THE N = 82 ISOTONES IN THE GENERALIZED SENIORITY SCHEME

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A calculation for the N = 82 isotones in the generalized seniority scheme is presented and compared with a shell model calculation.

As has been emphasized by Talmi [1], the generalized seniority scheme provides for semi-closed shell nuclei a good basis to label the states. The advantage of working directly in a generalized seniority basis is therefore that without losing any of the important physics, one can work in a much smaller basis than what is necessary in the shell-model. In this article we will show this by comparing a calculation in the generalized seniority scheme for the N = 82 isotones with a shell-model calculation.

The generalized seniority scheme as it was introduced by Talmi [1] is a generalization of the usual seniority concept [2,3] to the case of several non-degenerate single particle (s.p.) orbits. The seniority quantum number v counts the number of particles not pairwise coupled to angular momentum $J = 0$. Mathematically seniority is introduced via the operators

$$S_{+j} = (2J+1)^{1/2}(a_j^\dagger a_j^\dagger)^{(0)} , \tag{1a}$$

$$S_{-j} = (2J+1)^{1/2}(\tilde{a}_j\tilde{a}_j)^{(0)} , \tag{1b}$$

$$S_{0j} = \tfrac{1}{2}[S_{+j}, S_{-j}] = \tfrac{1}{2}[\hat{n}_j - \tfrac{1}{2}(2J+1)] , \tag{1c}$$

where $\hat{n}_j$ is the fermion number operator. It can easily be verified that these three operators form the generators of a quasi-spin [3] [SU(2)] algebra. The representations of this group are labeled by v and an n-particle state can be written as

$$|j^n, v, \alpha, J\rangle = S_{+j}^{(n-v)/2}|j^v, v, \alpha, J\rangle , \tag{2}$$

where α denotes the other quantum numbers necessary to label the state uniquely.

To generalize the seniority concept from the case of a single j shell to that of many orbits, one introduces the operators

$$S_+ = \sum_j \alpha_j S_{+j} , \tag{3a}$$

$$S_- = \sum_j \alpha_j S_{-j} . \tag{3b}$$

It can be shown [4] that it is possible to form from these operators (by introducing a proper S_0) a quasi-spin algebra if $\alpha_j^2 = 1$. This case corresponds to the case of degenerate s.p. levels. Similar to the case of normal seniority, the states with generalized seniority w are written as

$$|\tilde{j}^n, w, \alpha, J\rangle = S_+^{(n-w)/2}|\tilde{j}^w, w, \alpha, J\rangle , \tag{4}$$

where the ~ indicates that the particles are distributed over more than one j orbit, and furthermore

$$S_-|\tilde{j}^w, w, \alpha, J\rangle = 0 . \tag{5}$$

The definition of the generalized seniority basis as it is given here makes it very similar to that of the Broken Pair model [5].

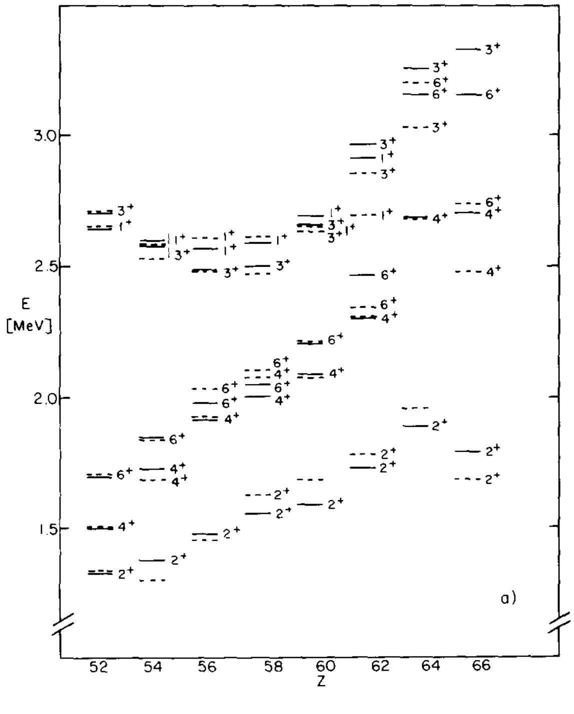
While in the normal seniority scheme, it is relatively simple to calculate the matrix elements of an operator, it is much more complicated in the generalized seniority scheme. Recently however analytic formulas have been derived [6] to calculate matrix elements for states in the generalized seniority scheme.

