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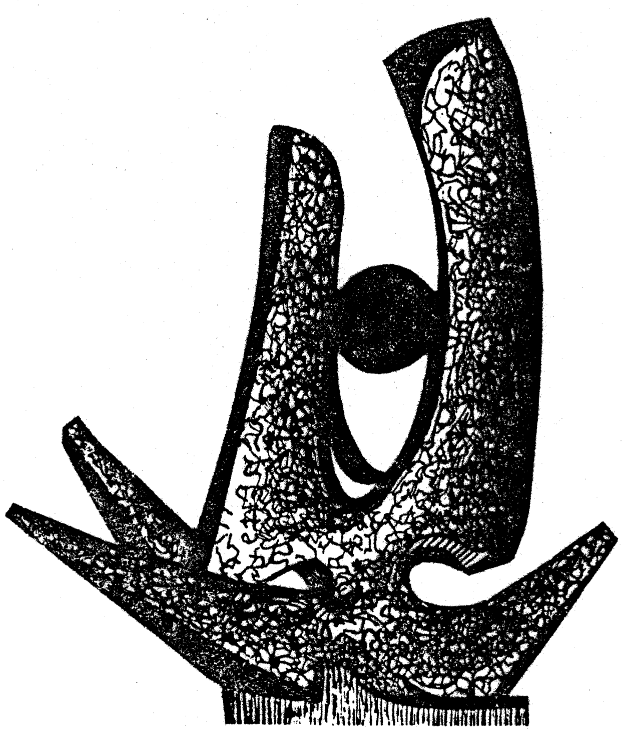
NUCLEAR SURFACE FLUCTUATIONS: CHARGE DENSITY

H. ESBENSEN and G.F. BERTSCH



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Nuclear Surface Fluctuations: Charge Density

H. Esbensen and G.F. Bertsch

Cyclotron Laboratory and
Department of Physics and Astronomy
Michigan State University
East Lansing, MI 48824-1321

Abstract

Fluctuations of the nuclear surface about the Hartree-Fock configuration can have a significant influence on ground state charge densities. We present simple formulas to describe the effects of zero-point motions of the surface due to RPA vibrations. We use a collective treatment, justifying it with a solvable model. We find that the main correction to the densities comes from low-lying collective states, whereas giant resonances are of minor importance.

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[NUCLEAR STRUCTURE Charge density in the correlated ground state obtained from Hartree-Fock theory and the RPA response.]

I. INTRODUCTION

Fluctuations of the nuclear surface due to the zero-point motions of collective surface vibrations can affect the ground state charge density. It is of interest to calculate these corrections in view of the apparent agreement between Hartree-Fock theory and experiment.[1] The usual approach to this calculation is via diagramatic expansion techniques, yielding a perturbation formula based on RPA y-amplitudes.[2] However, the set of graphs that needs to be included in a consistent treatment is larger than suggested in Ref. 2, and only recently a solution of the self-consistent equations has been attempted.[3] This microscopic approach has the disadvantage that it involves complicated numerical calculations.

Motivated by the collective model of Bohr and Mottelson, and the applicability of the coherent surface excitation model[4] to describe low-energy heavy ion reactions and sub-barrier fusion,[5] we present here a macroscopic treatment. We will calculate the charge density in the correlated ground state on the basis of the collective response obtained from RPA calculations,[6] and a Hartree-Fock description of the uncorrelated ground state.

II. EFFECTS OF COLLECTIVE SURFACE VIBRATIONS

In the collective model of Bohr and Mottelson, the surface radius of a nucleus is parametrized in terms of the amplitudes $\alpha_{n\lambda\mu}$ as follows

$$R = R_0 + s_0 + s = R_0 + s_0 + \sum_{n\lambda\mu} \alpha_{n\lambda\mu} Y^*_{\lambda\mu}(\hat{r}) \ . \qquad (1)$$

R_0 is an effective radius of the nucleus in the absence of fluctuations, s_0 is a correction needed to conserve the number of particles in the correlated ground state (see later). The distribuiton of s is determined by the zero-point motions of collective surface vibrations. Assuming that these vibrations are independent and harmonic the distribution of s is a Gaussian in a spherical nucleus. The square of its standard deviation is given by

$$\sigma^2_{ZPM} = \langle s^2 \rangle = \sum_{n\lambda} \sigma^2_{n\lambda} \; . \qquad (2)$$

The sum is over all collective states $(n\lambda)$ of all multipolarities λ. The $\sigma_{n\lambda}$ is related to the β-value and the isoscalar B-value for collective excitations by[7]

$$\sigma^2_{n\lambda}(\tau=0) = \frac{1}{4\pi} R_0^2 \beta^2_{n\lambda}(\tau=0) = B(\tau=0, 0\to n\lambda) 4\pi[(\lambda+2)A\langle r^{\lambda-1}\rangle]^{-2} \; . \qquad (3)$$

