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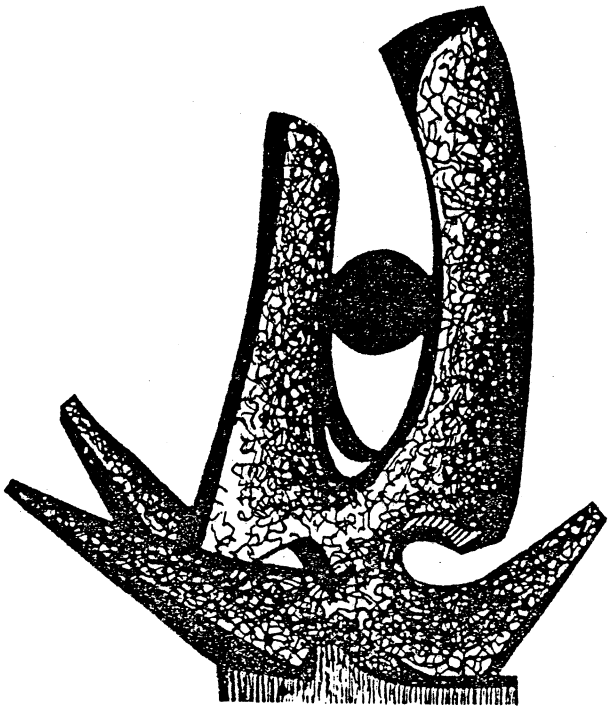

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Microscopic Calculations for the Interacting Boson Model

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Abstract

The parameters of the Interacting Boson Model are calculated in a shell model framework, using a generalized seniority basis. The effect of the neutron-proton interaction on the s- and d-boson structure is explicitly considered. The renormalization due to the truncation of the full fermion space to the S-D subspace is considered by taking the effects of the G-pair state into account using perturbation theory. It is found that this effects mainly the single boson energies and introduces a Majorana force that favors states symmetric in neutron and proton degrees of freedom over antisymmetric ones.

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[Nuclear Structure, Microscopic calculation of Interacting Boson Approximation parameters. Shell model calculations in the generalized seniority basis.]

I. Introduction

The Interacting Boson Model (IBM)[1] has been very successful in explaining the properties of low lying collective states in heavy and medium heavy nuclei.[2,3] In the phenomenological calculations, the IBM parameters are usually adjusted so as to give a best fit to a series of nuclei, with smoothly varying parameters. In this way parameters have been obtained for the majority of nuclei with mass A>100. In general the observed variations in the parameters, as a function of N and Z, agree qualitatively with a zeroth order estimate based on the seniority scheme.

Several attempts have been made to calculate the model parameters from a more detailed microscopic approach. The first and probably most extensive one was by Otsuka.[2,3,4,5] One of the largest deviations from the simple seniority estimate, the sharp decrease in the energy of the d-boson energy when going away from the closed shell, was explained in these calculations as coming from the coupling of the S-D subspace to the full shell model space. In these and subsequent calculations[6,7] the effects of the neutron-proton interaction on the microscopic structure of the bosons have not been considered. Furthermore the structure of the bosons has been obtained from a two fermion calculation and thus neglects the influence of the Pauli principle on the boson structure. This is not true for the calculations in the broken pair model presented in Ref. 8; where, as in the present approach, the structure is calculated as a function

of the number of particles in the valence shell. Calculations for deformed nuclei,[9] using HFB instead of the shell model, indicate that the microscopic structure of the bosons is strongly affected by the neutron-proton interaction. It is however not completely understood how one should relate the results of the HFB calculation to the IBM, especially the effect of the Pauli principle in the mapping procedure is not well formulated.

In this paper a detailed calculation of the parameters will be presented. It is based on a shell model calculation, but in order to keep the shell model calculations simple the model space is truncated on generalized seniority, as will be discussed in the second section. This truncation decreases the size of the model space while not affecting the calculated low lying states in which we are interested.

II. The Generalized Seniority Scheme

A major problem in shell-model calculations for heavy nuclei is the size of the model space. Even if the calculation is restricted to semiclosed shell nuclei where only particles of a single kind (neutrons or protons) are considered the dimension of the model space is easily of the order of several thousands in the J=2 subspace. However, it has been pointed out by Talmi[10] that for semiclosed shell nuclei the generalized seniority (g.s.) scheme is valid. Binding energy systematics and a constant excitation energy of the 2_1^+ level show that the breaking of the g.s. scheme is

only slight.

The g.s. scheme as it was proposed by Talmi is a generalization of the usual seniority concept[11,12] to the case of several nondegenerate single particle (s.p.) orbits. The seniority scheme can be introduced via the operators

$$S_{+j} = \frac{1}{2}\sqrt{2j+1}\,(a_j^\dagger a_j^\dagger)^{(0)} \qquad (2.1)$$

$$S_{-j} = \frac{1}{2}\sqrt{2j+1}\,(\tilde{a}_j \tilde{a}_j)^{(0)}$$

and

$$S_{0j} = \frac{1}{2}[S_{+j}, S_{-j}] = \frac{1}{2}(\hat{n}_j - (2j+1)/2)$$

where $\hat{n}_j$ is the fermion number operator. It can easily be verified that these three operators close under commutation and that they form the generators of a SU(2) quasi-spin algebra.[11] Different representations can be labelled using the seniority quantum number v which counts the number of particles not pairwise coupled to angular momentum J=0. The tools of group theory can now be used to obtain reduction formulas that relate matrix elements of operators between states of nonmaximal seniority to those for maximal seniority.[12] This formulation is however only valid for single-j configurations. In most problems in the description of medium heavy nuclei several single particle orbits enter. To generalize seniority to these cases one introduces the

