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Δ ISOBARS IN NUCLEI

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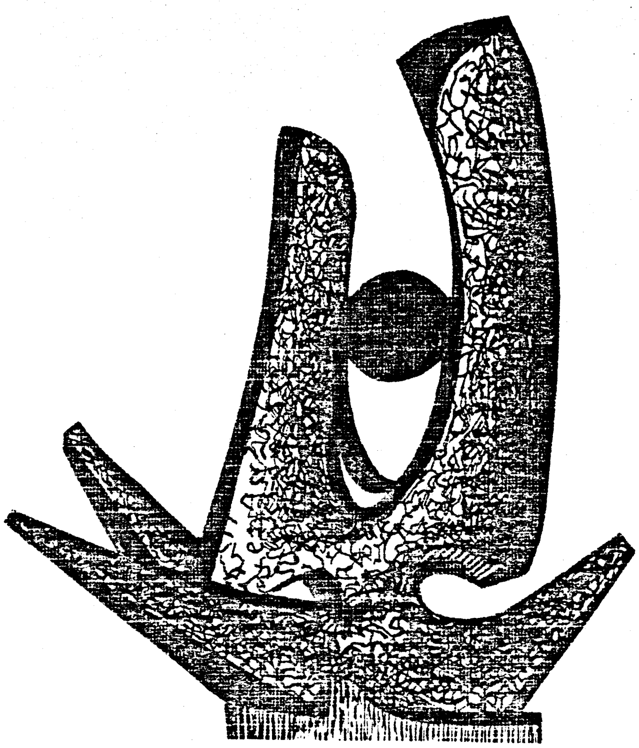

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by

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ABSTRACT

Δ isobars play important roles in many places in nuclear physics. As an example, the role of Δ isobars on pion absorption with the pion energy being in the resonance region is discussed. In addition to the standard two nucleon mechanism, a new process where two Δ isobars are excited following Δ isobar formation after pion absorption is proposed and claimed to be the dominant absorption process in heavy nuclei.

I. INTRODUCTION

There are many places where Δ isobars play important roles in nuclear physics. This is so because Δ isobar is strongly coupled to pion, which plays the central role in the nuclear many body problem, since it mediates interaction between nucleons. We may therefore group the appearance of Δ isobars into two categories according to the pion energy; formation of Δ isobar by a real pion [ω_π (pion energy) $\geq m_\pi$ (pion mass)] and that by a virtual pion [$\omega_\pi \sim 0$]. The phenomena in the first category includes pion elastic and inelastic scattering, pion absorption and production and pionic atoms. In particular, pionic phenomena with the pion energy being in the Δ isobar resonance region are directly related to the properties of Δ isobar in nuclear medium. Photon induced reactions in this energy region are also strongly influenced by the existence of Δ isobar.

In recent years, the role of Δ isobars in the low energy domain became one of the primary subjects in low energy nuclear physics, largely due to the possible occurrence of pion condensation at sufficiently high density and its precritical phenomena in ordinary nuclei. Furthermore, recent experimental development on detection of high energy neutron with great accuracy together with other techniques revealed systematically quenching of Gamow-Teller and magnetic transition strengths. In the pion condensation and its precritical phenomena expected at large pion momenta, Δ isobar is an additional important attractive component to provide the collectivity. On the other hand, in the long wavelength limit (small momenta) Δ isobar acts as a screening object in the spin-isospin response of nuclear medium in order to quench interaction strengths and therefore the transition matrix elements.

Invited talk at the LAMPF II workshop, Los Alamos (1982)

All of the above are very interesting to discuss, but I found it more relevant to concentrate on the high energy domain with the hope of having a bridge to the forthcoming experiments at LAMPF II. In particular, I chose a problem of pion absorption in the resonance energy region, which was motivated by the recent exciting data on pion absorption in light nuclei[1] and heavy nuclei.[2] These data seem to suggest a new interesting role of Δ isobars in nuclei. I believe the full understanding of the existing data should provide new directions for future experiments. The technique developed for dealing with Δ isobars will be used also for other hadronic excited states which will become important as the incident energy icreases and a different projectile is in use. Furthermore, the physics of the role of Δ isobars in the high energy domain has a direct consequence to that in the low energy domain, which is discussed in great detail in the recent review article by Oset et al.[3]

Hence, a question to ask here is what is a mechanism for a pion to be absorbed in a nucleus. From consideration of energy and momentum conservation, at least two nucleons have to be involved in the absorption process. Particularly, at the resonance energy the two nucleon mechanism with the Δ isobar intermediate excitation is the one believed to dominate. The recent pion absorption experiments performed at LAMPF in light as well as heavy nuclei should provide direct information on the absorption mechanism and thereby the role of Δ isobar in nuclei.

