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SUBBARRIER FUSION AND DYNAMICAL DEFORMATIONS

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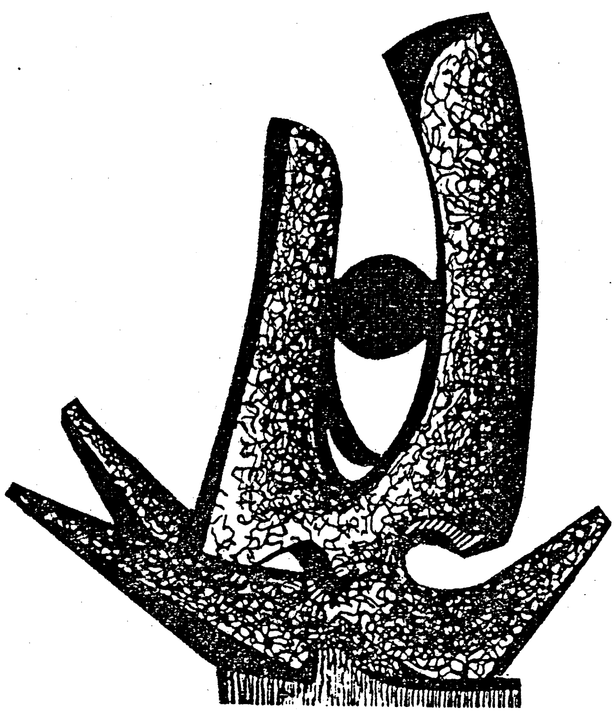

MAY 1983

MSUCL-414

MSUCL-414

Subbarrier Fusion and Dynamical Deformations

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Abstract. The effect of dynamic deformation degrees of freedom in subbarrier fusion is calculated numerically in the coupled channel representation to test some simpler approximations. In typical situations, about 10 to 15 channels are needed in each degree of freedom. We find that the frozen approximation is quite accurate for low-lying collective vibrations. Perturbation theory based on the eikonal approximation is remarkably accurate for excitation energies extending to the giant resonance frequencies. Giant resonances can give a significant contribution to the enhancement of subbarrier fusion.

1. Introduction.

Subbarrier fusion depends on energy and projectile/target combination in a way that cannot be explained by standard one-dimensional barrier penetration models[1-3]). The internal degrees of freedom of the colliding system evidently play an important role at the large separations associated with subbarrier fusion. Internal degrees of freedom which have been considered include static deformations[4]), zero-point motions of collective surface vibrations[5]), and dynamical deformations at the classical turning point[6]). Other processes such as transfer induced fusion, and neck formation[3,6]) have been suggested, but no quantitative calculations have yet appeared. It is possible to fit the fusion data with phenomenological optical potentials[7]), but this does not further the understanding of the dynamics of subbarrier fusion.

Other fusion reactions, with static deformed nuclei, seems to be well understood in terms of an orientation dependence of the fusion process [see e.g. ref.[8,9])]. The model for the enhancement of subbarrier fusion cross sections due to zero-point motions of collective surface vibrations studied in ref. [5]) is a natural extension of this treatment, averaging instead over all nuclear shapes compatible with the vibrational ground states of the interacting nuclei. However, this procedure is only justified for small one-phonon excitation energies $\hbar\omega$.

In the following we investigate the limitations of this model by considering the effect of finite excitation frequencies of surface modes. We study a two-dimensional barrier penetration problem, one dimension being the radial separation of colliding nuclei and the other a surface degree of freedom. We solve the problem using a coupled channel treatment and check the results in the limits $\hbar\omega = 0$ and $\hbar\omega = \infty$, where the exact results are known. Simple estimates of the effect of zero-point motions on fusion are discussed in section 4. A perturbation expansion based on the eikonal approximation is presented in section 5.

2. Coupled Channel Treatment.

The coupled channel formalism for barrier penetration has been used in connection with fission[10]) to test various semiclassical methods. We shall adopt a similar treatment for fusion and study the dynamical effects associated with the excitation of collective surface vibrations in a two-dimensional model. The relative motion of two ions, with reduced mass M, is governed by the Coulomb and nuclear interaction. We consider only head-on collisions ($\ell=0$) and assume that the nuclear interaction U_N depends on the surface-surface distance $r-R_1-R_2-s$ between the ions, which leads to a coupling of the relative motion to a surface mode with amplitude s. The total Hamiltonian is

$$H = -\frac{\hbar^2}{2M}\frac{\partial^2}{\partial r^2} + V(r,s) + H_{osc}, \tag{1}$$

where the total interaction is

$$V(r,s) = \frac{Z_1 Z_2 e^2}{r} + U_N(r-s), \tag{2}$$

and the intrinsic Hamiltonian for the surface vibration is

$$H_{osc} = -\frac{\hbar^2}{2D}\frac{\partial^2}{\partial s^2} + \frac{1}{2} D\omega^2 s^2. \tag{3}$$

Here D is the mass parameter and ω is the frequency of the surface mode.