operators

$$S_+ = \sum_j \alpha_j S_{+j} \qquad (2.2)$$

and

$$S_- = \sum_j \alpha_j S_{-j}$$

However it is not possible to introduce an operator $S_0 = \frac{1}{2}[S_+, S_-]$ which closes the algebra for arbitrary coefficients α_j. As a consequence g.s. is not related to a group algebra and reduction formulas are therefore more complicated to derive, but it can still be done.[13] In spite of the lack of an underlying group symmetry it is possible to introduce a g.s. quantum number, here denoted by w. In analogy to the case of normal seniority one can define a w=0; n=2N fermion state as

$$|\tilde{j}^n\ w=0, J=0\rangle = S_+^N|0\rangle/N \qquad (2.3)$$

where $\tilde{j}^n$ denotes n particles in a multi-j shell and N is an appropriate normalization factor, which of course depends on n and α_j. A w=2, n=2N state can now be defined as

$$|\tilde{j}^n\ w=2, J\rangle = (S_+^{N-1}|\tilde{j}^2\ J\rangle)_\perp/N' \qquad (2.4)$$

where $\perp$ denotes that the state is orthogonalized on the

$|w=0, n=2N\rangle$ state in the case that J=0. The need for an explicit orthogonalization is a result of the absence of a group structure. Proceeding in this way one can define states of higher g.s., in this paper we will deal with $w\leq 2$ states.

Shell model calculations can be done in the g.s. basis as outlined above, the matrix elements of the Hamiltonian can be calculated in the g.s. basis and the resulting matrix can subsequently be diagonalized to yield the energies and eigenvectors. The results will depend however on the coefficients α_j that enter in the definition of the S-pair. One way of determining the α_j is to minimize the energy of the w=0 state and thus make it the ground state. In the calculations presented in this paper a somewhat different approach has been taken. In order to minimize the coupling to states outside the $w\leq 2$ basis we are considering, the ground state should have w=0 (Eq. (2.3)). In lowest order the $\Delta w=2$ terms have thus been eliminated from the Hamiltonian. For an arbitrary choice of the α_j a diagonalization of the Hamiltonian in the J=0, $w\leq 2$ basis will however not give rise to a w=0 ground state. In the calculations presented in this paper the α_j coefficients have been determined in an iterative procedure, imposing as a self-consistency requirement that indeed the ground state in a $w\leq 2$ basis has g.s. w=0. A diagonalization in a J=0, $w\leq 2$ basis yields in general a ground state of the form $S_+' S_+^{(N-1)}|0\rangle$ where the coefficients α_j' that enter in S_+' are

different from the α_j that define S_+ (see Eq. (2.2)). The values α''_j, which replace α_j in the next iteration step, are taken as a weighted average of α_j and α'_j where the weighting factors are the number of particles.

$$\alpha''_j = ((N-1)\alpha_j + \alpha'_j)/N.$$

In the case that some of the orbits under consideration are almost filled, the weighting factors should be taken as the number of particles in the valence orbits to ensure a fast convergence. This procedure has been preferred since it has proven to be very fast. In most cases the self-consistency requirement is fulfilled in less than 5 iterations, if even the starting values of α_j differ by more than a factor 2 from the self consistent ones.

Once the α_j have been determined, the states with $J\neq 0$ can be obtained by diagonalizing the Hamiltonian in a J=J, w=2 basis. In principle one can readjust the α_j for each J, but this has not been attempted.

In this paper the g.s. scheme has been used for two reasons. There exists a very clear and simple relation between the g.s. scheme and the IBM.[5] The S-pair operator can be regarded as the microscopic equivalent of the s boson while the lowest J=2, w=2 state corresponds in the IBM to the $|s^{N-1}\ d\rangle$ state. A second reason is that due to the truncation on g.s. rather than normal seniority, the size of the dimension of the matrices to be handled in the shell-

model treatment is decreased by one or two orders of magnitude, while it does not affect the calculated properties of the low lying states as is shown below.

In order to check the validity of the scheme outlined above, calculations have been compared with the results of an extensive shell-model calculation for the N=82 isotones.[14] Excitation energies as a function of the number of protons have been compared in Ref. 15. In Fig. 1 the calculated levels for ^{144}Sm are compared with experiment. The interaction has been taken from Ref. 14 where it has been optimized so as to give a best fit to the even and odd-mass N=82 isotones in a full shell-model calculation. In the present calculation the largest matrix to be diagonalized was 9x9 in the $J^\pi = 2^+$ subspace. In spite of this extremely small basis the agreement is remarkable.

III. The coupling of neutrons and protons

In the procedures outlined in the previous section only the interaction between like particles was considered. In this case the generalized seniority scheme was extremely powerful since the interaction to a good extent conserves g.s. The neutron-proton force is dominated by a quadrupole-quadrupole interaction which strongly admixes states with different seniority. If we limit ourselves to nuclei with only a few neutrons or protons in the valence shell (vibrational nuclei), the admixture of higher seniority components in the ground and first excited state can

still be treated in lowest order.

The method proposed here is based on an iteration between a microscopic neutron and proton calculation, using the results of one to calculate the other. Due to the quadrupole character of the neutron-proton force components of the kind $S_\pi^{(N_\pi-1)} S_\nu^{(N_\nu-1)} (D_\pi D_\nu)^{(0)}$ will be admixed in the ground state where the collective fermion pair state D is defined via

$$D_\rho^+ = \frac{1}{2} \sum_{j,j'} \beta_{jj'} \sqrt{1+\delta_{jj'}} \, (a_j^\dagger a_{j'}^\dagger)^{(2)}; \rho = \nu, \pi \qquad (3.1)$$

and is the microscopic equivalent of the d-boson. In order to allow for this admixture and still keep the dimensionality of the basis small, we will assume while calculating the microscopic structure of the proton fermion pair states that the microscopic structure of the neutron fermion pair states is known and can be represented by boson degrees of freedom. Since there is also an appreciable hexadecupole component in the neutron-proton interaction, we will also include explicitly the G-pair state in the calculation,

$$G_\rho^+ = \frac{1}{2} \sum_{j,j'} \gamma_{jj'} \sqrt{1+\delta_{jj'}} \, (a_j^\dagger a_{j'}^\dagger)^{(4)}; \rho = \nu, \pi \qquad (3.2)$$

Representing the neutron degrees of freedom by bosons the ground state can be written as[17]

$$|0^+\rangle = a_0 |s_\nu^{N_\nu} S_\pi^{N_\pi}\rangle + a_2 |(s_\nu^{N_\nu-1} d_\nu S_\pi^{N_\pi-1} D_\pi)^{(0)}\rangle \qquad (3.3)$$

$$+ a_4 |(s_\nu^{N_\nu-1} g_\nu S_\pi^{N_\pi-1} G_\pi)^{(0)}\rangle,$$

where a_0, a_2, a_4 and $\alpha_j^\pi, \beta_{jj'}^\pi$, and $\gamma_{jj'}^\pi$, as enter in the definitions of the fermion pair states, Eqs. (2.2, 3.1 and 3.2) still have to be calculated.