A similar relation holds for the fluctuations in the charge radius

$$\sigma^2_{n\lambda}(E) = B(E\lambda, 0\to n\lambda) 4\pi[(\lambda+2)Z\langle r^{\lambda-1}\rangle_p]^{-2}. \qquad (4)$$

It is important to recognize that much of the fluctuation in Eq. (2) is in fact already contained in the uncorrelated ground state of the nucleus, in particular for the higher multipolarities. We shall therefore subtract this noncollective part σ_0 and use

$$\Delta\sigma^2 = \sigma^2_{ZPM} - \sigma^2_0 \qquad (5)$$

to determine the average density by the convolution

$$\rho(r) = (2\pi\Delta\sigma^2)^{-\frac{1}{2}} \int ds \, \exp\left(-\frac{s^2}{2\Delta\sigma^2}\right) \rho_0(r - s_0 - s) , \qquad (6)$$

where ρ_0 is the uncorrelated density.

The total fluctuation of the nuclear surface due to collective surface vibrations of multipolarity λ we relate to the "non-energy weighted" sum

$$S(E\lambda) = \sum_n B(E\lambda, 0\to n\lambda) , \qquad (7)$$

where the sum is over all RPA excitations. The noncollective part of this excitation strength is

$$S_0(E\lambda) = \sum_{ph} B(E\lambda, 0\to ph) , \qquad (8)$$

where the sum is over all particle-hole excitations. The difference between these two sums represents the increase in the fluctuations of the multipole moment. In our model we also associate this difference with the change in the ground state density due to correlations. We shall therefore determine $\Delta\sigma$ in Eq. (6) from the relation

$$\Delta\sigma^2(E) = \sum_\lambda [S(E\lambda) - S_0(E\lambda)] 4\pi[(\lambda+2)Z\langle r^{\lambda-1}\rangle_p]^{-2}. \qquad (9)$$

Equations (6) and (9) are our basic formulas. The reliability of these formulas is not obvious from our heuristic derivation, but it turns out that this prescription is exact in an example of a solvable model. This case is presented in the Appendix.

A. Characteristics of the density

Given the convolution in Eq. (6), it is simple to estimate how the parameters of the ground state density are affected. To zeroth order in the diffuseness of the surface and to second order in the surface fluctuations one finds that radial moments are related by

$$\langle r^n \rangle = \langle r^n \rangle_0 \left[1 + (n+3)\left(s_0/R_0 + \frac{n+2}{2}\left(\frac{\Delta\sigma}{R_0}\right)^2\right)\right] \; , \qquad (10)$$

where $\langle r^n \rangle_0$ is the moment in the uncorrelated ground state. This relation has also been obtained from a somewhat different parameterization of the density.[8] Nucleon number conservation requires no corrections to the $n=0$ moment, which is fulfilled by choosing $s_0=-\Delta\sigma^2/R_0$. Thus the mean square radius (for $n=2$) is

$$\langle r^2 \rangle = \langle r^2 \rangle_0 [1 + 5(\frac{\Delta\sigma}{R_0})^2]. \tag{11}$$

In the tail region, where the density falls off exponentially, one finds in particular

$$\rho(r) \cong \rho_0(r)\exp(s_0/a + \frac{\Delta\sigma^2}{2a^2}). \tag{12}$$

Here a is the diffuseness (or decay length) of the density. The dominant correction is due to the fluctuations in s, whereas the particle number conservation gives a minor correction. Effectively, the density is shifted outwards by the amount

$$\Delta r \cong \frac{\Delta\sigma^2}{2a} (1 - \frac{2a}{R_0}). \tag{13}$$

Around half-density these fluctuations lead to an increase in the effective diffuseness a_{eff}. For $\Delta\sigma^2 \ll a^2$ one finds that

$$a_{eff} \cong a(1 + \frac{\Delta\sigma^2}{2a^2}). \tag{14}$$

III. RESULTS

We have calculated fluctuation contributions to the charge density of ^{40}Ca and ^{208}Pb as described in the previous section. In the RPA calculations we used the Hartree-Fock model with the Skyrme I interaction to generate the uncorrelated ground state and the particle-hole excitations. In the residual interaction we

included the isoscalar and isovector channels, neglecting spin excitations. For details see Ref. 6.

In Fig. 1 we show the contributions to the quantity $\Delta\sigma^2$, defined in Eq. (9), from different multipolarities. We show the results both for the isoscalar and the electric response. The total result is given in Table I. In both cases the dominant contribution is due to octupole modes. In fact, the zero-point fluctuations, obtained from measured B- or β-values[9,10] of the low-lying octupole states in these nuclei, are quite substantial (see Table I).

The contributions from higher multipolarities are much less. This result is consistent with the experience that the strength of collective vibrations of high multipolarity is usally found at rather large excitation energies. The increase in zero-point fluctuation due to collectivity is therefore much less than for quadrupole or octupole vibrations.