The nuclear interaction is expressed in terms of the complementary error function

$$U_N(r-s) = -\frac{U_0}{2}\frac{R_1 R_2}{R_1+R_2}\,\mathrm{Erfc}\left(\frac{r-s-R_1-R_2-\Delta R}{a}\right), \tag{4}$$

where

$R_i = 1.233\, A_i^{1/3} - 0.98\, A_i^{-1/3}$, $\Delta R = 0.29$ fm,

$U_0 = 31.67$ MeV fm, and $a = 1.42$ fm.

We have chosen this parametrization because the matrix elements with the harmonic oscillator wavefunctions (eigenfunctions to eq. (3)) can be determined analytically through recursion relations. The parameters are chosen so that the maximum nuclear attraction is the same as that used in ref.[5]), and the two interactions will therefore give essentially identical fusion cross sections.

To solve the two-dimensional Schrödinger equation

$$H\psi(r,s) = (E + \frac{1}{2}\hbar\omega)\psi(r,s), \tag{5}$$

where E is the center of mass energy in the relative motion, we expand the wavefunction on a finite set of oscillator eigenfunctions $\phi_n(s)$

$$\psi(r,s) = \sum_n \psi_n(r)\phi_n(s), \tag{6}$$

and obtain the coupled equations

$$\{-\frac{\hbar^2}{2M}\frac{\partial^2}{\partial r^2} + \langle n|V|n\rangle - E + n\hbar\omega\}\psi_n(r)$$

$$= \sum_{m\neq n} \langle n|V|m\rangle\psi_m(r). \tag{7}$$

The matrix elements are

$$\langle n|V|m\rangle = \int ds\phi_n(s)V(r,s)\phi_m(s). \tag{8}$$

The boundary conditions are ingoing waves at a certain center of mass distance r_{min} near the minimum of the total interaction, - or decaying states, when $E - n\hbar\omega$ is less than the total interaction at r_{min}, and Coulomb wavefunctions at $r=r_{max}$ somewhat outside the barrier, where the nuclear field vanishes, i.e.

$$\psi_n(r) \rightarrow \begin{cases} T_n \exp(-ik_n(\min)r); \text{ for } r \leq r_{min} \\ \delta_{no}\psi^{(-)}_{c,n}(r) + R_n\psi^{(+)}_{c,n}(r); \text{ for } r \geq r_{max}. \end{cases} \tag{9}$$

Here $\hbar k_n(\min)$ is the radial momentum at r_{min} in the n'th

channel and $\psi^{(\pm)}_{c,n}(r)$ are the outgoing and ingoing Coulomb wavefunctions in the different channels with an energy of $E - n\hbar\omega$ in the relative motion. There is only an ingoing Coulomb wavefunction in the elastic channel. The fusion probability P_{fus} is the sum over all channels of the relative flux at r_{min} in each channel

$$P_{fus} = \sum_n |T_n|^2 \hbar k_n(\min)/\sqrt{2ME}\ . \tag{10}$$

To achieve a solution with these boundary conditions we use the method described in ref.[10]), which involves the inversion of a matrix. We also tried a perturbation expansion in the off-diagonal matrix elements, using the Green's functions method, but for realistic cases this procedure failed due to the rather strong coupling between different channels.

3. Exact solvable limits.

It is convenient to characterize the surface vibration in terms of the frequency ω and the standard deviation of the zero-point motion amplitude $\sigma = (\hbar/2D\omega)^{½}$. We can check the numerical results of the coupled channel calculations in two cases. In the sudden limit, i.e. for $\omega = 0$ and a finite σ, one can neglect the intrinsic oscillator Hamiltonian in the total Hamiltonian (1) and thus solve eq. (5) for frozen values of s. This leads to a solution of the form

$$\psi(r,s) = \psi_S(r,s)\phi_0(s), \tag{11}$$

where $\psi_S(r,s)$ is the solution to the equation

$$\{-\frac{\hbar^2}{2M}\frac{\partial^2}{\partial r^2} + V(r,s)\}\psi_S = E\psi_S , \tag{12}$$

which satisfies the boundary conditions

$$\psi_S(r,s) \rightarrow \begin{cases} T(s)\exp(-ik(s)r); & r \leq r_{min} \\ \psi^{(-)}_{c,o}(r) + R(s)\psi^{(+)}_{c,o}(r); & r \geq r_{max} . \end{cases} \tag{13}$$

Here $\hbar k(s)$ is the momentum in the relative motion at r_{min}, and it depends on s. The total fusion probability is therefore

$$P_{fus} = \int ds(\phi_0(s))^2 \, |T(s)|^2 \hbar k(s)/\sqrt{2ME}. \tag{14}$$

This is essentially the procedure used in ref.[5]), except that the transmission coefficients used there were obtained from the generalized WKB method.[11])