In the basis outlined above the Hamiltonian is written as

$$H = E_\nu^D n_{d_\nu} + E_\nu^G n_{g_\nu} + H_\pi + \kappa_2 q_\nu^{(2)} \cdot Q_\pi^{(2)} + \kappa_4 q_\nu^{(4)} \cdot Q_\pi^{(4)} \qquad (3.4)$$

where

$$q_\nu^{(2)} = A_2^\nu (s_\nu^\dagger \tilde{d}_\nu + d_\nu^\dagger s_\nu)^{(2)}/\sqrt{5N_\nu} + B_2^\nu (d_\nu^\dagger \tilde{d}_\nu)^{(2)}/\sqrt{5} \qquad (3.5)$$

$$+ C_2^\nu (d_\nu^\dagger \tilde{g}_\nu + g_\nu^\dagger \tilde{d}_\nu)^{(2)}/\sqrt{5} + D_2^\nu (g_\nu^\dagger \tilde{g}_\nu)^{(2)}/\sqrt{5},$$

$$q_\nu^{(4)} = A_4^\nu (s_\nu^\dagger \tilde{g} + g^\dagger s_\nu)^{(4)}/3\sqrt{N_\nu} + B_4^\nu (d_\nu^\dagger \tilde{d}_\nu)^{(4)}/3 \qquad (3.6)$$

$$+ C_4^\nu (d_\nu^\dagger \tilde{g}_\nu + g_\nu^\dagger \tilde{d}_\nu)^{(4)}/3 + D_4^\nu (g_\nu^\dagger \tilde{g}_\nu)^{(4)}/3$$

and

$$Q_\pi^{(2)} = - \sum_{jj'} \langle j||r^2 Y^{(2)}||j'\rangle (a_j^\dagger \tilde{a}_{j'})^{(2)} \left(\frac{m\omega_0}{\hbar}\right)/\sqrt{5}. \qquad (3.7)$$

$$Q_\pi^{(4)} = - \sum_{jj'} \langle j||r^4 Y^{(4)}||j'\rangle (a_j^\dagger \tilde{a}_{j'})^{(4)} \left(\frac{m\omega_\emptyset}{\hbar}\right)^2/3 \ . \quad (3.8)$$

The coefficients that enter in the neutron quadrupole operator can be calculated from the structure of the neutron fermion pair state. H_π is the shell model interaction for the protons. The Hamiltonian is diagonalized in the space that includes in the proton sector all (J=0, w≤2), (J=2, w=2) and (J=4, w=2) states, appropriately coupled with neutron s, d or g bosons. By imposing as a self-consistency condition that in the ground state no (J=0, w=2) components are admixed, the coefficients α_j^π and the other unknowns in Eq. (2.3) can be determined. In the next iteration step the protons will be dealt with as bosons and the parameters in the proton boson quadrupole operator can now be calculated as for example:

$$A_2^\pi = \langle s_\pi^{N_\pi} || q_\pi^{(2)} || d_\pi s_\pi^{N_\pi - 1} \rangle = \langle S_\pi^{N_\pi} || Q_\pi^{(2)} || D_\pi S_\pi^{N_\pi - 1} \rangle . \quad (3.9)$$

and

$$E_\pi^D = \langle S_\pi^{N_\pi - 1} D_\pi | H_\pi | S_\pi^{N_\pi - 1} D_\pi \rangle \quad (3.10)$$

The iteration is repeated until the parameters in the boson multipole operators are converged to within 1%, which in general takes no more than four iterations.

The effect of the neutron-proton interaction has been calculated for the case in which the protons occupy the 50-82 shell and the neutrons the lower part of the 82-126 shell. The interaction for the protons is taken from a shell-model best fit to the N=82 isotones.[14] For the neutrons a surface- delta interaction (SDI)[19] has been taken with a strength of $A_1 = 0.13$ MeV. In order to improve the agreement with experiment for the lead isotopes, an enhancement factor for the quadrupole component, F_2, in the multipole expansion of the SDI was introduced, F_2=1.4. The neutron single particle energies were taken linearly dependent on the number of proton pairs (N_π) outside the Z=50 closed shell, $\epsilon_j(N_\pi) = \epsilon_{\emptyset j} + \Delta_j N_\pi$. The values used for $\epsilon_{\emptyset j}$ and Δ_j are given in Table I. The neutron proton interaction is written as

$$V_{\nu\pi} = -\kappa_2 (r_\nu^2 Y_\nu^{(2)}).(r_\pi^2 Y_\pi^{(2)}) - \kappa_4 (r_\nu^4 Y_\nu^{(4)}).(r_\pi^4 Y_\pi^{(4)}). \quad (3.11)$$

The strengths were taken as the average of the quadrupole and hexadecupole components in the neutron-neutron and proton-proton interaction, giving $\kappa_2 = .0084\ (m\omega_\emptyset/\hbar)$ MeV and $\kappa_4 = 1.62 \times 10^{-3}\ (m\omega_\emptyset/\hbar)^2$ MeV. In the calculation of the radial matrix elements, harmonic oscillator wave functions were used. The strength of the quadrupole force is in between the values given in Refs. 9 and 20.