Let us mention that the subtraction of the fluctuations in the uncorrelated ground state is crucially important. Without this correction the total zero-point fluctuation (summed over all multipolarities) would in fact diverge, as can be seen for example in the liquid drop model. The total collective fluctuation $\sigma^2_{ZPM}(\lambda)$ can become rather large for high multipolarities λ, but in our treatment it is almost cancelled by the fluctuation $\sigma^2_0(\lambda)$ in the uncorrelated ground state, which leads to the trend towards convergence seen in Fig. 1. The importance of this correction is illustrated in Fig. 2 for quadrupole vibrations in ^{208}Pb. Here the collective

B(E2) value is shown as a function of excitation energy, together with the difference between the B-value for the collective and the free response, which enters into the calculation of $\Delta\sigma^2(E)$ through Eq. (9). It is seen that the low-lying collective state contributes almost fully, whereas the contributions from the isoscalar and isovector giant resonances are compensated to a large extent by the $\Delta N = 2$ transitions in the free response. The dominance of contributions from low-lying collective states is in fact a general feature of our calculations.

For ^{40}Ca one finds that the increase in the fluctuations of the charge density $\Delta\sigma^2(E)$ is given by the sum of the increase in the isoscalar and the isovector fluctuations, since we have used a spin-independent residual interaction in the numerical calculations. This relationship is also valid for the solvable model discussed in the Appendix. The isovector part is not very large, which can be seen from the differences between the isoscalar and the electric contributions shown in Fig. 1. When the collectivity for a definite multipolarity is very large, the $\Delta\sigma^2$ for the electric and in the isoscalar degrees of freedom are therefore about equal.

In the Appendix we also consider the spin degrees of freedom. The fluctuation correction there has contributions from the spin excitations as well as the density excitations as seen in Eq. (A.17). The spin modes, like the isovector mode, will have a small negative contribution to the overall fluctuation. We estimate that this effect can lead to a 25% reduction of $\Delta\sigma^2$ at most.

From the final results for $\Delta\sigma^2(E)$ given in Table I, we can now determine the modification of the Hartree-Fock charge density from Eq. (6). For ^{208}Pb the correction is almost insignificant compared to the uncertainties on the measured density (c.f. Ref. 1, and the estimates in Eqs. 13 and 14). For ^{40}Ca the correction is much larger, which can be seen in Fig. 3. There we show the charge densities, including the finite size of the proton as described in Ref. 11, before (dashed) and after (dotted) the Gaussian folding in Eq. (6), together with the density extracted from experiment.[12] The associated RMS radii are 3.40 fm for the uncorrelated ground state, 3.55 fm for the correlated ground state and 3.45 fm from experiment. The measured density is seen to fall in between the two calculated densities.

The measured charge density of ^{40}Ca is also compared to the result of mean field theory in Ref. 1. There a different interaction yielded a Hartree-Fock RMS radius of 3.50 fm and a calculated density that has an even larger diffuseness than the measured density, leaving no room for corrections due to collective vibrations.

We shall now compare our treatment of the fluctuation correction to the density with the microscopic calculation of Ref. 3. The effect of the low-lying octupole state in ^{40}Ca is shown in Fig. 4. The shapes and magnitude of $\Delta\rho_{ch}$ agree very well, confirming our macroscopic approach. The precise positions of the maxima in $\Delta\rho_{ch}$ differ slightly, but this could easily be due to differences in the underlying uncorrelated densities. Thus Khodel, et al., obtain a 40% lower shift in the RMS radius than we find. A precise comparison would be possible if both calculations were based on the same Hartree-Fock Hamiltonian.

IV. CONCLUSION

We have investigated the effect of ground state correlations on the charge density within a macroscopic model, based on the RPA response of a Hartree-Fock ground state. We have given a plausibility argument for our method, justifying it with an exactly solvable model.

We find that the dominant correction to the density arises from low multipolarities and that we do not need to impose an artificial cutoff at higher multipolarities in order to achieve convergence. Moreover, the main correction is due to low-lying collective states, while the contributions from giant resonances are compensated, to a large extent, by fluctuations already present in the uncorrelated ground state.

Although our results are model dependent (in several respects), they seem to indicate that there is a significant effect of ground state correlations on the charge density of ^{40}Ca, as is also seen in the microscopic results. This implies that the interactions used in mean field theory have to be modified, at least for ^{40}Ca, and the effect of ground state correlations should be taken seriously into account, before the agreement with measured charge densities can be properly interpreted.

ACKNOWLEDGMENTS

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APPENDIX. A SOLVABLE MODEL

In order to illustrate and justify our procedure for calculating the correlated ground state density, we consider here a proton and a neutron bound in a harmonic potential. The Hamiltonian is

$$H_0 = \sum_{i=n,p} \left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_i^2} + \frac{1}{2} K x_i^2\right), \tag{A.1}$$

and the one-particle ground state density is

$$\rho_0(x) = (2\pi\sigma_0^2)^{-\frac{1}{2}} \exp\left(-\frac{x^2}{2\sigma_0^2}\right), \tag{A.2}$$

where

$$\sigma_0^2 = \frac{\hbar}{2\sqrt{mK}} = \frac{\hbar^2}{2m\varepsilon}, \tag{A.3}$$

and ε is the common excitation energy.