The usual adiabatic limit, where the oscillator is assumed to be in the local ground state during the fusion process, is not useful in the domain of realistic ω values, because the adiabatic trajectory has a discontinuity in the (r,s) plane with the average s-coordinate of the local ground state jumping from one minimum in the potential to another. However, for sufficiently large values of ω the

adiabatic approximation is defined, and we shall make use of the extreme adiabatic limit (ω being very large) in which case the trajectory goes through the barrier in the elastic channel. Then the wavefunction satisfies $\psi(r,s) = \psi_{ad}(r)\,\phi_0(s)$, where $\psi_{ad}(r)$ is a solution to the equation

$$\{-\frac{\hbar^2}{2M}\frac{\partial^2}{\partial r^2} + \langle 0|V(r,s)|0\rangle\}\psi_{ad}(r) = E\psi_{ad}(r). \tag{15}$$

4. Simple formulas for $\omega = 0$.

In this section we give some simple formulas that can be used to estimate the effect of zero-point motions on subbarrier fusion cross sections in the limit $\omega = 0$. Including a centrifugal potential in the total interaction we can determine barrier penetration factors from the Hill-Wheeler formula[12])

$$P_\ell(s) = \left[1 + \exp(2\pi\frac{V_B(\ell,s)-E}{h\omega_0(\ell,s)})\right]^{-1} . \tag{16}$$

These penetration factors depend on the angular momentum ℓ and the amplitude s of a surface mode through the height of the barrier $V_B(\ell,s)$, and through the quantity $\omega_0(\ell,s)$, which is obtained from a parabolic approximation to the shape of the barrier and is given by

$$\omega_0(\ell,s) = \left[\frac{1}{M}\left|\frac{\partial^2 V(r,\ell,s)}{\partial r^2}\right|_{r=R_B}\right]^{1/2} . \tag{17}$$

The main dependence on s occurs through the height of the barrier. Following the derivation in ref. [8]) one finds (with certain approximations valid for energies near and below the Coulomb barrier) that the s-dependent fusion cross section is given by

$$\sigma_{fus}(s) = \pi R_{CB}^2 \frac{\varepsilon_0}{E} \log\left(1+\exp\left(\frac{E-V_{CB}(s)}{\varepsilon_0}\right)\right), \qquad (18)$$

where R_{CB} is the center of mass distance at the Coulomb barrier for $\ell=0$ and $s=0$, and

$$\varepsilon_0 = \hbar\omega_0(\ell=0,s=0)/2\pi. \qquad (19)$$

The enhancement of subbarrier fusion cross sections due to zero-point motions of surface modes is therefore essentially due to fluctuations in the height $V_{CB}(s)$ of the Coulomb barrier. Similar to eq. (14) we can determine the average fusion cross section from the equation

$$\langle\sigma_{fus}\rangle = (2\pi\sigma_\lambda^2)^{-\frac{1}{2}} \int ds\ \exp\left(-\frac{s^2}{2\sigma_\lambda^2}\right)\sigma_{fus}(s), \qquad (20)$$

where σ_λ is the standard deviation of surface fluctuations due to a collective vibration of multipolarity λ .

We can simplify these expressions further by expanding $V_{CB}(s)$ to first order in s. For the interaction used in eq. (2) one finds that

$$\left.\frac{\partial V_{CB}}{\partial s}\right|_{s=0} = \frac{\partial U_N}{\partial s} = -\frac{\partial U_N}{\partial r} = -\frac{Z_1 Z_2 e^2}{R_{CB}^2} \qquad (21)$$

at the Coulomb barrier. In general the nuclear force on a deformation degree of freedom may differ from the force on the relative motion, as e.g. in the proximity description[13]). Moreover, there will also be a contribution from the Coulomb monopole-multipole interaction, which for head - on collisions depends on the multipolarity λ and the amplitude $s_{i,\lambda}$ of the surface mode as follows

$$V_{i,\lambda}^{(Coul)}(r,s_{i,\lambda}) = \frac{Z_1 Z_2 e^2}{r^2}\frac{3}{2\lambda+1}\left(\frac{R_i}{r}\right)^{\lambda-1} s_{i,\lambda}\ . \qquad (22)$$

Instead of eq. (21) one should therefore use the expression

$$\frac{\partial V_{CB}}{\partial s_{i,\lambda}} = -\frac{Z_1 Z_2 e^2}{R_{CB}^2} f_{i,\lambda}\ , \qquad (23)$$

where

$$f_{i,\lambda} = -\frac{\partial U_N}{\partial s_{i,\lambda}} \Big/ \frac{\partial U_N}{\partial r} - \frac{3}{2\lambda+1}\left(\frac{R_i}{R_{CB}}\right)^{\lambda-1} \qquad (23')$$

is evaluated at the Coulomb barrier.

Far below the Coulomb barrier the fusion cross section is seen to have an exponential dependence on the center of mass energy. In this limit one finds that

$$\langle\sigma_{fus}\rangle = \sigma_{fus}(s=0)\exp\left(\frac{1}{2}\left(\frac{Z_1 Z_2 e^2}{\varepsilon_0 R_{CB}^2} f_{i,\lambda}\,\sigma_\lambda\right)^2\right). \qquad (24)$$

The enhancement factor can be quite large. It is more convenient to express the effect in terms of a modified effective Coulomb barrier, viz.

$$(V_{CB})_{eff} = V_{CB}(s{=}0) - \frac{1}{2\varepsilon_0}\left(\frac{Z_1 Z_2 e^2}{R_{CB}^2} f_{i,\lambda}\sigma_\lambda\right)^2 . \qquad (25)$$

When several surface modes are important, their contributions to the reduction of the effective Coulomb barrier should be added.

