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# The effective quadrupole force between like IBA-bosons

Olaf Scholten
Department of Physics-Astronomy and
National Superconducting Cyclotron Laboratory
Michigan State University
East Lansing, MI 48824-1321

In the Interacting Boson Approximation[1] (IBA) model the structure of even-even nuclei is described in terms of a system of interacting s- and d-bosons. A boson is regarded as a collective pair of two neutrons or two protons. From the spectra of semi-closed shell nuclei there is strong evidence that there is no, or at most a very weak, quadrupole force between like particles[2] and consequently no quadrupole force between like bosons. It is the strong neutron-proton quadrupole-quadrupole force that gives rise to the collective features of the spectra of medium heavy and heavy nuclei that have both valence neutrons and protons. In a recent paper, however, Dieperink and Bijker,[3] give strong evidence, on phenomenologic grounds, for a strong quadrupole force between like bosons in nuclei where the SU(3) or O(6) limits of the IBA model apply. In this letter it will be shown that this paradox can be resolved by considering the effective interaction which arises from the truncation of the full shell model space to the S-D pair subspace[4] which corresponds to the IBA boson space.

The consequence of the space truncation has been considered by Sage and Barrett,[5] where the effects of the G-pair are studied in a perturbative approach. The G-pair state is a collective $v=2$, $J=4$ state and is obviously outside the S-D fermion pair space. A parameter of the IBA-model that has been considered in Ref. 5

is $\varepsilon_d$, the energy difference between the s- and d-boson. Phenomenologic calculations[6] indicate that $\varepsilon_d$ decreases when adding both neutron and proton pairs to the closed shell. To explain this decrease in $\varepsilon_d$ essentially the diagram shown in Fig. 1 was considered, where $V^Q_{\pi\nu}$ is the proton-neutron quadrupole-quadrupole interaction.

The dominant part in the shell model neutron-proton interaction is the quadrupole term,

$$V_{\pi\nu} = F_2 Q^F_\pi \cdot Q^F_\nu \tag{1}$$

where the superscript F denotes an operator working in the fermion space. In general $Q^F_\rho$ can be written in terms of operators working on a boson space as

$$Q^F_\rho \rightarrow \kappa_\rho Q_\rho + \kappa'_\rho Q'_\rho + Q''_\rho; \quad \rho = \pi, \nu \tag{2}$$

where

$$Q_\rho = (s^\dagger \tilde{d} + d^\dagger s)^{(2)}_\rho + \chi_\rho (d^\dagger \tilde{d})^{(2)}_\rho \tag{3}$$

is the normal IBA quadrupole operator, and

$$Q'_\rho = (d^\dagger \tilde{g})^{(2)}_\rho + (g^\dagger \tilde{d})^{(2)}_\rho \tag{4}$$

is the part of the quadrupole operator that connects most strongly the IBA s-d space to states outside this space, while Q" contains all other terms not considered here. This form arises naturally from spin and parity considerations and the constants $\kappa_\rho$, $\kappa'_\rho$, $\chi_\rho$ can be determined by equating the matrix elements of Q in the boson space to the equivalent ones in the fermion space.[4] Due to the coupling to states outside the s-d space via $Q'_\rho$ the energy of the d-boson is renormalized, as is shown diagrammatically in Fig. 1 for the proton d-boson,

$$\Delta\varepsilon_\pi = \frac{F_2^2\kappa_\pi'^2\kappa_\nu^2}{\varepsilon_{d_\pi}+\varepsilon_{s_\nu}-\varepsilon_{g_\pi}-\varepsilon_{d_\nu}} \left[ \langle s^{N_\pi-1} d_\pi s_\nu^{N_\nu} | Q_\pi' \cdot Q_\nu | g_\pi s_\pi^{N_\pi-1} d_\nu s_\nu^{N_\nu-1} \rangle \right]^2 = \tag{5}$$

