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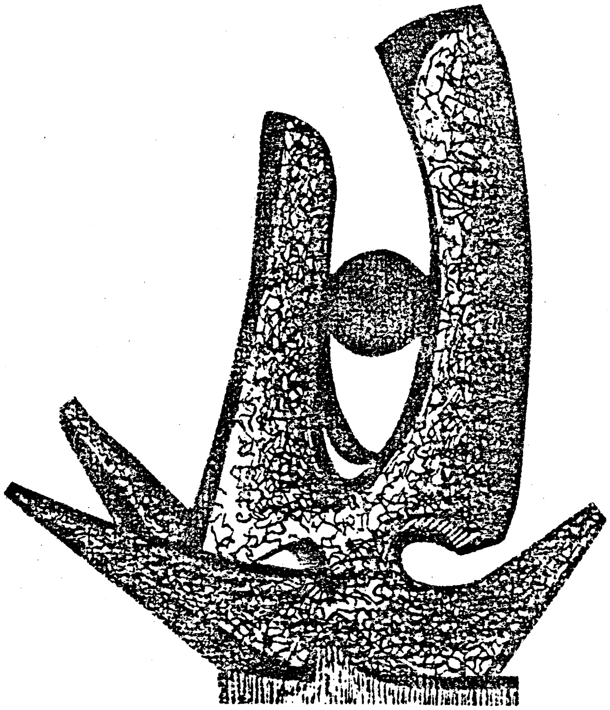

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Higher Order Deformations of ^{232}Th and ^{234,236,238}U

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We have measured the inelastic scattering of 35 MeV protons from ^{232}Th and ^{234,236,238}U. Angular distributions were extracted for $J^\pi = 0^+ - 8^+$ members of the ground state rotational bands, and were analyzed using coupled channels calculations for scattering from a deformed optical potential. The deformation parameter β_6 is positive for ^{232}Th and ^{234}U, nearly zero for ^{236}U, and negative for ^{238}U. The trends of the deformation parameters and multipole moments are explained qualitatively by a simple model.

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The actinide nuclei accessible to scattering experiments are known to be intrinsically deformed and thus their charge and matter (proton and neutron) distributions possess nonzero multipole moments. The moments best studied experimentally and theoretically are the quadrupole and hexadecapole ("2^λ-pole", where $\lambda=2$ and 4, respectively). Very little information currently exists on higher order deformations such as the $\lambda=6$ (hexacontatetrapole).[1] The present work[2] establishes the systematic variation of the $\lambda=6$ deformation for the nuclei ^{232}Th, and ^{234,236,238}U.

Apart from the fundamental interest in the nuclear shape the importance of studying deformations of higher order than quadrupole is reflected in the effects they have on nuclear properties. For example, in the heavy transition metals (e.g. W,Os) a stability against oblate ground state shapes may be attributed to the hexadecapole degree of freedom.[3] Indeed, experimental knowledge of higher order equilibrium deformations of the actinides can put stringent constraints on theoretical methods for calculating ground state properties (and thus influences predictions concerning the stability of heavy nuclei). For example, two approaches[4] to the liquid drop contribution to ground state energies which give almost the same quadrupole deformations yield substantially different higher-order equilibrium deformations. Furthermore, rotational bands built upon single particle states[5] and the sizes of gaps in single particle energies at Z=100 and N=152 depend[3] upon the amount of $\lambda=6$ deformation.

Empirical evidence for $\lambda=6$ deformations in actinide nuclei was first shown by Hendrie et al.[6] in (α,α') experiments and by Moss et al.[7] in a (p,p') experiment. But subsequent studies, using Coulomb excitation,[8] electron[9] and neutron scattering,[10] probed only $\lambda=2$ and 4 deformations, and thus only confirm theoretical expectations (eg. Ref. 3, 11) of how the deformation parameters β_2 and β_4 vary across a region of deformed nuclei: β_2 should attain a maximum value in the middle of the shell and β_4 should change sign near that region. For β_6, the early experimental work in the rare earth region and actinide nuclei used only zero or negative values in the data analysis whereas the calculations by Nilsson et al.[11] for the rare earth and transition metal nuclei suggest that β_6 should change sign twice in the region.