From these estimates one can see under what circumstances one would expect a large effect of zero-point motions on subbarrier fusion. The quantity ε_0, defined in eqs. (19) and (17), is rather insensitive to the projectile/target combination. The dominant factor on the relative change of the effective Coulomb barrier is the Coulomb force at the Coulomb barrier. The factor $f_{i,\lambda}$ defined in eq. (23) can be estimated, for example, from the proximity description[13]). Some examples of typical values are given in table I. They were obtained by integrating the proximity force and the forces on shape degrees of freedom over the area of the crevice between spherical nuclei. The factor $f_{i,\lambda}$ is generally largest for a light ion on a heavy target, the heavy ion containing the surface mode. In the example studied in ref.[6]), and for the interaction used there, the value of $f_{i,\lambda}$ was about 0.46. The effect of zero-point motions was therefore found to be small, and it was almost canceled by the Coulomb polarization of the quadrupole state that was studied. For more asymmetric collisions, such as the ^{16}O + Sm reactions studied in ref.[1,5]), the Coulomb polarization has a small effect on the subbarrier enhancement.

5. Finite ω.

We have simple and exact results for $\omega = 0$ and $\omega = \infty$, and it would be nice to understand how the penetrability at finite ω interpolates between these two limits.

For small values of ω we can treat the intrinsic Hamiltonian of the oscillator as a perturbation that acts on the solution in the sudden limit (ω =0). To pursue this idea it is convenient first to separate the total wavefunction $\psi(r,s) = \psi_S(r,s)\ \phi_0(s)$ as in eq. (11), and to rewrite the Schrödinger equation in terms of $\psi_S(r,s)$ as follows

$$\{-\frac{\hbar^2}{2M}\frac{\partial^2}{\partial r^2} + V(r,s) + \mathcal{V}\}\psi_S = E\psi_S, \qquad (26)$$

where

$$\phi_0 \mathcal{V}\psi_S = (H_0 - \frac{1}{2}\hbar\omega)(\psi_S\phi_0)$$
$$= -\phi_0\hbar\omega\sigma^2(\frac{\partial^2}{\partial s^2} - \frac{s}{\sigma^2}\frac{\partial}{\partial s})\psi_S. \qquad (27)$$

The oscillator mass has here been expressed by ω and the quantity σ defined in section 3.

We will estimate the effect of the operator $\mathcal{V}$ by evaluating it on the eikonal approximation to ψ_S. We shall only consider the case of subbarrier fusion and neglect the effect of excitations outside the barrier. In the classically forbidden region the wavefunction is

$$\psi_{eik} \sim \exp(-\Phi(r,s)), \qquad (28)$$

where

$$\Phi(r,s) = \int_{r}^{r_0} dr\sqrt{2M(V(r,s)-E)}/\hbar, \qquad (29)$$

r_0 being the outer turning point. We may then evaluate the derivatives in eq. (27). In the spirit of the eikonal approximation, we shall drop the second derivative term $\partial^2\Phi/\partial s^2$. This term produces a divergence at the classical turning points, just as in the ordinary WKB approximation consideration of $\partial^2\Phi/\partial r^2$ leads to a prefactor on the eikonal which diverges at turning points. We then find that the operator V is equivalent to the function

$$V_{eik} = -\hbar\omega\sigma^2\left(\frac{\partial\Phi}{\partial s}\right)^2 - \hbar\omega s\,\frac{\partial\Phi}{\partial s}, \qquad (30)$$

which we can include in the definition of an effective action integral

$$\Phi_{eff}(s) = \int_{r_i}^{r_0} dr\sqrt{2M(V(r,s) + V_{eik} - E}/\hbar\ . \qquad (31)$$

The limits of integration are the turning points. As in the generalized WKB approximation[11] we can now construct a penetrability; weighting it with the ground state distribution of s we obtain the average fusion probability

$$P_{fus} = \int ds(\phi_0(s))^2\,(1 + \exp(2\Phi_{eff}(s)))^{-1}. \qquad (32)$$

It is instructive to evaluate this expression with more simplifying assumptions. Let us assume that the barrier is

rectangular, that the force $F = -\,\partial V/\partial s$ in the s-direction is constant, and that the amplitude of zero-point motion is small. For energies far below the barrier the integrals can then be evaluated to obtain the result

$$P_{fus} = P_{fus}^{(0)}\exp\left(2\left(\frac{F\sigma\tau}{\hbar}\right)^2\left(1 - \frac{2\omega\tau}{3}\right)\right)\ . \qquad (33)$$

Here τ is the "time" that the particle spends under the barrier, defined as $\tau = ML/\sqrt{2M(V-E)}$, where L is the length of the barrier. We see that the correction due to a finite ω depends exponentially on the frequency, with a coefficient that depends quadratically on the force F, and cubically on the "time" τ.