II. PION ABSORPTION IN ^{3}HE AND ^{4}HE

Ashery et al.[1] performed coincidence experiments on outgoing nucleons after pion absorption with the pion energy of $T_\pi = 165$ MeV in ^{3}He and ^{4}He with the hope of obtaining information on pion absorption by a T = 1 nucleon pair in addition to that by a T = 0 pair, which can be obtained by pionic distintegration of a deuteron. The two reaction cross sections;

$$\pi^+ + {}^A\text{He} \rightarrow p + p + \text{anything}$$

$$\pi^- + {}^A\text{He} \rightarrow p + n + \text{anything}$$

are compared at several angles. They found that the ratio $d\sigma(\pi^+,pp)/d\sigma(\pi^-,pn)$ came out to be surprisingly large, on the order of ~ 100. If translated in terms of the isospin of initial nucleon pairs which absorb a pion, the ratio is $R = d\sigma(T{=}0)/d\sigma(T{=}1) \sim 50$.[1]

Let us consider just the isospins in the process where pion absorption goes through the formation of a Δ isobar as shown in Fig. 1. Since one of the initial nucleon pair ($t_N = 1/2$) has to combine with the incoming pion ($t_\pi = 1$) to form a Δ isobar ($t_\Delta = 3/2$), the cross section should be proportional to the vector recoupling coefficient (Unitary Racah coefficient);

$$|\langle(\tfrac{1}{2}\ \tfrac{1}{2})T,\ 1;\ 1|\tfrac{1}{2},(\tfrac{1}{2}\ 1)\tfrac{3}{2};\ 1\rangle|^2 = \begin{cases} \tfrac{1}{2} & \text{for } T=1 \\ 1 & \text{for } T=0 \end{cases} \qquad (1)$$

Here, the intermediate Δ isobar-nucleon state has to have the resultant spin 1 in order to decay into two nucleons. Hence, the ratio in this consideration is much too small; R = 2.

As the next consideration, let us take only the pion partial wave of $\ell_\pi = 1$. Maxwell et al.[4] and others demonstrated that $\ell_\pi = 1$ absorption is the dominant process in the pionic disintegration of a deuteron. The p-wave pion absorption ($J^\pi = 1^+$) by a T = 1 nucleon pair (S = 0, L = 0) leads to a final state with T = 1 and $J^\pi = 1^+$. The Pauli principle on the final nucleon pair does not allow these quantum numbers. This Pauli effect might be the reason for the large ratio between T = 0 and T = 1 pion absorption found experimentally.

We are then left with a task to show that other partial waves do not provide large cross sections, particularly for the T = 1 case. Since the radius of ^{3}He is about $R \sim 2$ fm and the momentum of the incoming pion with $T_\pi = 165$ MeV is $k_\pi = 1.4$ fm^{-1}, the partial waves of $\ell_\pi \leq k_\pi R \sim 3$ should be able to contribute to the absorption process.

We have therefore gone ahead and calculated the pion absorption cross sections explicitly using the model where Δ isobar is formed after pion absorption. This process is depicted in Fig. 2. The T-matrix is worked out using the Feynman graph techniques in momentum space;[5]

$$T(\vec{k}_1,\vec{k}_2;\vec{k}) = \frac{f(k_1)}{\mu}(\vec{\sigma}_1\cdot\vec{k}_1)\vec{\tau}_1\cdot D_\pi(k_1)\frac{f^*(k_1)}{\mu}(\vec{S}_2^{\dagger}\cdot\vec{k}_1)\cdot\vec{T}_1^{\dagger}G_D(P_\Delta)\frac{f^*(k)}{\mu}(\vec{S}_2\cdot\vec{k})T_{2\lambda}$$

$$+\frac{f(k_1)}{\mu}(\vec{\sigma}_1\cdot\vec{k}_1)\vec{\tau}_1\cdot D_\pi(k_1)\frac{f^*(k_1)}{\mu}(\vec{S}_2\cdot\vec{k}_1)\cdot\vec{T}_2 G_C(P_\Delta)\frac{f^*(k)}{\mu}(\vec{S}_2^{\dagger}\cdot\vec{k})T_{2\lambda}^{\dagger}+(1\leftrightarrow 2) \quad (2)$$