In Fig. 2 the effect of the neutron-proton force on the calculated values of α_j^π are shown for Z=62. Changing the

number of neutron pairs from 0 to 5 changes $\alpha^{\pi}_{g_{7/2}}$ and $\alpha^{\pi}_{d_{5/2}}$ by less than 5%. For the higher lying orbits, the change is larger, but still much less than what was determined in Ref. 9 from a HFB calculation. This difference could simply come from the fact that the single particle energies were taken constant in the present calculation, while they are taken N-dependent in Ref. 9. Another explanation could be that in the present calculation only a single D-pair state is considered, while nuclei with more than 4 neutron and 4 proton pairs in the valence shell are deformed and thus have several D-pairs admixed in the ground state, which makes the validity of the present approach questionable beyond N =4 for the Sm isotopes. This objection does not apply to the HFB calculations where a deformed intrinsic state is considered. Calculations in Chapter IV show however that the results obtained from the present calculation for the deformed Samarium isotopes reproduce the main features.

Figure 3 shows that the coefficients α^{π}_{j} depend strongly on N_{π}. The main origin of this strong dependence is in the subshell closure near Z=64. The qualitative dependence of α^{π}_{j} on N_{π} is independent of whether a best fit or a simple SDI interaction is used. The coupling to the neutron degrees of freedom leaves the qualitative features intact but smooths the N_{π} dependence.

In Table III, the influence of the neutron-proton interaction on the properties of the d-boson is shown. It can be seen that the ratio $B^{\pi}_{2}\sqrt{N_{\pi}}/A^{\pi}_{2}$, which is the lowest

order estimate for the parameter χ_{π} in the boson quadrupole operator, varies by more than a factor 2 due to the coupling of the neutrons. This shows that for determining the structure of the d-boson the consideration of effects from the neutron-proton interaction is crucial. The same also follows from Fig. 4, showing that its effects on the properties of the proton d-boson are more important when a system with only a few protons is considered. As a result of the neutron- proton force the changes in B^{π}_{2} as a function of N_{π} are smoothed out.

IV. The Boson Model Parameters

Unlike many other boson models (as for example Ref. 19), the mapping from the fermion space to the boson space is not done by mapping operators but rather by equating matrix elements between equivalent states in the two spaces. As was mentioned before the fermion space and the IBM space can be related via seniority. The state of Eq. (2.2) corresponds to $|s^{N}\rangle$ and the w=2 state of Eq. (2.4) to $|s^{N-1}d\rangle$. Similar relations exist for higher seniority states.

Two versions of the IBA-model exist , but here we will discuss only the IBA-2[2,3] since it is closest to a microscopic formulation. In the IBA-2 model neutron and proton degrees of freedom are taken into account explicitly. The Hamiltonian as it is generally used in phenomenolgical calculation can be written as

$$H = \epsilon_\nu n_{d_\nu} + \epsilon_\pi n_{d_\pi} + \kappa\, Q_\nu \cdot Q_\pi + F_s M_{\nu\pi} \qquad (4.1)$$

where

$$Q_\rho = (s_\rho^\dagger \tilde{d}_\rho + d_\rho^\dagger s_\rho)^{(2)} + \chi_\rho (d_\rho^\dagger \tilde{d}_\rho)^{(2)}; \rho = \nu, \pi \qquad (4.2)$$

and $M_{\nu\pi}$ is the Majorana force that shifts states according to their symmetry with respect to interchange of neutron and proton degrees of freedom (F-spin).[22]

In most phenomenological calculations ϵ_ν and ϵ_π, the neutron and proton d-boson energies are taken equal, however in a microscopic treatment one should distinguish them. In the IBA-2 model the interaction is taken as a pure neutron-proton quadrupole force. One of the reasons is that an important part of the interaction between like particles is taken into account by the energy difference between the S and D pair states (ϵ) and the remaining force is thus the neutron- proton interaction which has predominant quadrupole character. Furthermore an additional hexadecupole component in the neutron-proton interaction will not contribute much to the collectivity in the (s-d) boson space since it conserves the number of d-bosons. From the spectra of semi-closed shell nuclei there is also a strong indication that there is at most a small quadrupole interaction between like bosons, see for example Talmi in Ref. 2. For deformed nuclei one could expect such an interaction coming from the truncation of the full shell model space to the S-D

subspace,[23] but this will not be considered here. In numerous phenomenological applications[2,3] the Hamiltonian (4.1) has proven to be adequate.

To obtain the boson model parameters, the matrix elements between equivalent states of the boson operator and the fermion operator in the two spaces have to be put equal. The equivalent of the one-d boson state[5] is defined as the (J_π=2, w_π=2) component mixed in the ground state as defined in Eq. (3.3) and is calculated using the procedure outlined in the previous two chapters. The calculated parameters for the case corresponding to the Sm isotopes (Z=62) are shown in Fig. 5. Also shown in the figure are the parameters obtained from a best fit in the IBA-2 model,[24]. In the microscopic calculations the energy of the d-boson is constant at about 1.5 MeV; while in the phenomenological calculations, where the neutron and proton boson energies have been chosen equal, the d-boson energy drops sharply with increasing neutron number. This problem was also recognized by Otsuka[4] where it was solved by considering the renormalization of the parameters as a result of the basis truncation from the full shell-model space to the S-D pair subspace.