$$= \frac{F_2^2\kappa_\pi'^2\kappa_\nu^2}{\varepsilon_{d_\pi}+\varepsilon_{s_\nu}-\varepsilon_{g_\pi}-\varepsilon_{d_\nu}} \frac{1}{5}\left[ \langle d_\pi s_\pi^{N_\pi-1} || Q_\pi' || g_\pi s_\pi^{N_\pi-1} \rangle \right]^2 \times \langle s_\nu^{N_\nu} | Q_\nu \cdot Q_\nu | s_\nu^{N_\nu} \rangle$$

as is used in Ref. 5. In the present paper we will use this expression to arrive at an effective interaction, V', whose matrix elements are equal to those calculated from diagrams like Fig. 1. For this reason we will rewrite

$$\frac{1}{5}\left[ \langle d_\pi s_\pi^{N_\pi-1} || Q'' || g_\pi s_\pi^{N_\pi-1} \rangle \right]^2 = \langle d_\pi s_\pi^{N_\pi-1} || (Q_\pi' Q_\pi')^{(0)} || d_\pi s_\pi^{N_\pi-1} \rangle \tag{6}$$

$$= \langle d_\pi s_\pi^{N_\pi-1} | \hat{n}_{d_\pi} | d_\pi s_\pi^{N_\pi-1} \rangle$$

By inspection we now can write

$$V' = V_\nu \hat{n}_{d_\pi} Q_\nu \cdot Q_\nu + V_\pi Q_\pi \cdot Q_\pi \hat{n}_{d_\nu} \tag{7}$$

where a term has been added in which neutrons and protons are interchanged and

$$V_\nu = \frac{F_2^2\kappa_\pi'^2\kappa_\nu^2}{\varepsilon_{d_\pi}+\varepsilon_{s_\nu}-\varepsilon_{g_\pi}-\varepsilon_{d_\nu}} \tag{8}$$

to shorten notation.

The strength parameters $V_\nu$, $V_\pi$ can be estimated from the decrease $\Delta\varepsilon_\pi$ of the energy of the d-boson from its value at the semi-closed shell, as can be determined from phenomenological calculations,

$$\Delta\varepsilon_\pi = \langle s_\pi^{N_\pi - 1} d_\pi s_\nu^{N_\nu} | V' | s_\pi^{N_\pi - 1} d_\pi s_\nu^{N_\nu} \rangle = 5\, V_\nu N_\nu \tag{9}$$

A typical value for $\Delta\varepsilon_\pi = -0.8$ MeV in the 50-82 major shell.[6] From the interaction (1) an effective quadrupole interaction $V^Q_{\rho\rho}$ between like bosons can be derived by replacing $\hat{n}_d$ by its expectation value in the ground state. In the SU(5) limit of the IBA model this is zero and there is therefore no effective quadrupole force. This is different in the SU(3) and O(6) limits where $\langle \hat{n}_d \rangle = \frac{2}{3} N$ and $\frac{1}{2} N$. In the SU(3) limit we thus obtain

$$V^Q_{\nu\nu} = V_\nu \frac{2}{3} N_\pi Q_\nu^{(2)} \cdot Q_\nu^{(2)} = \frac{2N_\pi \Delta\varepsilon_\pi}{3 \cdot 5 \cdot N_\nu} Q_\nu^{(2)} \cdot Q_\nu^{(2)} = \kappa_{\nu\nu} Q_\nu^{(2)} \cdot Q_\nu^{(2)} \tag{10}$$

For a typical deformed nucleus we have $N_\pi \simeq N_\nu$, giving a strength of $\kappa_{\nu\nu} \simeq -0.1$ MeV for the quadrupole interaction between like bosons. This is of the same order of magnitude as the strength of the neutron-proton quadrupole-quadrupole force used in phenomenologic IBA calculations, ranging from -0.1 to -0.2 MeV.[6]

In conclusion, it has been shown that, although there is no quadrupole force between like fermions, in the SU(3) and O(6) limits of the IBA model there exists an effective quadrupole force between like particles resulting from the use of a restricted model space. The strength of this effective quadrupole force is comparable in magnitude to the neutron-proton quadrupole force, as is required for the occurrence of the SU(3)* symmetry in the IBA model as discussed in Ref. 3.

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Figure Caption

Figure 1 - The diagram used in Ref. 5 to explain the renormalization of the d-boson energy due to the couplings to states outside the S-D model space.

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