Here $\vec{k}$ is the incoming pion momentum in the laboratory frame, which after absorption gets distributed as $\vec{k}_1$ and $\vec{k}_2$ among the two nucleons 1 and 2 involved in the process. $\vec{\sigma},\vec{\tau}$ are the spin, isospin Pauli matrices and $\vec{S},\vec{T}$ the corresponding transition operators between a nucleon and a Δ isobar. The πNΔ coupling constant f^* is related to the πNN coupling constant f through $f^* = 2f$ (Chew-Low relation) and has the monopole form factor $f^*(k) = (\Lambda^2 - \mu^2/\Lambda^2 - k^2)f^*$, where Λ is the cutoff mass. D_π is the pion propagator and μ the pion mass. The isobar propagators for the direct (Fig. 2(a) and (c)) and the crossed (Fig. 2(b) and (d)) graphs are

$$G_D(P_\Delta)^{-1} = m_\Delta - m + \vec{P}_\Delta^2/2m_\Delta - \omega - \frac{1}{2}i\Gamma$$

$$G_C(P_\Delta)^{-1} = m_\Delta - m + \vec{P}_\Delta^2/2m_\Delta + \omega, \quad (3)$$

where the Δ isobar mass and nucleon mass are denoted by m_Δ and m and the pion energy by ω. We take the static nucleon approximation and therefore $|\vec{P}_\Delta|^2 = |\vec{k}|^2$ for G_D and $|\vec{P}_\Delta|^2 = m\omega$ for G_C. For the width of the Δ isobar, we take the empirical relation

$$\Gamma = \frac{0.47}{1+0.6(q/\mu)^2}\frac{q^3}{\mu^2} \quad (4)$$

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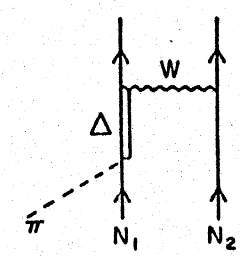

Fig. 1.
Standard p-wave pion absorption leading to two nucleon emission.

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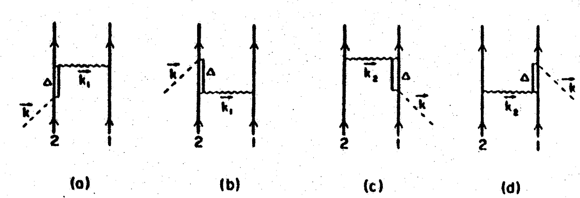

Fig. 2.
p-wave pion absorption mechanism with Δ isobar intermediate excitation. (a), (c) direct graph. (b), (d) crossed graph.

The pion-nucleon center of mass momentum q is related to the incoming pion momentum $\vec{k}$ through $\vec{q} = m/\sqrt{S}\,\vec{k}$ where S is the S-channel invariant mass.

The T-matrix (Eq. 2) together with the similar expression for rho meson exchange can be further worked out using the technique developed in Ref. 6. A lengthy expression is then obtained for pion absorption in terms of two-body transition operators in the nuclear space.[5]

In the $\ell_\pi = 1$ case, the ratio $R = d\sigma(T=0)/d\sigma(T=1)$ does not depend on any details of the model parameters and is simply given by the ratio of the squares of the Δ isobar propagators G_D and G_C of Eq. (3);

$$R = \frac{d\sigma(T=0)}{d\sigma(T=1)} = \left|\frac{G_D}{G_C}\right|^2 \quad (5)$$

The calculated ratio using Eq. (5) is shown by the dotted line in Fig. 3. Because of the resonant behavior in the direct channel, the ratio has a peak at $k \sim 2\mu$ ($T_\pi \sim 180$ MeV). At the experimental energy ($T_\pi = 165$ MeV), this ratio is ~200.

We show in Table I all the results of the pion absorption cross sections in several channels for $\ell_\pi \leq 2$ as a function of the pion momentum k. The $\ell_\pi = 0$ and 2 contributions are indeed very small as compared to the dominant one ($\ell_\pi = 1$; $T = 0 \to T' = 1$) even up to large momenta considered here particularly for the initial isospin $T = 1$ case. This is due to the small matrix elements for $\ell_\pi = 0$ and 2 as compared to those for $\ell_\pi = 1$, typically about 1/5 in the resonance region, and strong cancellation between two competing terms for $T = 1$. Higher partial wave contributions ($\ell_\pi \geq 3$) are further smaller by an order of magnitude.

The summed values for the $T = 0$ and $T = 1$ initial pairs are then used to derive the ratio R as a function of pion momentum, which is depicted by the solid line in Fig. 3. The ratio is now down to about 40 at $T_\pi = 165$ MeV. The experimental results taken at different angles are also shown with error bars.[1]

I did not discuss the S-wave rescattering process in the above. We get a feeling of the S-wave contribution by looking at the deuteron disintegration data and the calculation.[4] The cross section decreases with increasing pion momentum from the threshold, which is due to the diminishing contribution of the S-wave rescattering process with k above $T_\pi \geq 50$ MeV.

To conclude this section, the surprisingly large ratio between the $T = 0$ and $T = 1$ pion absorption cross sections is understood in terms of the two nucleon rescattering mechanism where Δ isobar plays the central role. It would be interesting to measure the ratio at several pion energies and also the angular distribution of the outgoing nucleons.

