SHORTENING OF THE NUCLEON'S MEAN FREE PATH IN HEAVY ION COLLISIONS

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The mean free path of nucleons in heavy ion collisions is most essential for the development of nuclear collective phenomena. We discuss the effect of the nucleon Fermi motion in nuclei for shortening the mean free path. This Fermi motion together with the prior Pauli effect makes nuclei nontransparent for heavy ions in the medium energy domain.

When the bombarding energy of heavy ion projectiles incident on heavy target nuclei well exceeds the Coulomb barrier ($E/A > 20$ MeV), the nuclei interpenetrate each other. The simplest picture for the dynamics of such heavy ion collisions is based on the assumption of independent nucleon–nucleon scattering taking place in the collisions, which finds its manifestation in the nuclear cascade model [1,2] used to describe the time development of the process. Here, the mean free path λ of the nucleons plays an essential role: if $\lambda \gg d_0$ (the internucleon spacing or the range of the nuclear force), the quasi-free scattering model should be valid. On the other hand, if $\lambda \sim d_0$ and $\lambda \ll R$, where R is the nuclear radius, nuclear matter behaves like a quantum liquid and a hydrodynamical description [3,4] may be applicable. Of particular importance is the mean free path of nucleons in nuclear matter at zero temperature. For low energy nucleons impinging on nuclei λ is large ($\lambda \gtrsim 5$ fm) because of the Pauli blocking effect, which forbids nucleon–nucleon scattering. This is the reasoning often used to argue that nuclei should be transparent to medium energy heavy ions ($E/A \sim 20$–100 MeV/A).

What do the experimental data tell us? Fig. 1 shows the total reaction cross section [5] for $^{12}C + ^{12}C$ and the inclusive cross section [6] of protons produced in asymmetric heavy ion collisions (e.g. Ne + Au) as a function of the bombarding energy. The total reaction cross section closely reflects the energy dependence of the nucleon–nucleon cross section. The dip at medium energy has been attributed to the expected rather large transparency on nuclei at this energy. We want to argue that the dip is solely caused by the peripheral collisions. On the other hand, the cross section of the emitted protons does not exhibit this dip but in fact increases monotonically as a function of E_{lab}. This finding indicates that nuclei are not transparent to each other in the energy range of $E/A \sim 20$–300 MeV/A. The smooth increase of the proton cross section, however, is not the only indication of nontransparency at $E/A \lesssim 300$ MeV/A. A recent analysis [7] of high multiplicity triggered data [8], i.e. central collisions, also indicates the absence of transparency for large nuclear overlaps. A typical data sample [7,8] is shown in fig. 2: the energy distribution of the emitted fragments is plotted as a function of the kinetic energy per nucleon of the ejectile in the laboratory frame for ^{12}C (70–80 MeV/A) + (Ag and Br). The spectrum shows no sign of a projectile remnant peak; i.e. uninteracted nucleons. However, the mean free path of a nucleon calculated with the Pauli blocking effect included in this regime is around 5 fm. Hence a large fraction of the protons should be able to interpenetrate the target nucleus without collision. In fact, cascade model calculations [1] using the standard Pauli blocking prescription [9]

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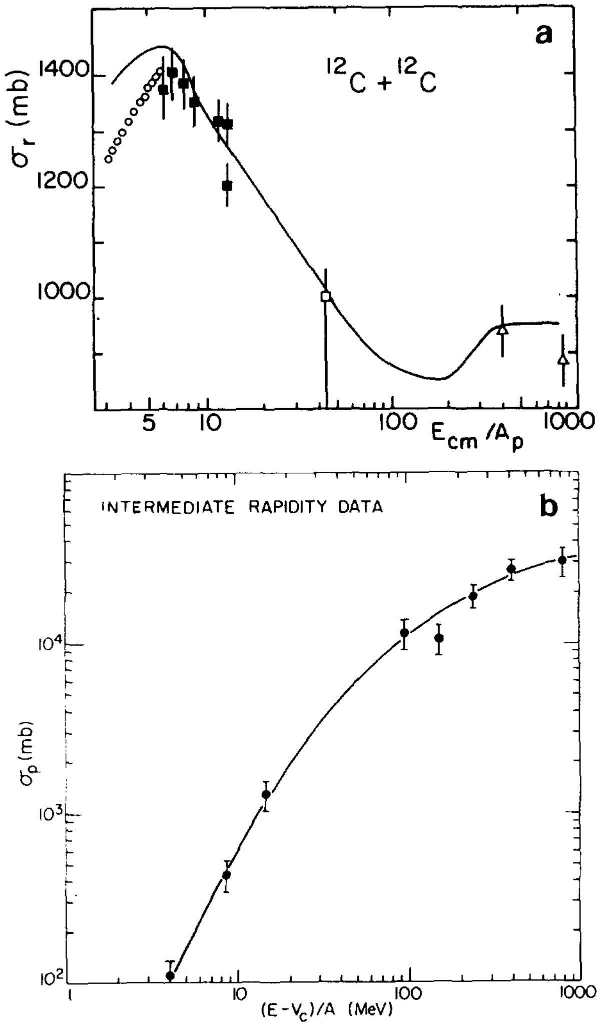