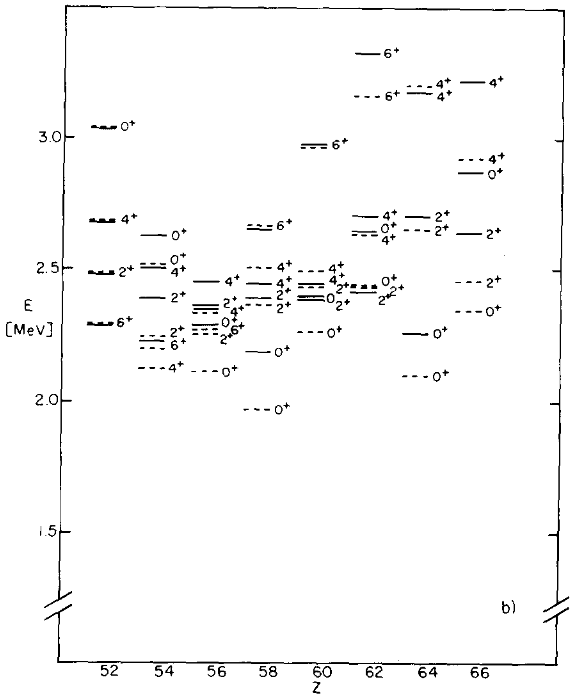
The calculations presented here are done in a basis that includes all states with $w \leqslant 2$. In this basis the ground-state is, by definition, a $w = 0$ state and can therefore be written as

$$|\text{g.s.}\rangle = S_+^N |0\rangle \,, \qquad (6)$$

where N is the number of fermion pairs outside the closed shell $|0\rangle$. The S operator was defined in eq. (3a). To determine the coefficients α, one can minimize the expectation value of H for the state (6). In our work we have followed a somewhat different procedure which is based on the fact that the eigenvector belonging to the lowest eigenvalue of H in the space spanned by the vectors $j^2|S^{(N-1)}\rangle$ is in fact the state given in eq. (6).

To test the validity of the generalized seniority scheme, we have applied it to the case of the even mass $N = 82$ isotones with $50 < Z < 68$ and compared the results with those of a shell-m model calculation





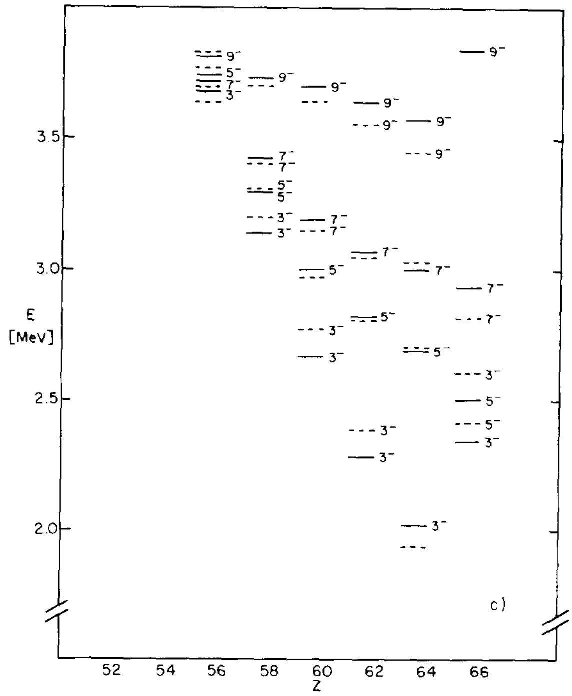


Fig. 1. Comparison of excitation energies calculated in the shell-model and the generaliezd seniority scheme, as discussed in the text in (a) for the $1_1^+, 2_1^+, 3_1^+, 4_1^+$, and 6_1^+, in (b) for the $0_2^+, 2_2^+, 4_2^+$ and 6_2^+ and in (c) for the $3_1^-, 5_1^-, 7_1^-$ and 9_1^- levels. Solid lines represent the results of the generalized seniority model, the dashed lines those of the shell model. If the two results are very close, only one is labeled.