We present here measurements of the $\lambda=2$, 4 and 6 deformations of ^{232}Th and ^{234,236,238}U using proton inelastic scattering at 35 MeV. This is the first systematic study of the $\lambda=6$ deformations in the actinide region and gives the first evidence for a change in sign of β_6 in a deformed region. A 35 MeV proton should be sensitive to the β_6 deformation because its wavelength $\lambda\!\!\!^{-}$ is less than 1 fm whereas the β_6 deformation induces 6 surface lobes (see Fig. 1), with about 8 fm between lobes in ^{238}U.

The elastic and inelastic scattering reactions using 35 MeV proton beams from the Michigan State University

cyclotron were measured in the focal plane of the Enge split-pole spectrometer using a proportional counter, as previously described.[2] Thorium and uranium tetraflouride materials were used to make[2] targets of ^{232}Th and ^{238}U. Targets of ^{234,236}U were made[8] in an isotope separator. Angular distributions for elastically and inelastically scattered protons leading to states in the ground state rotational band with $J^{\pi}=0^{+}-8^{+}$ in ^{232}Th are shown in Fig. 1. Our analysis of the data follows closely our previous study[2] where we fit the data using a parameterized deformed optical model potential (DOMP) to describe the scattering of the protons from the target nucleus modelled by a rigid rotor. In this model inelastic scattering transitions to rotational states are determined by the intrinsic deformations of the nucleus. The deformations are introduced by replacing the radii in the nuclear and Coulomb potentials by $R(\theta) = r_o A^{1/3}(1+\sum_{\lambda}\beta_{\lambda}Y_{\lambda o}(\theta))$ where $\lambda=2$, 4, 6, and 8. The deformation parameters β_{λ} of the real and imaginary parts of the nuclear potentials were assumed to be equal. In the nuclear potential, a Woods-Saxon radial form was assumed whereas the Coulomb potential was for a deformed, uniform charge distribution whose moments reproduce those measured by Coulomb excitation.[8] Searches on the standard optical model parameters V, W_d, V_{so}, a_r, and a_i as well as the deformation parameters β_2, β_4 and β_6 were carried out. Both the full deformed (DSO) and spherical (SSO) spin-orbit interactions were used: SSO calculations allowed all levels of the ground band to 8^{+} to be coupled to L=10 (with β_8=0) and extensive searches to be made; DSO calculation searches were made using the values of β_6 from the SSO searches; finally, for ^{232}Th and ^{238}U, several "one-shot" DSO calculations were performed on a large computer with couplings first up to the 6^+ state and then up to the 8^+ state. The 0^+, 2^+, 4^+, and 6^+ data were then scaled by ratios of the calculated cross sections at each point and then used in searches (DSO) coupling only the 0^+, 2^+, 4^+, and 6^+ states. The results of these approaches were not very different. Table I gives the deformation parameters (for the DSO calculations only). We note that the sign of β_6 is positive for ^{232}Th and ^{234}U, nearly zero for ^{236}U, and negative for ^{238}U.

Figure 1(b) shows the angular distributions for the 6^+ states in ^{232}Th and ^{238}U. The difference in the phases of the oscillations is the most striking feature, and, as the calculations show, help indicate the difference in the signs of β_6 in these two nuclei. Note that the fits to these data are significantly better than in Reference 2 where β_6 deformations were not included.