If we now include the coupling

$$V = -Cx_p x_n \tag{A.4}$$

between the neutron and the proton, the total Hamiltonian H can be separated into two terms, using the transformation

$$\xi = (x_p + x_n)/\sqrt{2} \quad \text{and} \quad \eta = (x_p - x_n)/\sqrt{2}. \tag{A.5}$$

Thus we obtain

$$H = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial\xi^2} + \frac{1}{2}(K - C)\xi^2 - \frac{\hbar^2}{2m}\frac{\partial^2}{\partial\eta^2} + \frac{1}{2}(K + C)\eta^2, \tag{A.6}$$

and the correlated two-particle ground state density is

$$\rho(\xi,\eta) = (2\pi\sigma(\xi)\sigma(\eta))^{-1}\exp\left(-\frac{\xi^2}{2\sigma^2(\xi)} - \frac{\eta^2}{2\sigma^2(\eta)}\right), \tag{A.7}$$

where

$$\sigma^2(\xi) = \sigma_0^2/\sqrt{1 - C/K} \quad \text{and} \quad \sigma^2(\eta) = \sigma_0^2/\sqrt{1 + C/K} \tag{A.8}$$

The one-particle density ρ_{1p} is also a Gaussian, and it has the standard deviation

$$\sigma^2_{1p} = \sigma^2_0 \frac{1}{2}[1/\sqrt{1-C/K} + 1/\sqrt{1-C/K}] . \qquad (A.9)$$

We note that this density can be obtained from the uncorrelated ground state density by the folding

$$\rho_{1p}(x) = (2\pi\Delta\sigma^2)^{-\frac{1}{2}} \int dx' \exp(-\frac{x'^2}{2\Delta\sigma^2})\rho_0(x-x'), \qquad (A.10)$$

where

$$\Delta\sigma^2 = \sigma^2_{1p} - \sigma^2_0 .$$

We shall now show that all the fluctuations in the correlated ground state can be extracted from the RPA Green's function, and thus the procedure described in Section 2 is in fact exact in the present model.

Without the coupling V, the Green's function is given by

$$G_0(x,x',\omega) = \sum_{i=n,p} \frac{2\varepsilon}{\varepsilon^2-\omega^2} \Psi^*_{i0}(x)\Psi_{i1}(x)\Psi^*_{i1}(x')\Psi_{i0}(x') \quad (A.11)$$

where ε is the common excitation energy, and Ψ_{i0} and Ψ_{i1} are the wave functions for the ground state and the first excited state of particle i, respectively. In the present model it is convenient to separate the RPA Green's function into two terms, consisting of even and odd powers of V, as follows

$$G_{RPA} = G_0(1+VG_0)^{-1} = (G_0 - G_0VG_0)\sum_{m=0}^{\infty}(VG_0VG_0)^m . \qquad (A.12)$$

In this expression the residual interaction $V = -Cx_px_n$ replaces a proton excitation by a neutron excitation, and vice versa, so one can effectively replace VG_0VG_0 by

$$\left[-C\langle p1|x_p|p0\rangle\langle n1|x_n|n0\rangle \frac{2\varepsilon}{\varepsilon^2-\omega^2}\right]^2 = \left(\frac{\varepsilon^2}{\varepsilon^2-\omega^2}\cdot\frac{C}{K}\right)^2 . \qquad (A.13)$$

Thus we obtain the RPA Green's function

$$G_{RPA} = (G_0 - G_0VG_0)\frac{(\varepsilon^2-\omega^2)^2}{(\varepsilon^2-\omega^2)^2 - (\frac{C}{K}\varepsilon^2)^2} , \qquad (A.14)$$

and the poles are seen to be identical to the exact energies of excitations from the correlated ground state.

In Sec. 2 we associate the sum

$$S(f) = \frac{1}{\pi}\,\mathrm{Im}\int_0^{\infty} d\omega \int dx dx' \; f^*(x) G_{RPA}(x,x',\omega) f(x') \qquad (A.15)$$

with the total zero-point fluctuation of the field f in the correlated ground state. We can illustrate this by choosing $f = x_p$. In this case the contributions from odd powers of V in Eq. (A.14) vanish, and we find

$$S(x_p) = 2\varepsilon|\langle p1|x_p|p0\rangle|^2 \frac{1}{\pi}\,\mathrm{Im}\int_0^{\infty} d\omega \frac{\varepsilon^2-\omega^2}{(\varepsilon^2-\omega^2)^2 - (\frac{C}{K}\varepsilon^2)^2}$$

$$= \sigma^2_0 \frac{1}{2}[1/\sqrt{1-C/K} + 1/\sqrt{1+C/K}] , \qquad (A.16)$$

which is identical to the standard deviation of the one-particle density in the correlated ground state (cf. Eq. (A.9)). Note that the change $\Delta S(x_p)$ from the free response is of second order in C, as it must be to represent a correlation effect.