Eq. (33) has the correct functional dependence on the parameters of the system, but unfortunately is not quantitatively accurate for our purpose. This is due to the fact that the momentum transfer to the oscillator, represented by $\partial\Phi/\partial s$, is essentially accumulated in a region near the inner turning point, where $\partial V/\partial s$ is largest. Thus the effect of V is poorly represented by its value in the middle of the barrier. We shall therefore use the expressions in eqs. (29-32) when comparing with the coupled channel results discussed in the next section.

In the limit of high frequencies, the adiabatic approximation becomes applicable. This yields a penetrability whose ω-dependence arises from the dependence of the adiabatic path on ω. We find that the deviation from the extreme adiabatic limit behaves as $1/\omega^2$. From the numerical calculations, discussed in the next section, this behaviour applies to $\hbar\omega > 10$ MeV.

6. Coupled Channel Results.

We study the reaction $^{16}O + ^{148}Sm$ for head-on collisions ($\ell=0$), in order to see how much the results obtained in ref.[5]) are affected by excitations during the fusion process due to a finite value of the one-phonon excitation energy $\hbar\omega$.

The fusion probability is shown in fig. 1 as function of the center of mass energy. Both the extreme adiabatic and the sudden limit are shown together with the results for $\hbar\omega$ = 1 MeV and 10 MeV. The value of σ was 0.27 fm, which is close to the value obtained both for the low-lying quadrupole and octupole states in ^{148}Sm. Since the excitaion energy $\hbar\omega$ for these states are 0.55 MeV and 1.16 MeV, respectively, we can conclude that the dynamical effect due to the finite value of $\hbar\omega$ will not lead to a major reduction of the subbarrier fusion cross sections obtained in ref.[5]).

In fig. 2A we show in more detail the dependence on the values of σ and $\hbar\omega$, at a center of mass energy of 57 MeV. The results are seen to decrease smoothly with increasing values of $\hbar\omega$ from the sudden limit towards the adiabatic limit, where the effect of zero-point fluctuations only enters through the average potential in the elastic channel. The coupled channel calculations were checked against the exact results for $\hbar\omega = 0$, eqs. (12-14). To achieve

agreement we should include about 10-15 channels in the coupled channel calculations. According to eq. (24) one would expect that the $\log(P_{fus})$ should be a straight line for $\hbar\omega = 0$. The deviations from this dependence in fig. 2 are mainly due to the fact that for large values of σ there are contributions to the the fusion probability from large values of s for which the exponential approximation to the Hill-Wheeler formula is not very good.

For comparison we show in fig. 2B the results obtained from the eikonal approximation, eqs. (29-32). The agreement is remarkably good for small values of $\hbar\omega$. Some deviations from the coupled channel results are seen for $\hbar\omega$ = 10 MeV and large values of σ. For even larger values of $\hbar\omega$ the eikonal approximation fails more dramatically.

For realistic values of the parameters the numerical coupled channel calculations can be quite difficult. The reason for this is seen in a comparison with the exact results for $\hbar\omega = 0$. The location of the maximum of the density $|\psi(r,s)|^2$ in the s-direction is shifted inside the barrier due to the strong s-dependence of the penetration factor. The shift s_0 can be estimated from the Hill-Wheeler approximation used in section 4, and for energies far below the barrier one finds that

$$s_0 \cong \frac{Z_1 Z_2 e^2}{\epsilon_0 R_{CB}^2} \sigma^2 . \tag{34}$$

This is illustrated in fig. 3, where some contours of equal density are shown. They were obtained from the exact treatment for $\hbar\omega = 0$, i.e. from eqs. (11-13). The shaded area inside the dashed curve is the classically forbidden region. In order to reproduce this figure in detail from a coupled channel calculation one would have to include many more channels than mentioned above.

For heavy systems, where the shift given in eq. (34) and thus the enhancement factor in eq. (24) can be much larger than in the present example, more serious numerical difficulties can arise, in particular when many decaying states become important [c.f. ref.[10])]. The numerical treatment of these states requires a high precission, since the wavefunctions of decaying states varies over many orders of magnitude inside the barrier. Such problems already occur in the reaction $^{16}O + ^{148}Sm$ for $\hbar\omega \cong 4 - 8$ MeV. Thus the results in fig. 2A are somewhat uncertain for $\hbar\omega = 3$ MeV and large values of σ.

7. Conclusion.

We have studied subbarrier fusion and the dynamical effect associated with the nuclear excitation of collective surface vibrations during the fusion process. Previous calculations[5]) that neglect these excitations but include the effect of ground state fluctuations of collective vibrations, showed a large enhancement of subbarrier fusion cross sections compared to standard one-dimensional barrier penetration models. Our present results show that nuclear excitations during the barrier penetration only lead to a

minor reduction of these results, when the collective state is low-lying (say $\hbar\omega \lesssim 1$ MeV). The reduction is much larger for high-lying states and it tends to wash out the effect of ground state fluctuations. Some net enhancement, however, still persists in the giant resonance region. Coulomb excitations, which we have neglected here, also reduce the subbarrier enhancement.