Deformation parameters and even deformation lengths ($\beta_\lambda R$) are model and reaction dependent quantities. The multipole moments of an intrinsically deformed charge or matter distribution are more fundamental. Assuming axial symmetry, these are

$$q_{\lambda o} = \int r^{\lambda} Y_{\lambda o}(\theta)\rho(r,\theta,)d\vec{r} \quad .$$

In Coulomb excitation measurements, for example, using the rotational model,

$$B(E\lambda;0^+ \rightarrow J^\pi = \lambda^+) = q^2_{\lambda o} \quad .$$

Our analysis follows a procedure suggested by Mackintosh[12] following work by Satchler[13]: To the extent that the DOMP we use can be derived from folding a realistic nucleon-nucleon interaction with the matter distribution (assuming a density-independent interaction and equal proton and neutron deformations), the moments of our DOMP should be equal to those of the underlying matter distributions. Thus, we calculate

$$q_{\lambda o} = \frac{Z\int r^{\lambda} V(r,\theta) Y_{\lambda o}(\theta) d\vec{r}}{\int V(r,\theta\ ) d\vec{r}}$$

for λ=2, 4, and 6, using the parameters in Table I (for the DSO DOMP only, those from the SSO DOMP are the same within statistical uncertainties). These are also given in Table 1.

Some time ago, Bertsch proposed[14] a simple picture of how, to first order, intrinsic moments and deformation parameters should vary for nuclei within a major shell ($q_{\lambda o}, \beta_\lambda \propto \int_\mu^1 P_\lambda(\mu')d\mu'$, where $P_\lambda(\mu)$ is a Legendre polynomial and μ is the cosine of the angle between the symmetry axis and the orbital being filled). He was thus able to explain the existence of positive values of β_4 at the beginning of a deformed region and negative values at the end.

Qualitatively, as shown in Fig. 2, the variations of q_{20} and q_{40} values across the region are described by

Bertsch's model. (The apparent quantitative disagreement in locations of maxima, minima, and crossing points presumably is because the simple picture neglects shell, surface, and Coulomb energy effects.) Quantitatively, the percentage differences between the values from Coulomb excitation and from our study, which has sensitivity to neutron distributions, are nearly the same as what Brack et al.[15] predicted for differences between reduced proton and neutron moments. But, here for the first time in either the rare earth or actinide region, a change in sign of β_6 is observed, along with an indication that q_{60} changes sign in the vicinity of $A \approx 240$. It is rather remarkable that the simple model of Bertsch predicts this same effect for this order of deformation. The more sophisticated calculations of Nilsson et al.[11] also predict the same trend. Further studies of higher order deformations are needed both in the actinide and rare earth regions to extend the present results.

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(b) Operated by Union Carbide Corporation under contract W-7405-eng-26 with the U.S. Department of Energy.