Fig. 1. (a) Total reaction cross section [5] for $^{12}C + ^{12}C$ (b) Inclusive cross section of protons [6] in Ne + Au.

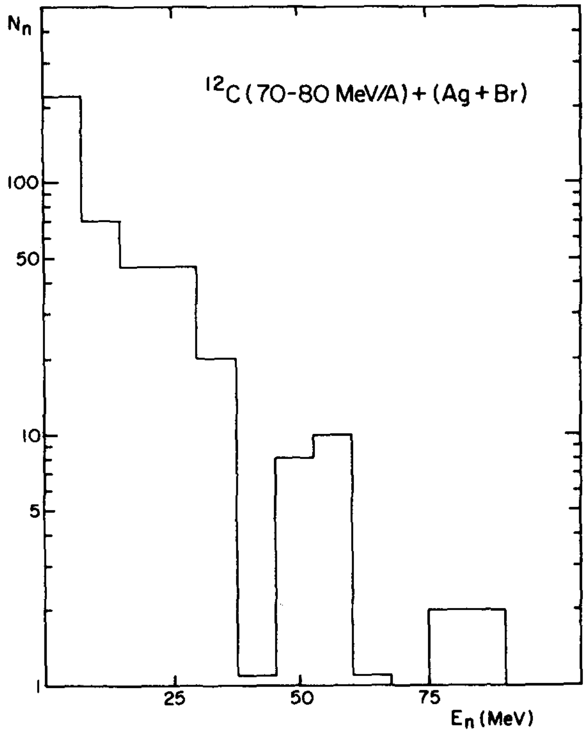

Fig. 2. Energy distribution of the emitted nucleons [8] with the trigger of 20% highest multiplicity (143 prongs) in ^{12}C(70–80 MeV/A) + (Ag and Br).

result in a peak of uninteracted nucleons at the beam energy per nucleon [7].

How can we understand this discrepancy? The question one has to ask is how reasonable is the use of the nucleon mean free path with the Pauli effect in the above consideration. Nucleons move randomly in accord with their Fermi momenta and bounce back and forth within the potential boundary. If a nucleon enters into this nuclear system with a cross section σ, would this random Fermi motion change the number of collisions while it goes through the nucleus?

The number of collisions is proportional to the volume spanned by the cross section σ and the length of the incoming particle motion through the nuclear medium. Writing the velocity of the incoming particle as $\boldsymbol{v}_i$, the volume change dV_0 within small time change dt is written by

$$dV_0 = \sigma|\boldsymbol{v}_i|\, dt. \tag{1}$$

The entire volume is then

$$V_0 = \int \sigma|\boldsymbol{v}_i| dt \equiv \sigma L, \tag{2}$$

where $\int|\boldsymbol{v}_i|dt \equiv L$ = the length of the particle motion through the nuclear medium. When, on the other hand, the nucleons inside the nucleus are moving uniformly with velocity $\boldsymbol{v}_f$, the effective volume change dV for getting new particles becomes

$$dV = \sigma|\boldsymbol{v}_i - \boldsymbol{v}_f| dt. \tag{3}$$

Hence the number of collisions gets modification according to the effective volume

$$V = \int \sigma |\boldsymbol{v}_i - \boldsymbol{v}_f| dt = \sigma L |\boldsymbol{v}_i - \boldsymbol{v}_f| / |\boldsymbol{v}_i|. \tag{4}$$

Now, the velocity of nucleons $\boldsymbol{v}_f$ is not unique and is randomly distributed within the Fermi velocity

$$|\boldsymbol{v}_f| = v_f \leqslant v_f. \tag{5}$$

We shall, therefore, take the average of the effective volume

$$\overline{V} = \int V d^3 V_f \Big/ \int d^3 V_f.$$

We can work out this integral easily and find

$$R \equiv \overline{V}/V = 1 + \tfrac{1}{5}(\boldsymbol{v}_F/\boldsymbol{v}_i)^2 \tag{6}$$

for $v_F < v_i$, which is the case we are interested in. If $v_F \gg v_i$, then this ratio approaches v_F/v_i.