In the present calculation two sources for renormalization were considered, the coupling of the G-pairs state to the S-D subspace via the neutron-proton quadrupole and the hexadecupole force. The renormalized parameters were obtained by calculating the matrix elements of the operators in fermion states between perturbed states, i.e., the

equivalent of the one d_ν boson state is taken as

$$|s_\pi^{N_\pi} s_\nu^{N_\nu-1} d_\nu\rangle = N_\nu[|s_\pi^{N_\pi} s_\nu^{N_\nu-1} D_\nu\rangle + \delta_1^\nu |s_\pi^{N_\pi-1} s_\nu^{N_\nu-1} (D_\pi G_\nu)^{(2)}\rangle \quad (4.3)$$

$$+ \delta_2^\nu |s_\pi^{N_\pi-1} s_\nu^{N_\nu-1} (G_\pi D_\nu)^{(2)}\rangle + \delta_3^\nu |s_\pi^{N_\pi-1} s_\nu^{N_\nu-1} (G_\pi G_\nu)^{(2)}\rangle]$$

instead of Eq. (2.4). The coefficients δ are calculated using standard perturbation theory, for example

$$\delta_1^\nu = \langle s_\pi^{N_\pi} s_\nu^{N_\nu-1} D_\nu | H | s_\pi^{N_\pi-1} s_\nu^{N_\nu-1} (D_\pi G_\nu)^{(2)}\rangle / \Delta E_1 \quad (4.4)$$

where $\Delta E_1 = E_\nu^D + E_\pi^S - E_\pi^D - E_\nu^G$. The matrix elements for the coupling to the G-pair state were calculated using the microscopic structure of the G-pair state that enters in the description of the ground state, Eq. (3.2). This approach is equivalent to second order perturbation theory for diagonal matrix elements. The advantage of the present approach is that there is no ambiguity in the choice of energy denominators in the calculation of off-diagonal matrix elements. The effects of coupling to S' or D' states, collective (w=2, J^π=0,2) states different from the ones considered, have not been taken into account. This is expected to give only a small effect since--contrary to the case of the G-pair state--by construction these states do not couple to the ground state.

From the calculation it follows that both of the effects for renormalization that were considered contribute

about equally to the renormalization of ϵ, κ, χ_ν and χ_π. This is not true for the Majorana force, which comes completely from renormalizations due to the hexadecupole force. In Fig. 6 the calculated spectra, using the renormalization parameters as given in Fig. 5, are compared with experiment. The decrease in the energy of the 2_1^+ and 4_1^+ level with increasing number of neutrons is well reproduced. This decrease is closely related to the decrease in ϵ. For ^{146}Sm (N_ν=1) the 4_1^+ level is calculated too high since the calculated level is a two-d boson state which corresponds to a generalized seniority w=4 state. In ^{144}Sm (see Fig. 1) and ^{146}Sm the 4_1^+ has a predominant w=2 character and will thus not be reproduced in the IBA model description unless a g-boson degree of freedom is included explicitly. For the heavier isotopes the 0_2^+ state is calculated too high in the spectrum. This indicates that the value used for κ is too large as can be seen by comparing with the phenomenological value. This can be seen as an indication that the strength of the neutron-proton quadrupole force in the microscopic calculation was chosen too strong. Weakening this force will directly give rise to a lower value of κ since in first approximation they are proportional. Since this force is at least partially responsible for the renormalization of ϵ, a reduction will give rise to increased values of ϵ, which is in contradiction to that data. In the renormalization calculation, however, states of higher angular momentum ($J^\pi=6^+$ for example) and other multipoles in the neutron-proton

interaction have not been considered. This might very well compensate for the effect of a reduction of the shell-model neutron-proton interaction on the boson energies but is beyond the scope of this work. It should be noted that the phase transition from spherical to deformed, which is observed experimentally by the dip in the energy of the 0_2^+ level, is reproduced in the present calculation.

In Fig. 7 the calculated parameters for the N=84 and N=86 isotones are compared with those obtained from phenomenological fits.[24,25] In the phenomenolgical calculations the parameters $\chi_\nu(\chi_\pi)$ were restricted to be a function of the number of neutrons (protons) only, which is not supported by the present microscopic calculations. In the present calculations χ_ν varies more than a factor two when changing the proton number from Z=52 to Z=66. Since the IBA calculations have only been done for nuclei with two or more neutron bosons ($N \geq 86$), the quoted values in Fig. 7 are for the N=86 isotones. Some of the trends in the parameters are well reproduced in the calculation, ϵ reaches a maximum around $N_\nu=7$, $|K|$ decreases with increasing neutron number and χ_π shows a steep increase for larger neutron numbers. The lower values for χ_ν which are obtained from the fits could find their origin in neutron excitations across the N=82 shell closure, an effect which has not been considered in the present calculations.

V. Conclusions

In the absence of the neutron-proton interaction the structure of the IBM s- and d- bosons can be calculated in a generalized seniority basis. Unlike the pure generalized seniority scheme the S-pair operator is allowed to change from nucleus to nucleus so as to give a w=0 ground state. In the 50-82 proton shell this gives rise to coefficients d_j that vary considerably due to seniority breaking terms in the Hamiltonian. However for each isotope separately the g.s. basis provides a powertul truncation scheme. The neutron-proton interaction can only be treated in a lowest order approximation in this basis since it strongly admixes states with different seniority. This mixing of different seniorities is fully considered in the Interacting Boson Model where d-boson number and generalized seniority are closely related.[4] In this paper a method has been developed for calculating the effect of the neutron-proton interaction on the microscopic structure of the bosons. The method is derived for cases near the SU(5) vibrational limit of the the IBM where the admixtures of higher seniority components in the ground state are small. The success of the application of the procedure to the Samarium isotopes seems to indicate that it is also applicable in the deformed region.

The renormalization of the parameters due to the coupling of the S-D subspace to the full space via the neutron-proton interaction is crucial in order to understand the decrease in the d-boson energy. The renormalization

also introduces terms in the boson-boson interaction, such as the Majorana force, which do not have a direct counterpart in the shell model interaction. The Majorana force results from renormalizations due to the multipoles in the interaction other than the quadrupole.