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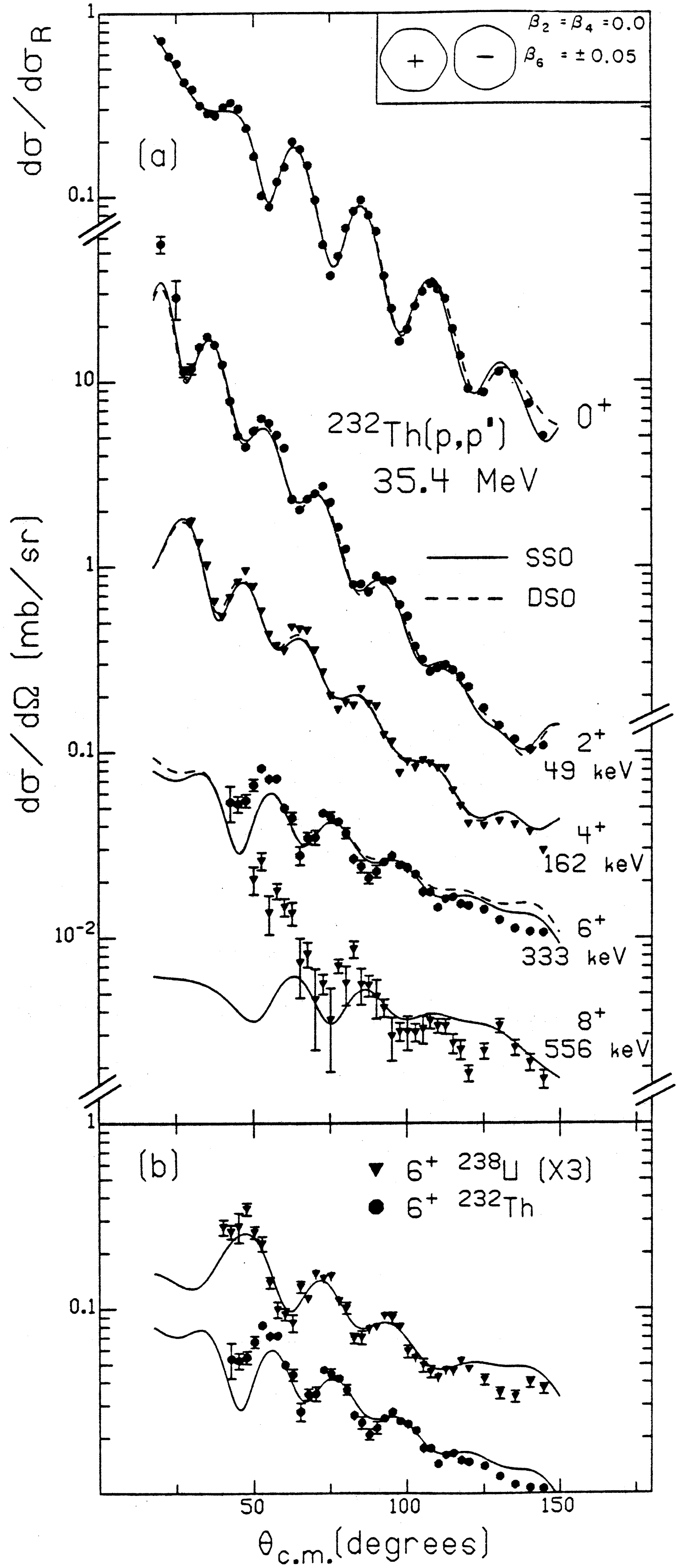
Table 1. Deformation parameters and multipole moments for ^{232}Th and ^{234,236,238}U from coupled channels calculations using a deformed spin-orbit interaction.