For $f = (x_p + x_n)/2$ or for $f = (x_p - x_n)/2$, one finds similarly that the sum S(f) above gives the exact fluctuations of f in the correlated ground state.

We can easily generalize this model to deal with the spin degrees of freedom. Considering four particles with a Hamiltonian analogous to Eqs. (A.1, A.4), the one particle density is given by the convolution (A.10) with

$$\Delta\sigma^2 = \sigma_0^2 \, \frac{1}{4} \left[\sum_{i=1}^{4} 1/\sqrt{1 - C_i/K} - 4 \right] , \qquad \text{(A.17)}$$

where the C_i are the restoring force coefficients for the four different spin-isospin modes, and $\Sigma C_i = 0$ for Hamiltonians with interactions of the type (A.4). The appropriate response sum will now contain four contributions, and so in the generalization to the many particle case we should add contributions from the spin and spin-isospin response.

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Table I. Calculated values of $\Delta\sigma^2$, defined in Eq. (9), are given for ^{40}Ca and ^{208}Pb. Both the isoscalar and the isovector results are shown, as well as the results for neutrons and protons. The electric zero-point fluctuation, defined in Eq. (4), for the low-lying 3^- state alone is also shown. The numbers (exp. and calc.) are based on the measured B(E3) or β_3-value of Refs. 9 and 10, and the B-values obtained from our calculations, respectively.

$\Delta\sigma^2$ (fm^2)	^{40}Ca	^{208}Pb
isoscalar	0.463	0.189
isovector	-0.075	-0.089
neutrons	0.388	0.096
protons	0.388	0.087
low-lying 3^-:		
calc.	0.184	0.041
exp.	0.203	0.056

Figure Captions

Figure 1 - Contributions to $\Delta\sigma^2$ in Eq. (9) from different multipolarities in ^{208}Pb and ^{40}Ca, both for the isoscalar (●) and the electric (▲) response. The number of configurations included in the calculation of the free response is determined by the size ΔL of the radial interval in which the nucleus is placed, and the energy cutoff in particle-hole excitations E_{max} (cf. Ref. 6). We used $\Delta L = 9.5$ fm and $E_{max} = 40$ MeV for ^{208}Pb, and $\Delta L = 6.9$ fm and $E_{max} = 100$ MeV for ^{40}Ca.

Figure 2 - B(E2) values for quadrupole excitations in ^{208}Pb obtained from the RPA response (full drawn curve) as a function of the excitation energy. The difference between this quantity and the B(E2) value for the free response is also shown (dot-dashed curve).

Figure 3 - Charge density of ^{40}Ca. The full drawn curve is extracted from experiment (Ref. 12). The dashed curve is obtained from our Hartree-Fock density, including corrections for the finite size of the proton as in Ref. 11. From the convolution in Eq. (6) of the latter density, we obtain the density in the correlated ground state (dots) using $\Delta\sigma = 0.62$ fm. The differnce $\Delta\rho_{ch}(r)$ between the two calculated densities is also shown (dot-dashed curve).

Figure 4 - Comparison of calculated changes in the ground state charge density due to the low-lying 3^- state in ^{40}Ca. The dashed curve is from Ref. 3, and the full drawn curve is our result obtained from Eq. (6) with $\Delta\sigma = 0.38$ fm, which is the total fluctuation due to the low-lying 3^- state with $B^{\dagger}(E3) = 1.31\ 10^4\ e^2 fm^6$. Corrections for the finite size of protons were neglected.