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Table I. The proton single particle energies used in the calculation (from Ref. 14)

j	ϵ_j
$0g_{7/2}$	-9.596
$1d_{5/2}$	-8.676
$1d_{3/2}$	-6.955
$2s_{1/2}$	-6.928
$0h_{11/2}$	-6.838

Table II. The relative neutron single particle energies, $\epsilon_j = \epsilon_{0j} + \Delta_j N_\pi$ as used in the calculation (from Ref. 18)

j	ϵ_{0j}	Δj
$1f_{7/2}$	0	0
$0h_{9/2}$	2.22	-.2056
$2p_{3/2}$	1.52	-.005
$0i_{13/2}$	1.33	-.0389
$1f_{5/2}$	2.877	-.06919
$2p_{1/2}$	2.22	0.0075

Table III. The determined values of E^d, A_2 and B_2 as defined in Eqs. 3.4-10 for neutrons and protons

N_ν	0	1	2	3	4	5
E^d_ν	-	1.35	1.41	1.44	1.42	1.39
A^ν_2	-	5.63	7.61	8.99	10.04	10.89
B^ν_2	-	-4.42	-3.96	-3.34	-2.78	-2.34
E^d_π	1.74	1.98	1.99	2.00	2.00	2.01
A^π_2	7.05	7.99	8.09	8.16	8.21	8.25
B^π_2	-1.61	-0.93	-0.87	-0.84	-0.81	-0.79

Figure Captions

Fig. 1. The spectrum of ^{144}Sm calculated in the generalized seniority scheme is compared with a shell model calculation[14] and the experimental spectrum.[16]

Fig. 2. Calculated values of α^π_j for the Sm isotopes as a function of N_ν, the number of neutron pairs in the 82-126 shell.

Fig. 3. Calculated values of α^π_j for the N=82 and N=86 isotones as a function of N_π, the number of proton pairs in the 50-82 shell.

Fig. 4. Calculated values of E^d_π, A^π_2 and B^π_2, see Eqs. 4-10, as a function of N_π. The difference in the results for the N=82 and N=86 calculations shows the influence of the neutron-proton interaction.

Fig. 5. The calculated values of the IBM parameters for the Sm isotopes are compared with those obtained from a best fit.[2,4]

Fig. 6. A comparison of the experimental spectra of the Sm isotopes with the calculation using the renormalized parameters of Fig. 5.

Fig. 7. The calculated renormalized IBM parameters for N=84 and N=86 are compared with a best fit for the N=86 isotones.

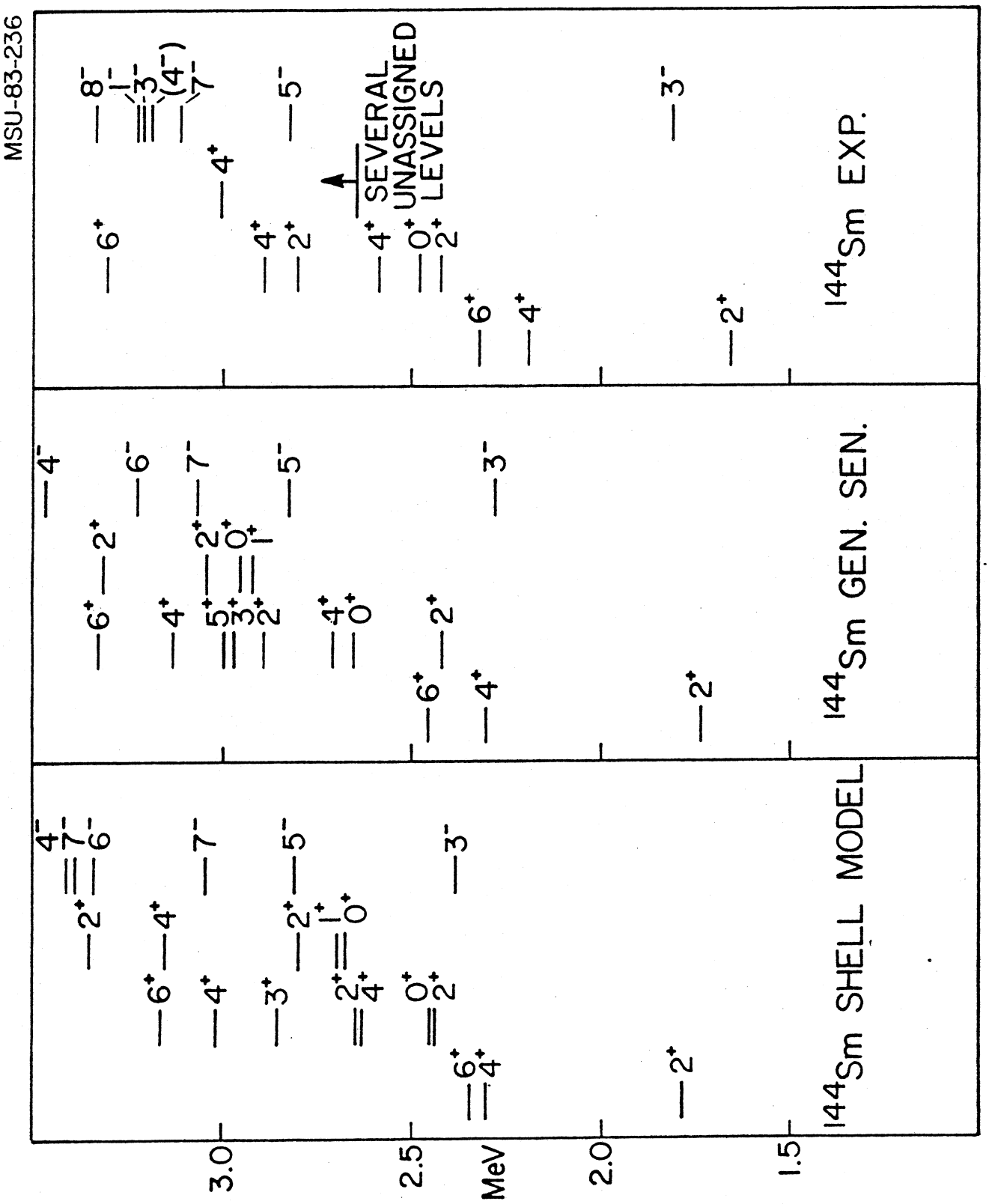
MSU-83-236
144Sm SHELL MODEL
144Sm GEN. SEN.
144Sm EXP.
SEVERAL UNASSIGNED LEVELS
MeV
3.0
2.5
2.0
1.5