III. PION ABSORPTION IN HEAVY NUCLEI

The success of the two nucleon absorption mechanism of a pion through Δ isobars makes us expect the same thing to happen in heavy nuclei. McKeown et al.[2] performed a systematic study on pion absorption in nuclei between ^{12}C and ^{181}Ta. The proton inclusive spectra after absorption of 220 MeV π^+ and π^- show features which are unexpected from the 2N absorption mechanism.

A typical proton spectrum after absorption is depicted in Fig. 4. We do not see a pronounced peak around $E_p \sim \omega_\pi$ (incoming pion energy)/2 but rather find large cross sections of protons at lower energies. When the same data are

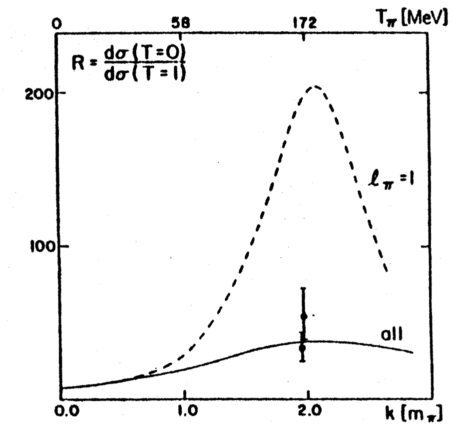

Fig. 3.
The pion absorption ratio $R = d\sigma(T=0)/d\sigma(T=1)$ as a function of incoming pion momentum k in pion mass unit. The dashed line denotes the result with $\ell_\pi = 1$ only and the solid line that with all partial waves. Experimental data obtained at two different angles are also shown at $k \sim 2m_\pi$ ($T_\pi = 165$ MeV) with error bars (taken from Ref. 5).

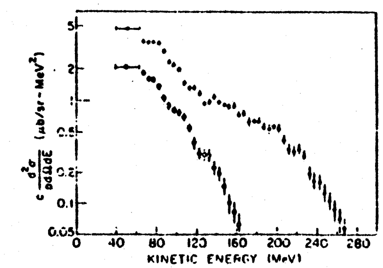

Fig. 4.
Proton spectra from 160 MeV π^+ on Ni at 30° (solid points) and 150° (open circles) (taken from Ref. 2).

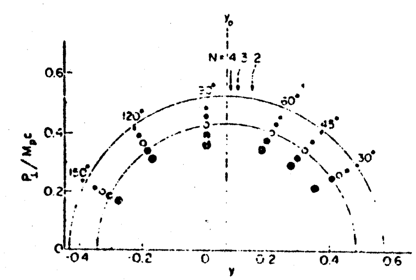

Fig. 5
Contours of the invariant cross section, $\frac{c}{p}\frac{d^2\sigma}{d\Omega dE}$, in the plane of rapidity y and $P_\perp$, for 220 MeV π^- on ^{181}Ta. The rapidity of the frame in which the cross section is most nearly isotropic is indicated by a dashed line (y_0) (taken from Ref. 2).

plotted in the plane of rapidity y_0 (approximately the velocity parallel to the beam) and perpendicular velocity $v_\perp$, the invariant cross section, $\frac{c}{p}\frac{d^2\sigma}{dEd\Omega}$ exhibits isotropy around some rapidity y_0 as shown in Fig. 5. Assuming several nucleons share the energy (which includes the pion rest mass) and momentum brought in by a pion, we can extract the effective number of nucleons involved in pion absorption from the rapidity y_0. The number comes out ranging between 3 and 5. Furthermore, the ratio of proton yields from π^+ and π^- absorption, $Y_p(\pi^+)/Y_p(\pi^-)$, turns out to about 4, independent of proton angle.

Can we understand these features by considering secondary (final state) interactions for primary nucleons in the 2N absorption mechanism? First of all, several experimental data indicate that the mean free path of a nucleon with energy of 100∿200 MeV is rather large, $\lambda \sim 5$-6 fm.[7] This long mean free path makes rescattering of the primary nucleons unlikely. Direct evidence against the final state interaction picture is found in the comparison of the proton inclusive cross sections σ_p after pion and photon absorption with the same energy,[3] because a pion is absorbed right at the nuclear surface while a photon is annihilated anywhere in the nucleus. The photon induced cross section does not indicate any significant difference except the overall magnitude from that induced by a pion.[7]

The next consideration might be a statistical model, where in average four nucleons participate for pion absorption. The ratio $Y_p(\pi^+)/Y_p(\pi^-)$ in this picture is 2, which contradicts the experimental number, $R_p \sim 4$.