In addition to the effect of the Fermi motion of the target nucleons, we have to consider the same effect of the projectile nucleons for the heavy ion collision case. It will become complicated integrals to work out. Since the modification is a fraction for the actual case, we shall assume that the effect is the same and simply double the change:

$$R = 1 + \tfrac{2}{5}(v_F/v_i)^2. \tag{7}$$

Hence in the range we are considering, $v_i = v_F \sim 2v_F$ (remember the relativistic effect) and the number of collisions would increase by 10%–40% for the heavy ion collisions. If expressed in terms of the mean free path λ, which is reciprocally proportional to the number of collisions, we find

$$\lambda_{\text{eff}} = [1 + \tfrac{2}{5}(v_F/v_i)^2]^{-1}\lambda. \tag{8}$$

We have checked the above expressions by explicitly performing cascade model calculations in two dimensions.

This shortening effect for the mean free path is not taken into account in the Cugnon cascade code [1], where the nuclear surfaces are not constructed and nucleons move out freely. In a modified version, this undesired aspect is removed by simply freezing the initial nucleon's motions (Cugnon(frozen)) unit they meet with the incoming particles. This modified version also does not take the above mentioned shortening effect into account. Kitazoe et al. [18], on the other hand, have constructed a new cascade program, in which they have tried to consider most of the realistic aspects in some way. In particular, nuclear surfaces are constructed, which are able to confine nucleons within the boundary. Hence, the above discussed shortening mean free path effect is automatically included in the Kitazoe code. In this respect, fig. 5 in ref. [10] is instructive. They show the number of particles, which have made collisions, as a function of the impact parameter. Cugnon's results are always smaller than the geometrical ones, which correspond to the ones of the frozen version of the Cugnon program. Kitazoe's results are always larger than the geometrical ones.

For incident energies below $E_P < 40$ MeV, however, the Fermi motion effect alone is not sufficient for nuclei to be nontransparent. However, we want to recall the Pauli blocking effect: it can be stated as: "particles cannot enter a cell in phase space, which is already occupied by another like-particle". Above, we have considered the Pauli effect for nucleons after two-body collisions only, which makes λ (Pauli) in nuclear matter much larger than λ (free). This is what we want to call post Pauli blocking. We have neglected another important role of the Pauli effect, which is particularly important for heavy ion collisions. The Pauli effect must also be considered prior to any nucleon–nucleon scattering for nucleons which are about to enter the other nucleus. Nucleons with momenta smaller than the Fermi momentum P_F of the other nucleus are not allowed to enter this nucleus, hence they must either scatter back into their original nucleus (in configuration space) or they must scatter to high lying states in momentum space by a collective repulsion from the nuclear field – this can be understood as squeezing the wave functions into higher orbits [11] giving rise to some contribution to the phenomenological compression potential. This is what we call the prior Pauli effect – to be distinguished from the post Pauli effect. Only those nucleons with momenta exceeding P_F, which corresponds to $E_F = 40$ MeV, are allowed to remain in their state upon entering the target (projectile). The average energy of the nucleons which are allowed to enter the target is therefore higher than the laboratory energy. It is the combination of the Fermi motion and the prior Pauli effect which make nuclei nontransparent to heavy ion projectiles even at medium energy.

We would like to point out that this prior Pauli effect corresponds to the Pauli hard core commonly discussed quantum mechanically in the α-cluster mode [12]. When the incident energy is very low, the projectile Fermi sphere overlaps almost completely with that of the target and hence the projectile is pushed back by the target. As the incident energy increases, the number of repelled nucleons gets smaller and the repulsive effect decreases, and so does the Pauli hard core [12]. It is also important to note that the prior Pauli effect plays no role in nucleon–nucleus encounters, since the incoming nucleon has a large momentum (energy) when measured from the bottom of the target potential well.

Finally, we want to mention that for those nucleons which enter the other nucleus after the particles which entered before, have already experienced several collisions; the Pauli blocking is weakened, because the momenta are randomized and the Fermi sphere is depleted to some extent. Hence, the mean free path for the second generation nucleons is even shorter than that for the first generation of particles. Therefore the number of collisions that occur increases. This consideration indicates that after a few collisions the system consists of intermingled projectile and target nucleons with randomized momenta, which may be describable by means of a thermodynamical statistical approach, e.g. a fluid dynamical description.

In conclusion, we have demonstrated that medium and heavy nuclei ($A > 50$) are nontransparent to heavy ion projectiles in the intermediate energy region. This is due to the nucleon Fermi motion and due to the prior Pauli blocking of the target nucleus.

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