[7]. In the shell-model calculation the basis was truncated so as to include only states with $v \leqslant 4$, while only states with $w \leqslant 2$ were considered in the generalized seniority basis. The largest matrix in the latter case had a dimension of only 9, while in the shell model the largest dimension was of the order of 3500. The shell model interaction was fitted as to reproduce the known experimental energies. In the generalized seniority calculation this interaction was used without modification. In fig. 1 the results of the two calculations are compared. For the case of $Z = 52$ the shell model description and the generalized seniority scheme are identical since there are only two particles outside the $N = 82$ closed shell. For $Z > 52$ the results of the two calculations agree quite well. Also the trends in the energies with mass number, such as the steep decrease in the energy of the 3^- level are well reproduced. In general the differences in energies (which are larger for the second level of a given spin than for the first), can be related to admixtures of a $v = 4$ component in the shell model wave functions. The largest differences can be found in the energies of the 4_2^+ level in ^{136}Xe and the 0_2^+ level in ^{148}Dy. The reason is that these states contain an extremely large admixture of $v = 4$ components. In fact in Xe the 4_2^+ level calculated in the shell model is almost a pure $v = 4$ state and the 4_3^+ level coincides with the 4_2^+ level as calculated in the generalized seniority scheme. The energy differences that occur in the calculated spectra for Dy and the 1_1^+ level in Sm cannot be explained by $v = 4$ admixtures and further studies will be necessary to determine the nature of these differences.

In conclusion, it has been shown that the generalized seniority concept generally provides a valid truncation scheme of the full shell-model basis, in the sense that a calculation in a much smaller basis using the same interaction gives essentially the same excitation energies for the first few levels of each spin. In a pure generalized seniority scheme one expects that the coefficients α_j do not vary from isotope to isotope. In the present calculation these were obtained for each isotope separately. For $Z \leqslant 58$ the α_j were indeed rather constant but from $Z = 58$ to $Z = 66$, $\alpha_{d_{5/2}}$ changed by as much as 50%. Electromagnetic properties will be compared in a forthcoming paper. Comparison with experiment could possibly be improved by constructing an effective interaction for the $w \leqslant 2$ basis. It should be noted that a similar truncation will not be valid in the cases where the neutron–proton interaction is important, i.e. where there are both valence neutrons and protons present. In this case it is possible to do a calculation in the IBA [8,9] model and use the results of a generalized seniority calculation for the valence neutrons and protons separately to calculate the properties of the s and d bosons where the s boson is related to the S-pair introduced in eq. (3a).

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References

[1] I. Talmi, Nucl. Phys. A172 (1971) 1.
[2] G. Racah, Phys. Rev. 63 (1943) 367; 76 (1949) 1352; A.R. Edmonds and B.H. Flowers, Proc. Roy. Soc. A214 (1952) 515.
[3] A. de Shalit and I. Talmi, Nuclear shell theory (Academic Press, New York, (1963).
[4] A. Arima and M. Ichimura, Prog. Theor. Phys. 36 (1966) 296.
[5] S. Haq and Y.K. Gambhir, Phys. Rev. C16 (1977) 2455; Y.K. Gambhir, S. Haq and J.K. Suri, Phys. Rev. C20 (1979) 381; J.N.L. Akkermans, Ph.D. Thesis, University of Amsterdam (1981).
[6] O. Scholten and S. Pittel, Phys. Lett. 120B (1983) 9.
[7] H. Kruse and B.H. Wildenthal, Bull. APS 27 (1982) 533, 725.
[8] A. Arima and F. Iachello, Ann. Phys. (NY) 99 (1976) 253; 111 (1979) 201; 123 (1979) 468.
[9] T. Otsuka, A. Arima and F. Iachello, Nucl. Phys. A309 (1978) 1.