Figure 1

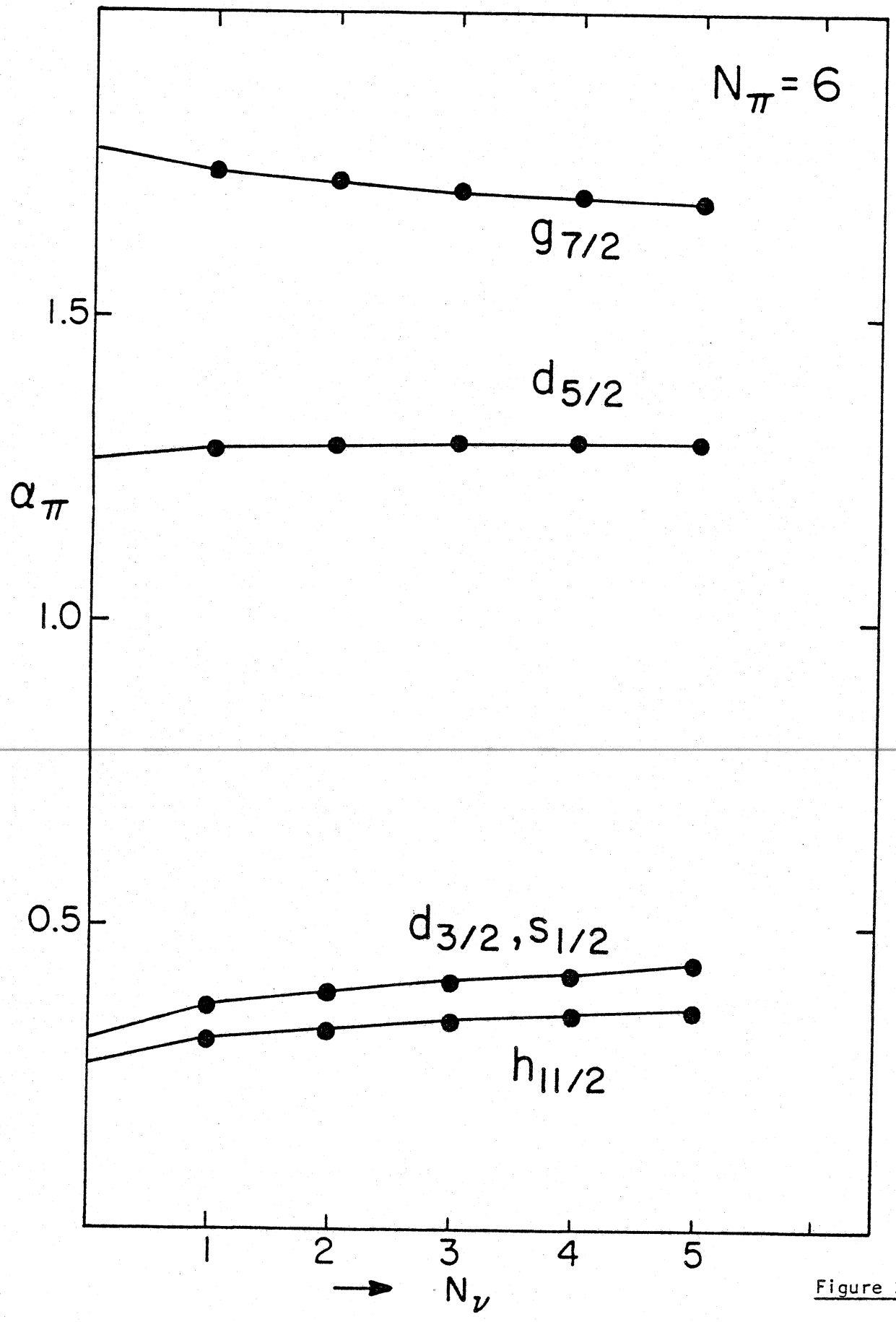

Figure 2

MSU-83-237

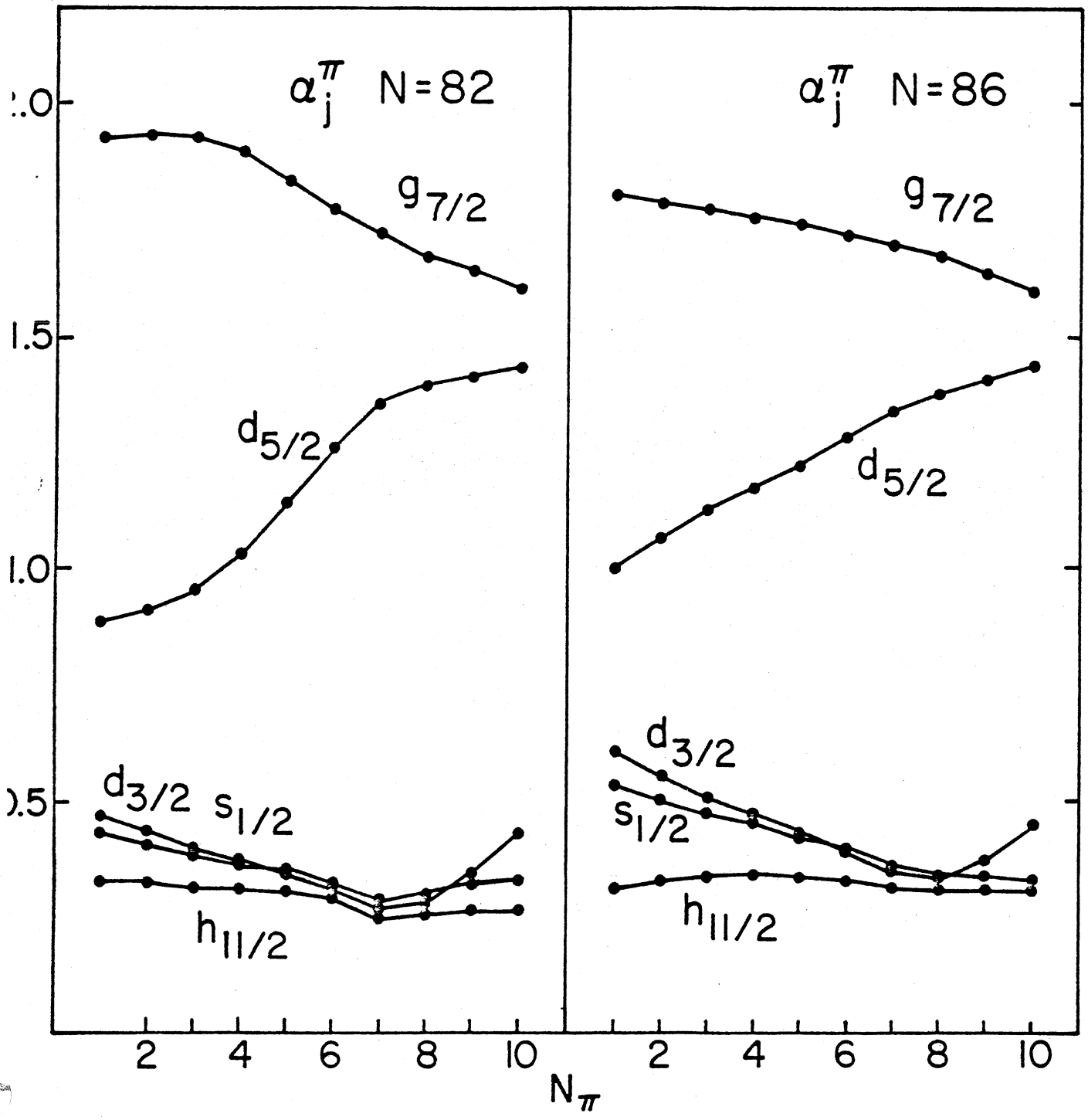