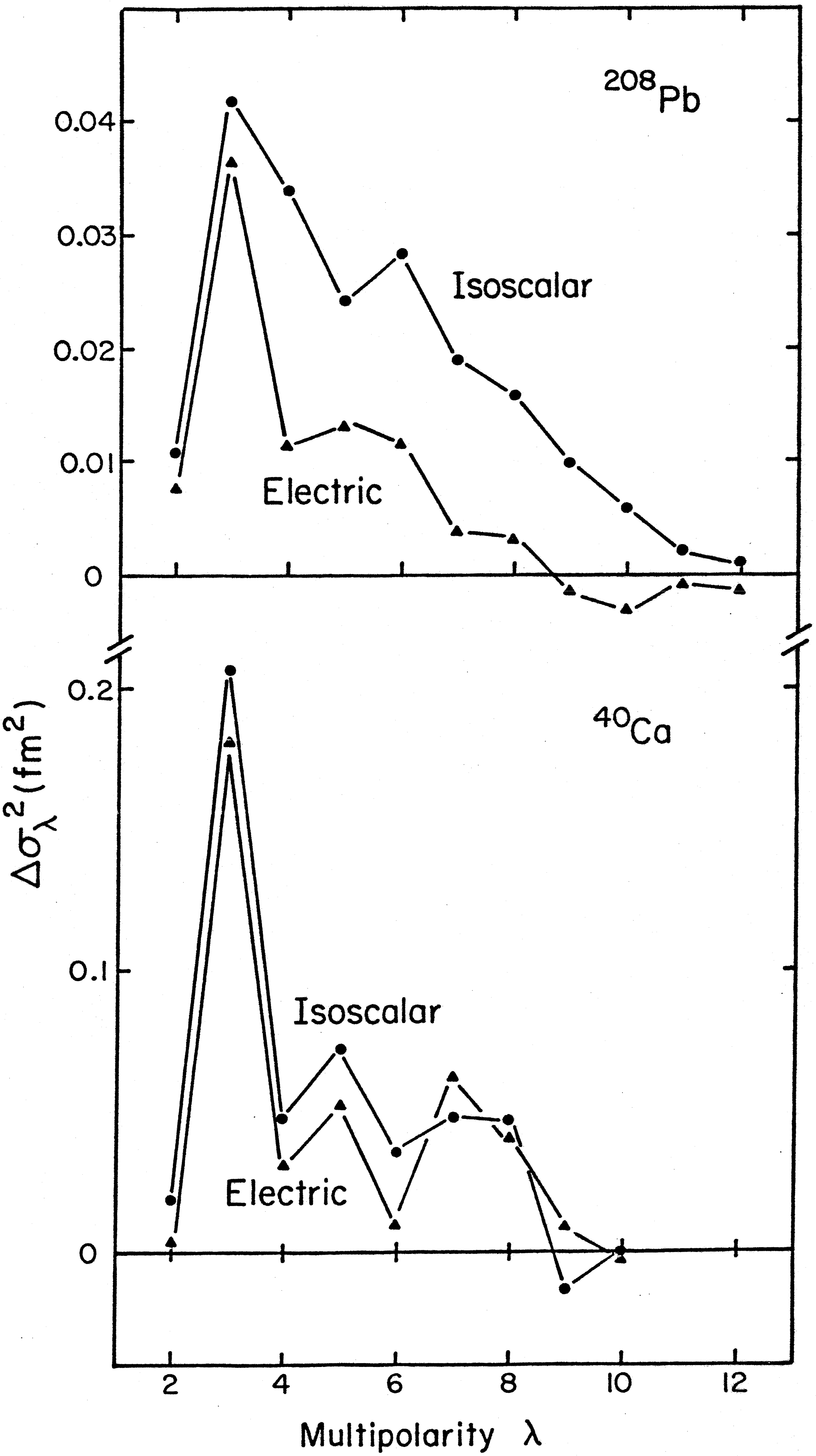

208Pb
0.04
0.03
0.02
0.01
0
Isoscalar
Electric
40Ca
0.2
0.1
0
Isoscalar
Electric
Δσλ2 (fm2)
2
4
6
8
10
12
Multipolarity λ