From estimates of the proximity forces acting on surface modes we conclude that the most significant effect of zero-point motions on subbarrier fusion should be seen in asymmetric collisions, where the heavy reaction partner contains very collective low-lying surface modes. In this case the effect of long range Coulomb polarizations will be relatively unimportant.

We used both a coupled channel treatment and a perturbation expansion based on the eikonal approximation. A key parameter is the product of the zero-point fluctuation of the nuclear potential, $F\sigma$ in our notation, and the barrier tunnelling time τ, which in turn relates directly to the steepness of the penetrability as function of energy. When this parameter is large, as is the case in realistic situations of interest, the enhancement is large but also the coupled channel calculations become difficult. Also, the perturbation expansion for the ω-dependence is proportional to this parameter. However, the eikonal approximation gives remarkably accurate results for low-lying collective states.

Our present study does not provide an explanation of the detailed features of the subbarrier fusion observed in ref.[illegible]). Particle transfer processes are probably important[14]). However, presently there is no alternative to the coupled channel technique for calculating their influence on subbarrier fusion.

Acknowledgment.

We acknowledge support by the National Science Foundation under grant PHY-80-17605.

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Figure captions.

Fig. 1 Fusion probabilities for the reaction $^{16}O + {}^{148}Sm$ are shown as function of the center of mass energy. The results are obtained from coupled channel calculations, for various values of the one-phonon excitation energy $\hbar\omega$. The standard deviation of the zero-point motion amplitude is $\sigma = 0.27$ fm.

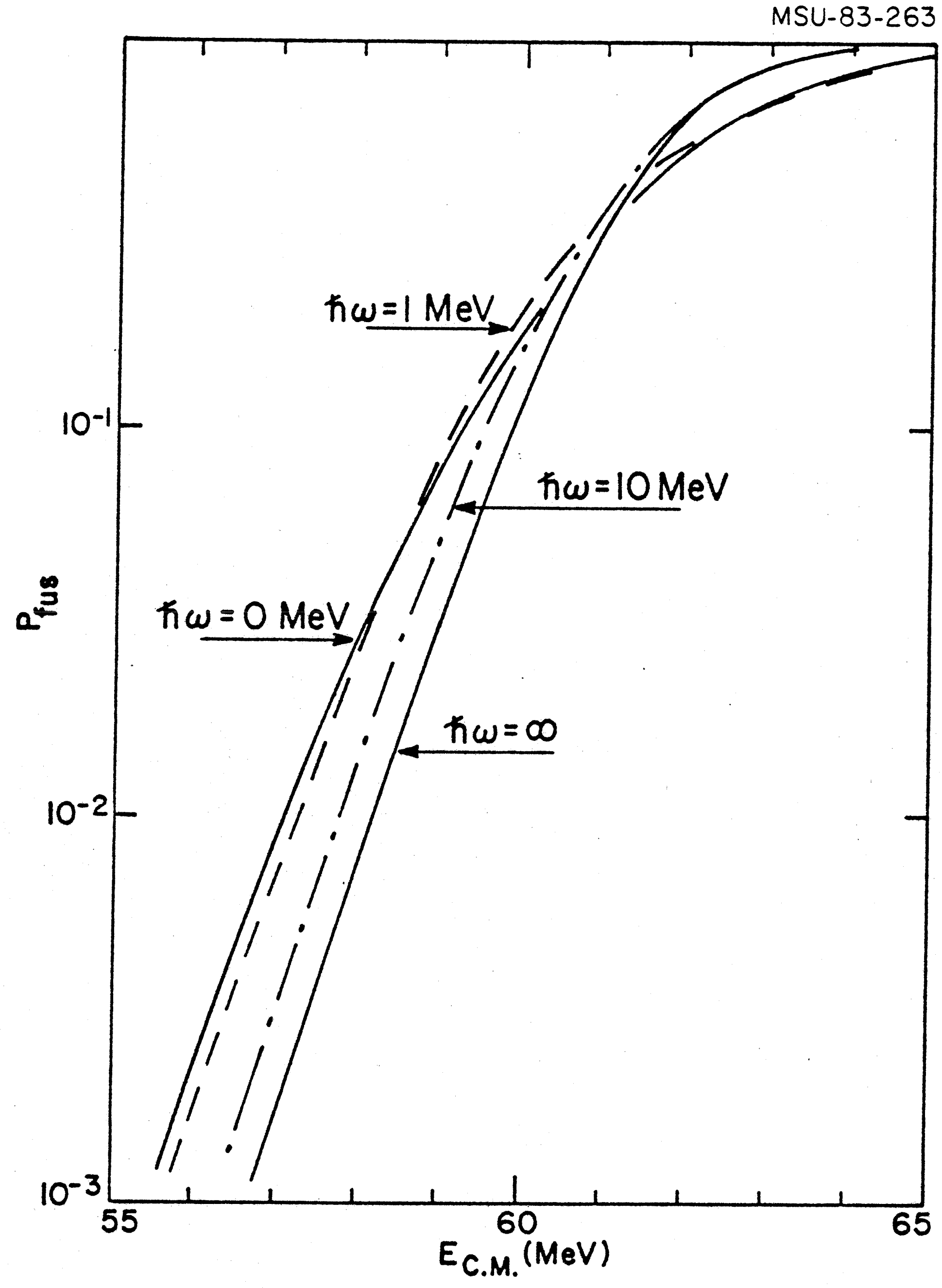
Fig. 2 Fusion probabilities for the reaction $^{16}O + {}^{148}Sm$ at a center of mass energy of 57 MeV are shown as a function of σ^2, for different values of $\hbar\omega$. Coupled channel results are displayed in (A), and the results from the eikonal approximation, eqs. (29)-(32), are shown in (B).

