashed lines are to guide the eye. The points surrounded by open circles are from the (p, d) reaction on ^{119}Sn and ^{117}Sn .

attempts to calculate the widths of the $1qp$ states microscopically,²¹⁻²³ but no correspondingly detailed calculations exist for the $2qp$ states. In the $1qp$ calculations, the $1qp$ states are coupled to the collective excitation of the core, which admixes $3qp$ configurations. The results of these calculations are in qualitative agreement with the magnitudes and trends of the $(1d)$ width. According to Koeling and Iachello,²² the increase in width in heavier Sn isotopes is due to the change in the level density of $3qp$ states in the vicinity of the decaying $1qp$ state.

For the $2qp$ state, the width can be crudely estimated in terms of the $1qp$ width, once the structure of the $2qp$ state is specified. In the $(\nu + d)$ model, if we assume that only the deep hole is active in mixing with higher qp configurations, then the width should be the same as that of the $(1d)$ state,

$$\Gamma(\nu + d) = \Gamma(1d). \quad (3)$$

Clearly this model is inadequate, as the measured $2qp$ width is substantially larger than the $1qp$ widths.

In the $(2d)$ model of the peak, an estimate of the width can be made along the lines given by Bohr and Mottelson.²⁴ The two holes are assumed to decay independently, thus the width is twice that of a $(1d)$ state. The width of a $(1d)$ state depends on its available energy, and this is assumed to obey the Fermi gas behavior, with $\Gamma(1d) \approx (e_{1d})^2$. Finally, if the two holes are assumed to share the excitation energy equally, the predicted relation of widths is

$$\Gamma(2d) = \Gamma(1d) \times \left[\frac{E(2d)}{2E(1d)} \right]^2. \quad (4)$$

This formula, as it stands, predicts much too small a width, but it can be criticized on several grounds. First, as was mentioned in connection with the $1qp$ width, there are large fluctuations in the $3qp$ level density, which invalidate the simple E^2 dependence of the Fermi gas model. This is seen explicitly in Fig. 1 of Ref. 25. Second, there can be an important coherence effect between the two quasiparticles that can easily change the width by a factor of 2.²⁵ In our case of two correlated holes, the width would be increased by a substantial amount. If this is put together with the energy estimate of the $(2d)$ state from Table III, the predicted width would be so large as to make the peak too broad to be observed.

We can also compare the $2qp$ widths to the widths observed in the (p, d) reaction on odd- A isotopes as we did for the excitation energies. The width of the peak observed in both ^{116}Sn and ^{118}Sn in the (p, d) reaction is less than that observed in the same nuclei in the (p, t) case, but appears to be somewhat broader than that observed in one neutron pickup reactions on the even-even targets ^{116}Sn and ^{118}Sn (see Fig. 7). However, the comparison in this case is less precise, first because there is more spread in the values reported from different reactions, and second because the $l=4$ and $l=1$ structures were not separated, so that there is some angle dependence to the extracted width. Since no double peak structure could be identified in the states populated in the (p, t) reaction, we felt that a better comparison would be made by reporting the total full width at half maximum for both experiments.

The fact that, in spite of the uncertainties, the structure in the (p, d) is narrower than that observed in the (p, t) reaction in the same nuclei at about the same excitation energy suggests that the configuration populated in the (p, t) reaction is more complex than in the (p, d) case and mixes with more of the background states. These more complex configurations may arise either because one has more choice of valence neutrons to pick up in (p, t) or because there is some contribution

to the structure from two particles picked up from deep orbits.

VII. CONCLUSION

A marked broad structure is observed in (p, t) reactions on six even-even tin isotopes. The excitation energy of the structure increases with increasing mass with the same general trend as is observed in one neutron pickup reactions. The width generally increases with mass, although it is also large for ^{112}Sn . The angular distributions of the gross structure agree with the one step DWBA calculation, assuming an incoherent sum over various states. No single l transfer appears to dominate the angular distribution. It is not possible to definitively distinguish between the possibilities of both particles being picked up from deep holes or only one particle from a deep hole

with one valence particle. The excitation energy of the structure, particularly the behavior with A and the comparison with the excitation energy observed in the (p, d) reaction on the odd mass tin isotopes, strongly suggests that at least some of the strength arises from a one valence-one deep hole configuration. The width is not well predicted by a simple Fermi gas model and probably requires shell corrections. The comparison of the width in (p, d) with that in (p, t) implies that more complex configurations are present in the structure observed in (p, t) .

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*Present address: Institut de Physique Nucleaire, Orsay, France.

† Present address: The Aerospace Corp., P.O. Box 92957, Los Angeles, Cal. 90009.

‡ Present address: Institute of Nuclear Research, Warsaw, Poland.

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