Therefore we need to find a new mechanism, which leads to an average of about 4 nucleons involved more or less directly in the pion absorption process at energies in and slightly above the resonance region. Brown et al.[9] suggest a process of direct four nucleon emission following virtual double Δ isobar excitation, as shown in Fig. 6. This might appear to be rather exotic at first sight, but there are a few facts to consider which favor this process. First, initially there are two protons to interact for an incoming positively charged pion, where the $\pi^+ + p$ cross section is 9 times larger than the $\pi^+ + n$ cross section. Second, the two Δ isobars share on average the incoming pion energy, $E_\Delta \sim \omega_\pi/2 \sim m_\pi$. Hence only a small phase space is available for the pion decay. Third, a Δ isobar of this energy range has a rather large decay width into a nucleon by interacting with another nucleon, where the width is about $\Gamma_{\Delta N \to NN} \sim 40$ MeV.[10,11] Therefore, although the 2Δ formation brings the system into largely off-shell by about the Δ isobar nucleon mass difference, the above mentioned points might favor the 2Δ process over the standard 2N process.

It turns out to be most convenient to calculate pionic response functions for the two processes as shown in Fig. 7. The imaginary part of the response function corresponds to the absorption width

$$\Gamma = -2\pi \, \text{Im} \, R \qquad (6)$$

The ratio of absorption rates for the 2Δ process ($\Gamma^{\Delta\Delta}_{abs}$) and the standard 2N process (Γ^{NN}_{abs}) is given by

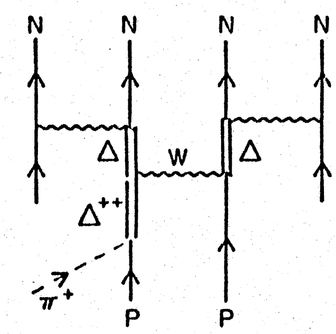

Fig. 6.
π^+ absorption by a two proton pair through formation of a 2Δ state which subsequently decays by preferentially emitting four nucleons.

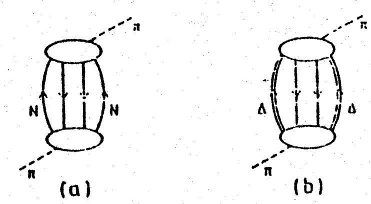

Fig. 7.
Pion-nuclear response functions due to (a) the standard 2N process and (b) the 2Δ formation process.

$$\frac{\Gamma^{\Delta\Delta}_{abs}}{\Gamma^{NN}_{abs}} = \frac{\mathrm{Im}\int d^3p_1 d^3p_2 \delta^3(\vec{p}_1+\vec{p}_2-\vec{k})G_{\Delta\Delta}|\langle\pi NN|\delta H|\Delta N\rangle\langle\Delta N|V|\Delta\Delta\rangle|^2}{\mathrm{Im}\int d^3p_1 d^3p_2 \delta^3(\vec{p}_1+\vec{p}_2-\vec{k})G_{NN}|\langle\pi NN|\delta H|\Delta N\rangle\langle\Delta N|V|NN\rangle|^2} \tag{7}$$

Here the summations represent the properly weighted sums over all possible spins and isospins of NN, ΔN and ΔΔ pairs. The integration leads over the two-body phase space of the excited ΔΔ or NN pair, where we have neglected corrections from binding and Fermi motion in the momentum conserving δ-function, $\vec{k}$ being the incoming pion momentum. The δH generates the $\pi N\to\Delta$ transition at either one of the two nucleons and the interaction V changes the ΔN into ΔΔ or NN pairs. The product of energy denominators is

$$G^{-1}_{\Delta\Delta} = (\omega_\Delta - \omega - \frac{i}{2}\Gamma_\Delta(\omega))^2 (\omega - 2\omega_\Delta - \frac{p_1^2}{2M_\Delta} - \frac{p_2^2}{2M_\Delta} + \frac{i}{2}\Gamma_{\Delta N\to NN} + \frac{i}{2}\Gamma_{\Delta N\to NN}) \tag{8}$$

for the 2Δ process. $\omega_\Delta = M_\Delta - M$ is the ΔN mass difference, while Γ_Δ the free $\Delta\to\pi N$ decay width. $\Gamma_{\Delta N\to NN}$ is the decay width of the Δ isobar by the interaction with a nucleon into two nucleon states, the information on which can be obtained from the analysis of pion scattering;[10, 11] $\Gamma_{\Delta N\to NN} \sim 40$ MeV. The 2N process has

$$G^{-1}_{NN} = (\omega_\Delta - \omega - \frac{i}{2}\Gamma_\Delta(\omega))^2 (\omega - \frac{p_1^2}{2M} - \frac{p_2^2}{2M} + i\delta) \tag{9}$$