FIGURE 1

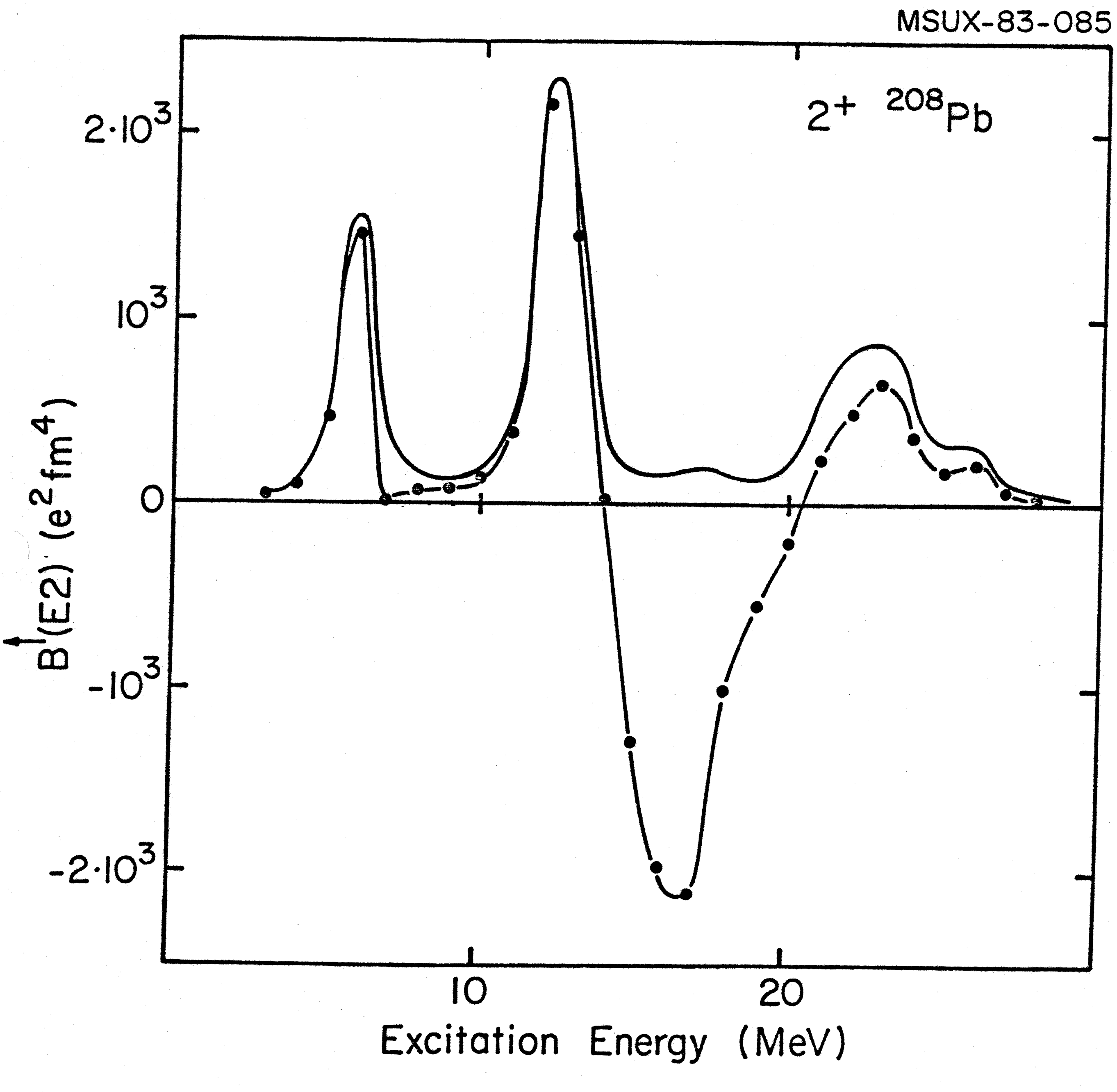
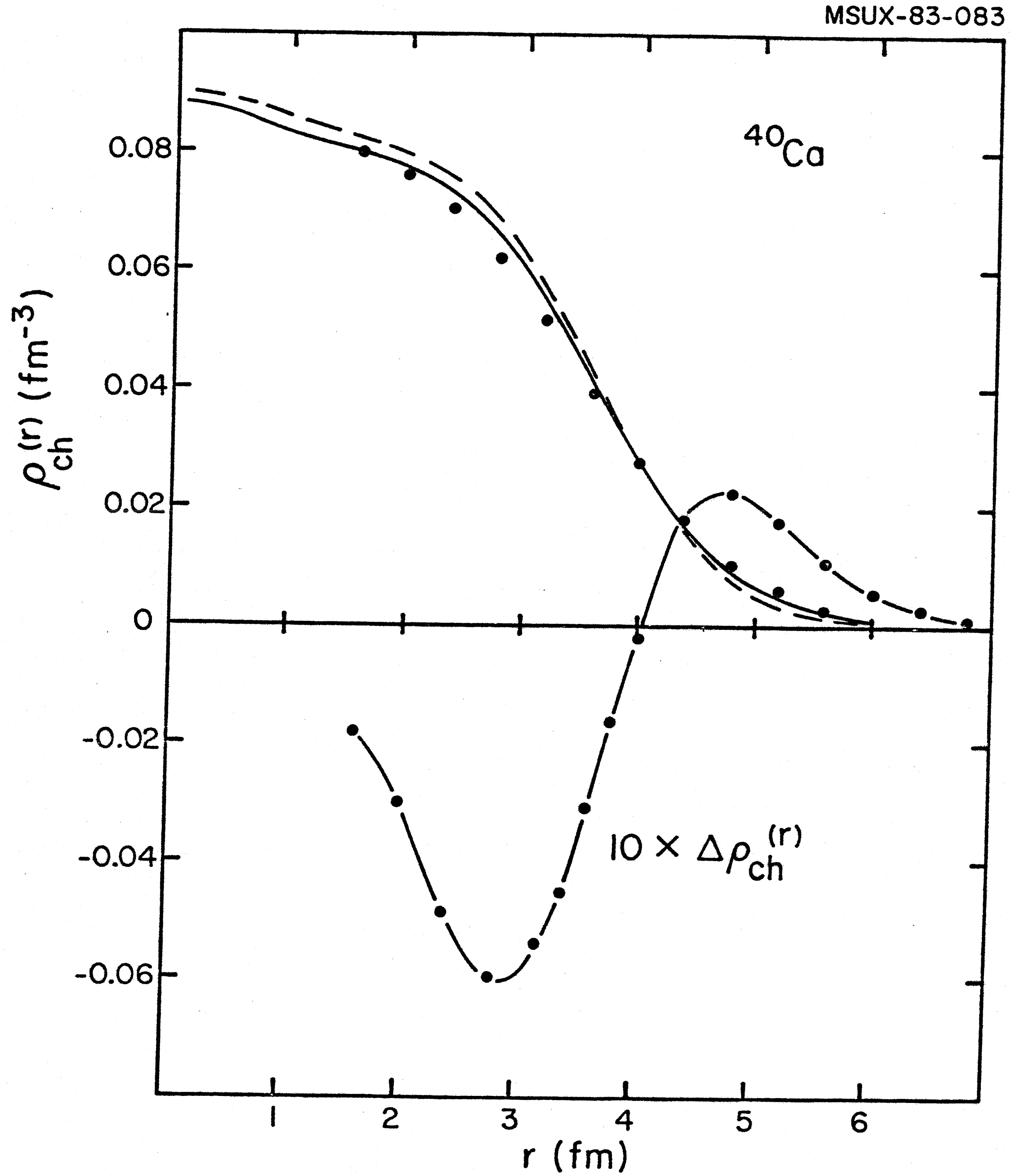