Fig. 3 Contours of equal density $|\psi(r,s)|^2$ are shown in the (r,s)-plane. The wavefunction is a solution to eqs. (11)-(13), the sudden limit, for the reaction $^{16}O + {}^{148}Sm$ at a center of mass energy of 57 MeV, with $\sigma = 0.27$ fm. The shaded area inside the dashed curve is the classically forbidden region. The wavefunction is essentially a standing wave outside the barrier. Note that s shifts from zero to about 0.5 fm in the penetrating wave.

Table I.

Values of $f_{i,\lambda}$, defined in eq. (23) and calculated from the proximity description, are given for different multipolarities λ, $\lambda = 2$ to 5, and for various reactions. The collective mode is in the nucleus (i) shown in the second column.

Reaction	(i)	$\lambda = 2$	3	4	5
O + Sm	Sm	0.60	0.73	0.72	0.65
O + Sm	O	0.43	0.21	0.06	7.10^{-4}
Ar + Sm	Sm	0.59	0.66	0.59	0.46
Ar + Sm	Ar	0.51	0.37	0.17	0.04
Ni + Ni	Ni	0.57	0.52	0.36	0.20


MSU-83-263
ħω=1 MeV
ħω=10 MeV
ħω=0 MeV
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P_{fus}
10^{-1}
10^{-2}
10^{-3}
55
60
65
$E_{C.M.}$ (MeV)

Figure 1

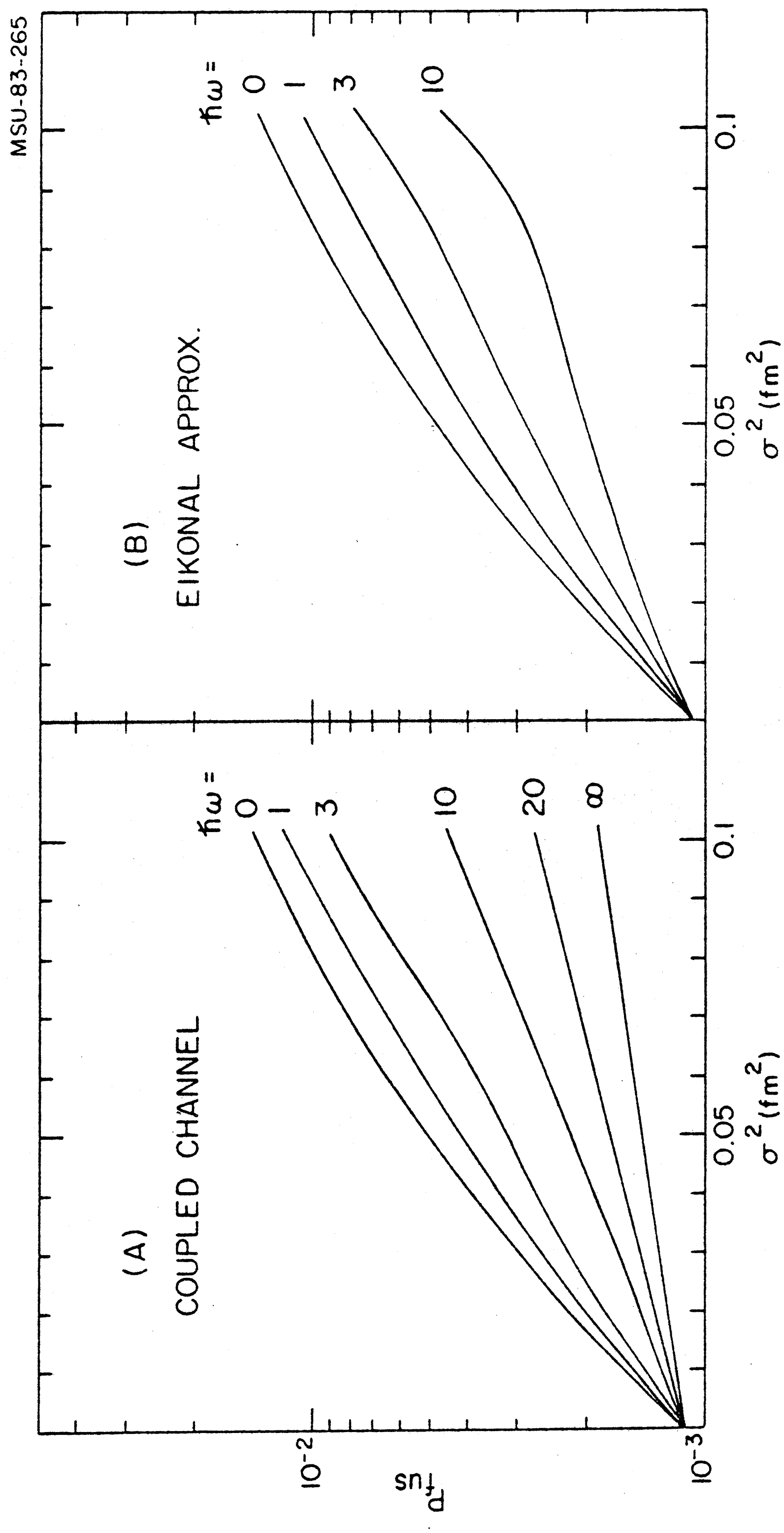
MSU-83-265
(A)
COUPLED CHANNEL
(B)
EIKONAL APPROX.
ħω =
0
1
3
10
20
∞
10^{-2}
10^{-3}
P_{fus}
0.05
0.1
σ^2 (fm^2)