The ratio of absorption rates (Eq. 7) may be factorized as

$$R = \frac{\Gamma^{\Delta\Delta}_{abs}}{\Gamma^{NN}_{abs}} = R_{phase}\cdot R_{isospin}\cdot R_{spin}\cdot R_{space} \quad , \tag{10}$$

where R's are, respectively, due to the phase space, isospin, spin and coordinate spaces. The phase space ratio may be obtained by taking the squared matrix elements of V at a proper kinematical average and performing the remaining phase space integrations. At $\omega = \omega_\Delta$, the result is

$$R_{phase} = \frac{1}{2}\left(\frac{M_\Delta}{M}\right)^{3/2} \frac{\Gamma_{\Delta N\to NN}}{\omega_\Delta} \sim 0.1 \tag{11}$$

In working through the isospin matrix elements for $R_{isospin}$, we find the leading channel in the 2Δ process is π^+ absorption on a pp pair; $T = 1 \to T' = 2$, whereas that in the 2N process is π^+ absorption on p_n; $T = 0 \to T' = 1$. The isospin part gives the ratio,

$$R_{isospin} = \frac{27}{16} \tag{12}$$

For comparison, $T = 1 \to T' = 1$ transitions in the ΔΔ process are suppressed by a factor of 3.

The spin and space parts need a further consideration. The standard 2N process involves high momentum transfers; $q \sim \sqrt{M\omega_\pi}$, which is required by the energy and momentum conservation. On the other hand, the 2Δ process proceeds with low q, because the sharing of momenta among the four nucleons (Fig. 6) does not require the exchange of high momenta along with the interaction, $V(\Delta N\to\Delta\Delta)$. The short range of the standard process favors absorption on T = 0 pairs in relative ^3S state. The T = 0 absorption is about 50 times larger than the T = 1 absorption as discussed in the previous section.[1] The tensor part of $V(N\Delta\to NN)$ dominates by far in the 2N process. On the other hand, the 2Δ process is expected to proceed by the spin-spin part of the interaction $V(N\Delta\to\Delta\Delta)$. Again the leading channel in the 2Δ process is the S = 1 (NN pair) → S' = 2 (ΔN and ΔΔ) transition. The combined ratio turns out to be

$$R_{spin}\cdot R_{space} = 3\left(\frac{f_{\pi\Delta\Delta}}{f_{\pi NN}}\right)^2 \left(\frac{C}{W_T(q^2\sim M\omega_\pi)}\right)^2 \sim 6 \tag{13}$$

Here C corresponds to the coupling constant of the spin-spin interaction and W_T the tensor interaction. With the use of the SU(4) ratio for the coupling constants, $f_{\pi\Delta\Delta}/f_{\pi NN} = \sqrt{4/5}$, a reasonable estimate on the spin-spin and the tensor coupling constants lead to the ratio, $R_{spin}\cdot R_{space} \sim 6$. Hence the ratio R (Eq. 10) comes out to be $R \sim 1$.

Let us summarize what we have learned in Table II. In the 2Δ process, an initial nucleon pair has S = 1 and T = 1 and therefore the relative angular momentum L = odd. The interaction range is very large on the order of the inverse of the pion mass ($m_\pi^{-1} \sim 1.4$ fm), since four nucleon emission does not require large momentum transfer. The 2N process proceeds with an initial pair of S = 1 and T = 0 in the relative S state whose interaction range is very short, ∿0.5 fm.

Finally for the full absorption ratio, we have to multiply the ratio of the numbers of nucleons, R_{pair}, to the ratio R obtained by calculating the matrix elements. Since the number of nucleons should be proportional to the interaction volume, we can write

$$R_{pair} = \frac{(\frac{4\pi}{3} d_\Delta^3)\ \frac{Z(Z-1)}{2}\ \frac{1}{2}}{(\frac{4\pi}{3} d_N^3) N \cdot Z \cdot \frac{1}{2}} = \left(\frac{d_\Delta}{d_N}\right)^3 \frac{Z-1}{2N} \qquad (14)$$

In the numerator, the number of protons $(\frac{Z(Z-1)}{2})$ is divided by 2, because only the odd partial waves for the relative motion are utilized. The 2N process, appears in the denominator, is further proportional to the number of proton-neutron pairs, N·Z, divided by 2 to select T = 0 pairs. If we take a heavy nucleus like ^{208}Pb, the ratio is $R_{pair} \sim 10$. Therefore, the 2Δ process is about 10 times more probable than the 2N process in heavy nuclei.