MSUX-83-085
2^+ ^{208}Pb
$2\cdot10^3$
10^3
0
-10^3
$-2\cdot10^3$
$B^{\uparrow}(E2)$ (e^2fm^4)
10
20
Excitation Energy (MeV)

Figure 2



MSUX-83-083
^{40}Ca
0.08
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
$\rho_{ch}(r)$ (fm^{-3})
$10 \times \Delta\rho_{ch}(r)$
1
2
3
4
5
6
r (fm)

Figure 3

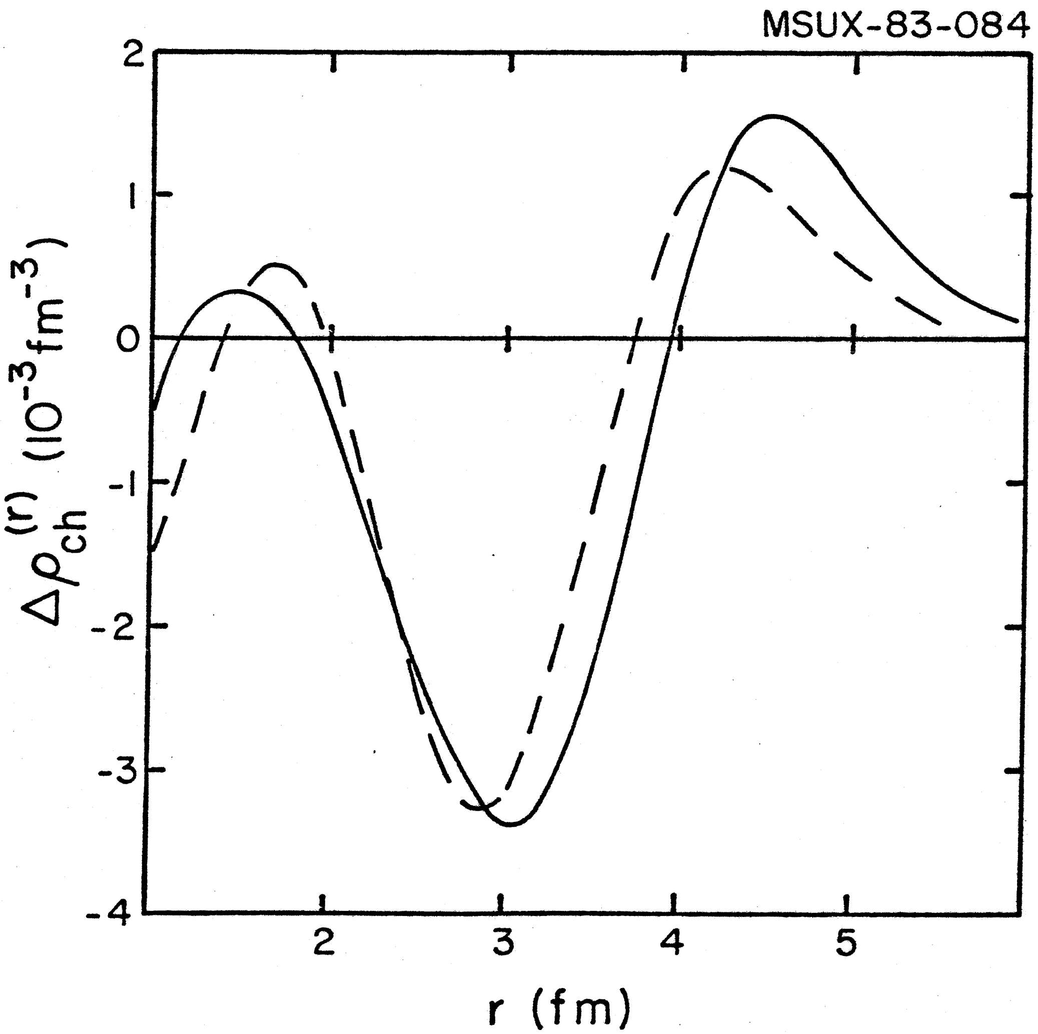


Figure 4