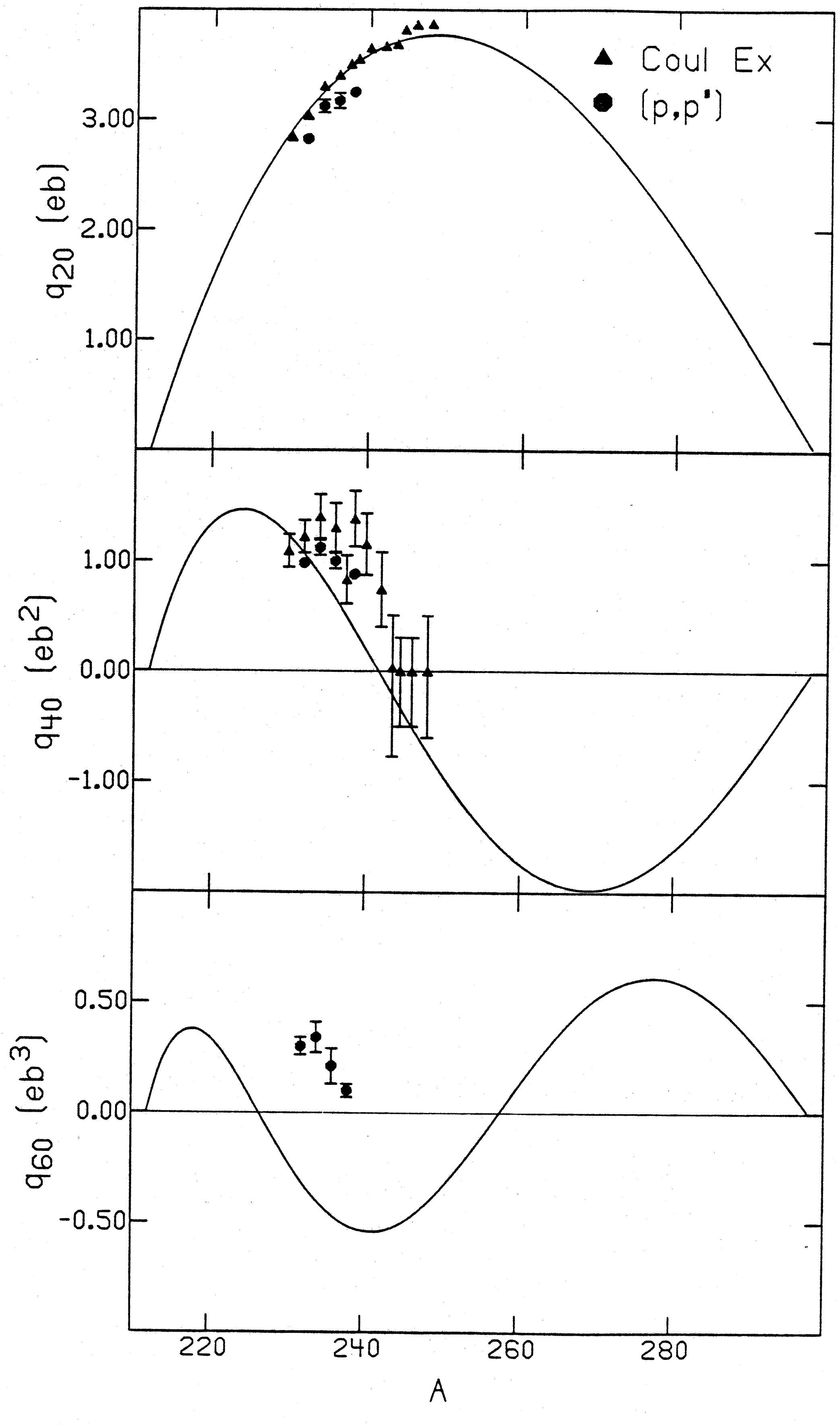
Quantity	^{232}Th	^{234}U	^{236}U	^{238}U
β_2	0.202(2)	0.214(2)	0.220(2)	0.226(1)
β_4	0.068(1)	0.072(2)	0.063(2)	0.052(1)
β_6	0.009(2)	0.007(4)	-0.003(5)	-0.011(2)
q_{20} (eb)	2.82(4)	3.12(6)	3.17(7)	3.25(3)
q_{40} (eb^2)	0.98(4)	1.12(7)	1.00(7)	0.88(3)
q_{60} (eb^3)	0.30(4)	0.34(7)	0.21(8)	0.10(3)

Figure Captions

FIG. 1. The inset panel to Fig. 1(a) shows hexacontatetrapole deformations of a sphere. Figure 1(a) shows data and "best fit" coupled channels calculations for the ^{232}Th(p,p') reaction, using a full deformed spin-orbit interaction (DSO) and a spherical spin-orbit interaction (SSO). Figure 1(b) shows the phase (and magnitude) difference in the 6^+ data for ^{232}Th and ^{238}U, this due to the different signs of β_6.

FIG. 2. Values of the quadrupole, hexadecapole, and hexacontatetrapole moments in the actinide region from Coulomb excitation (Coul Ex) (Ref. 8) and the present study (p,p'). The average percentage differences between q_{20} and q_{40} values from these two studies (6.7% and 17%, respectively) are consistent with the predictions of Brack et al. (Ref. 15) for differences between reduced neutron and proton moments (6.6% for q_{20} and 22% for q_{40}).


dσ/dσ_R
1
0.1
(a)
β₂ = β₄ = 0.0
β₆ = ± 0.05
+
−
0⁺
²³²Th(p,p')
35.4 MeV
10
1
0.1
10⁻²
SSO
DSO
dσ/dΩ (mb/sr)
2⁺
49 keV
4⁺
162 keV
6⁺
333 keV
8⁺
556 keV
1
0.1
(b)
6⁺ ²³⁸U (X3)
6⁺ ²³²Th
50
100
150
θ_c.m. (degrees)


Coul Ex
(p,p')
q_{20} (eb)
3.00
2.00
1.00
q_{40} (eb^2)
1.00
0.00
-1.00
q_{60} (eb^3)
0.50
0.00
-0.50
220
240
260
280
A