How about the mass dependence? Since the incoming pion is absorbed at the nuclear surface, the absorption cross section should grow as $\sigma_{abs} \propto A^{2/3}$. In the 2Δ process, however, which has a large interaction volume ($d_\Delta \sim 1.4$ fm), we have to correct for the surface effect, since a nucleon which absorbs the incoming pion at the edge of the nucleus does not find nucleons in the outer half of the interaction volume. This might be corrected phenomenologically by replacing the radius r by an effective one;

$$r_{eff} = r - d \qquad (15)$$

With this surface correction, the absorption cross section as a function of the mass number is obtained as shown in Fig. 8. While the 2N process is larger in light nuclei, the 2Δ process should dominate in heavy ones. Altogether the absorption cross section, $\sigma_{abs}(\Delta\Delta) + \sigma_{abs}(NN)$, behaves as $\sigma_{abs} \propto A^\alpha$, where α is slightly larger than 2/3.

Assuming the dominance of the 2Δ process, let us estimate the ratio of protons emitted after absorption of π^+ and π^-. The leading processes are $\pi^+ pp \to \Delta^{++}\Delta^+ (S = T = 2)$ and $\pi^- nn \to \Delta^- \Delta^0$ (S = T = 2). The Δ's combine with surrounding nucleons in either one of the processes $\Delta^{++} n \to pp$, $\Delta^+ p \to pp$, $\Delta^+ n \to pn$ or np, or $\Delta^- p \to nn$, $\Delta^0 p \to pn$ or np. Collecting the Clebsch-Gordan weights for these channels and assuming otherwise statistical distributions of nucleons, we find

$$\frac{Y_p(\pi^+)}{Y_p(\pi^-)} = \frac{Z(Z-1)}{N(N-1)} \frac{4N(\frac{Z-2}{3})\ \frac{3}{4}\cdot\frac{1}{4} + 3\ \frac{N(N-1)}{2}\ \frac{3}{4}\ \frac{1}{2}}{\frac{Z(Z-1)}{2}\ \frac{3}{4}\ \frac{1}{2}} = \frac{4}{3}\ \frac{Z-2}{N-1} + 3 \qquad (16)$$

where the Z(Z - 1)/N(N - 1) factor in front comes from the ratio of initial pp or nn pairs. The first term in the numerator comes from $\Delta^{++}\Delta^+$ combining with a neutron and a proton with four protons in the final state, while the second term corresponds to $\Delta^{++}\Delta^+$ combining with two neutrons to form three protons. The process involving $\Delta^- \Delta^0$ in the denominator leads to emission of one proton in combination with two surrounding protons. The remaining factors are squared C.G. coefficients.

Considering the simplicity of the assumptions underlying Eq. (16), the result for $Y_p(\pi^+)/Y_p(\pi^-)$ is surprisingly close to the experimental findings (see

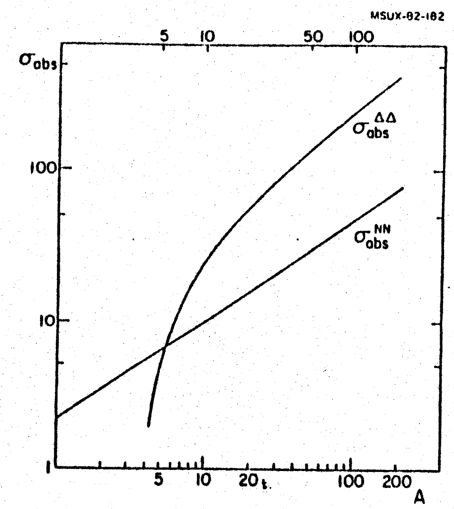

Fig. 8.
Pion (π^+) absorption cross section σ_{abs} in arbitrary units is plotted as a function of mass number A. The 2N absorption cross section σ_{abs}^{NN} is proportional to $A^{2/3}$. The 2Δ absorption cross section $\sigma_{abs}^{\Delta\Delta}$ includes the surface correction (d = 1.5 fm) in Eq. (14) and it is obtained with the assumption that the ratio R in Eq. (10) is R = 1. $\sigma_{abs}^{\Delta\Delta} = 0$ is used for A = 4 (^{4}He).

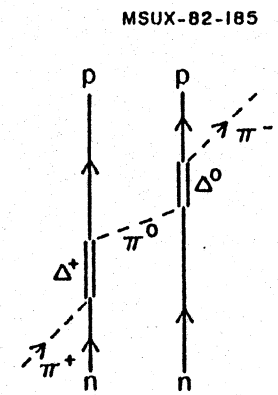

Fig. 9.
Successive charge exchange process for double charge exchange reaction (π^+,π^-) through formation of Δ isobar.

Table III). Additional contributions from all $T=1$ NN pairs leading to $T=2\Delta\Delta$ pairs not taken into account in Eq. (16), reduce these numbers by only about 10%.