Figure 2

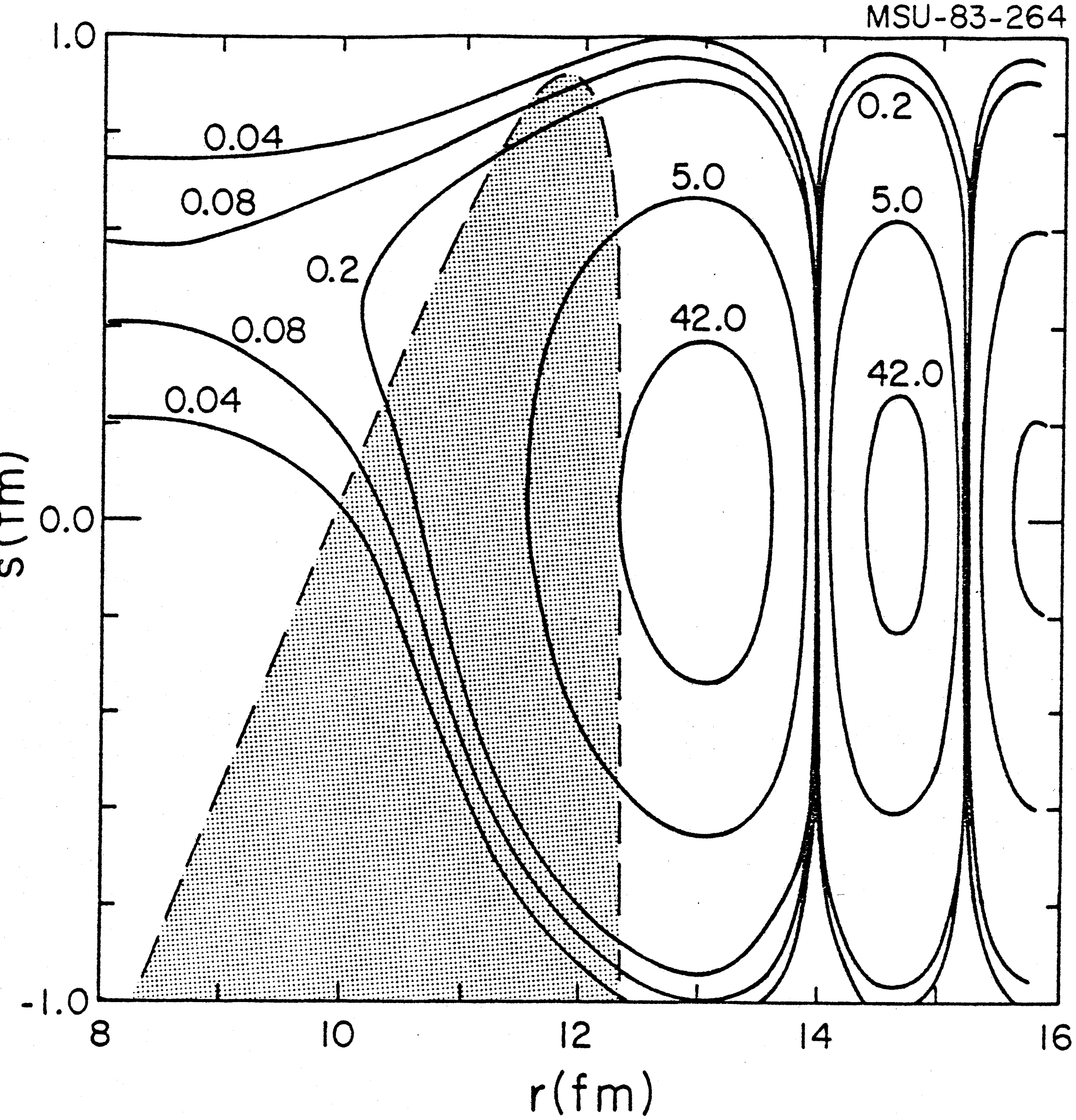
MSU-83-264
1.0
0.0
-1.0
s(fm)
8
10
12
14
16
r(fm)
0.04
0.08
0.2
0.08
0.04
5.0
42.0
0.2
5.0
42.0

Figure 3