The simple estimates presented above seem to indicate that indeed the 2Δ process is the dominant mechanism for pion absorption in heavy nuclei.

IV. CONCLUSION AND FUTURE

I have discussed possible mechanisms for pion absorption at the energy in the Δ resonance region in light and heavy nuclei. The surprisingly large ratio, $d\sigma(T=0)/d\sigma(T=1)$, between the $T=0$ and $T=1$ nucleon pairs for pion absorption in the He isotopes is reproduced naturally within the 2N mechanism through the Δ isobar formation. Experimental data for pion absorption in heavy nuclei, however, demanded a new mechanism. The 2Δ process after Δ isobar formation seems to fit very well to all the experimental requirements. We need to have more experimental data and also further theoretical works to establish the 2Δ absorption mechanism.

The success of the 2Δ mechanism in explaining pion absorption data in heavy nuclei suggests interesting processes in other channels. The double charge exchange reaction, (π^+,π^-), requires at least two nucleons and has been considered to go by successive charge exchange through Δ formation as depicted in Fig. 9. It is quite probable also for double charge exchange to happen in one nucleon and leave a Δ^{++} state, which then decays into pp by combining with another neutron (see Fig. 10). The similarity of this process to the 2Δ absorption process should be obvious.

Another interesting process expected to happen is pion production at high energy ($E_\pi \geq 400$ MeV). A pion production process depicted in Fig. 11 resembles the 2Δ pion absorption process up to the 2Δ formation, but when the incoming pion energy is large, each one of the Δ's has sufficient energy to decay by pion emission. It is very interesting to investigate this change of pion absorption into pion production through 2Δ formation by gradually increasing the incoming pion energy.

There seem to be many more interesting phenomena associated with Δ isobars and also with other hadronic resonances unexplored yet at the presently available energy region provided by LAMPF. The mechanisms discussed in this note are certainly relevant to the K-on physics where the Λ and Σ baryon resonances will play the similar role as Δ isobar. Finally, a nucleus provides a rare and important experimental ground for us to gain valuable information on elementary particles which interact with a couple of nucleons at the same time.

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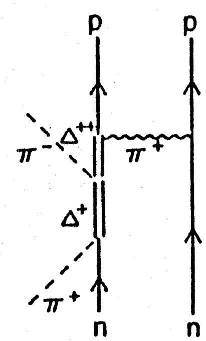

Fig. 10.
Double charge exchange process by a single nucleon leaving Δ^{++}, which successively decays into two protons for (π^+,π^-) reaction.

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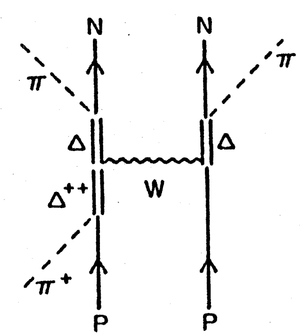

Fig. 11.
2Δ process for pion production, which is expected to occur at higher energy ($T_\pi \geq 400$ MeV). Notice the similarity with Fig. 6.

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TABLE I

Pion absorption cross sections in several channels as a function of pion momentum k are compared, where ℓ_π is the angular momentum of the incoming pion. These numbers are normalized to the (T,S,L) = (0,1,0) to (T',S',L') = (1,0,2) transition at k = 0.5μ as indicated by *.

k[μ]					0.5	1.0	1.5	2.0	2.5
Initial	ℓ_π	T'	S'	L'					
T=0 S=1 L=0	0	1	1	1	0.027	0.40	2.9	9.7	7.3
	1	1	0	2	1*	4.9	21	52	30
	2	1	1	3	0.006	0.11	1.0	4.2	3.6
T=1 S=0 L=0	0	1	1	1	0.008	0.086	0.44	1.1	0.63
	1	0	1	2	0.076	0.17	0.24	0.26	0.26
	2	1	1	3	0.002	0.023	0.14	0.40	0.26

TABLE II

The quantum numbers of initial nucleon pair and the range of interaction between the pair nucleons for the 2Δ and 2N pion absorption processes.

Mechanism	Range	Initial pair T	S	L
ΔΔ	1.4 fm	1	1	odd
NN	0.5 fm	0	1	0

TABLE III

Ratio of proton yields from π^+ or π^- absorption on three different nuclei, following from the double-Δ mechanism prediction Eq. (15) [taken from Ref. 9]. Experimental data are taken at a pion energy of 220 MeV [deduced from Ref. 2].

$\frac{Y_p(\pi^+)}{Y_p(\pi^-)}$	ΔΔ-mechanism	exp. (Ref. 2)
^{27}Al	4.1	3.8 ± 0.8
^{58}Ni	4.2	3.7 ± 0.7
^{181}Ta	3.9	3.4 ± 0.6