Figure 3

MSU-83-235

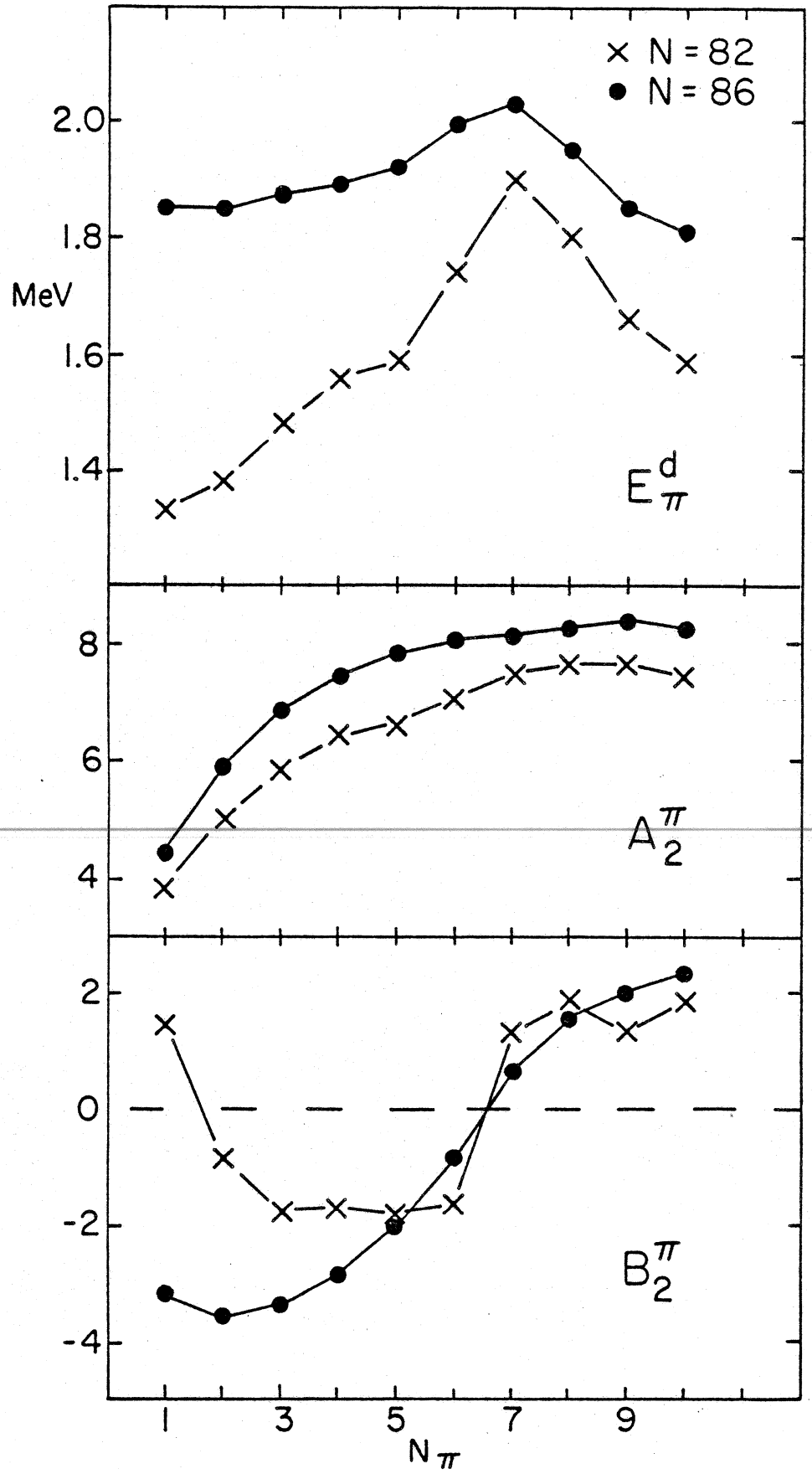

Figure 4

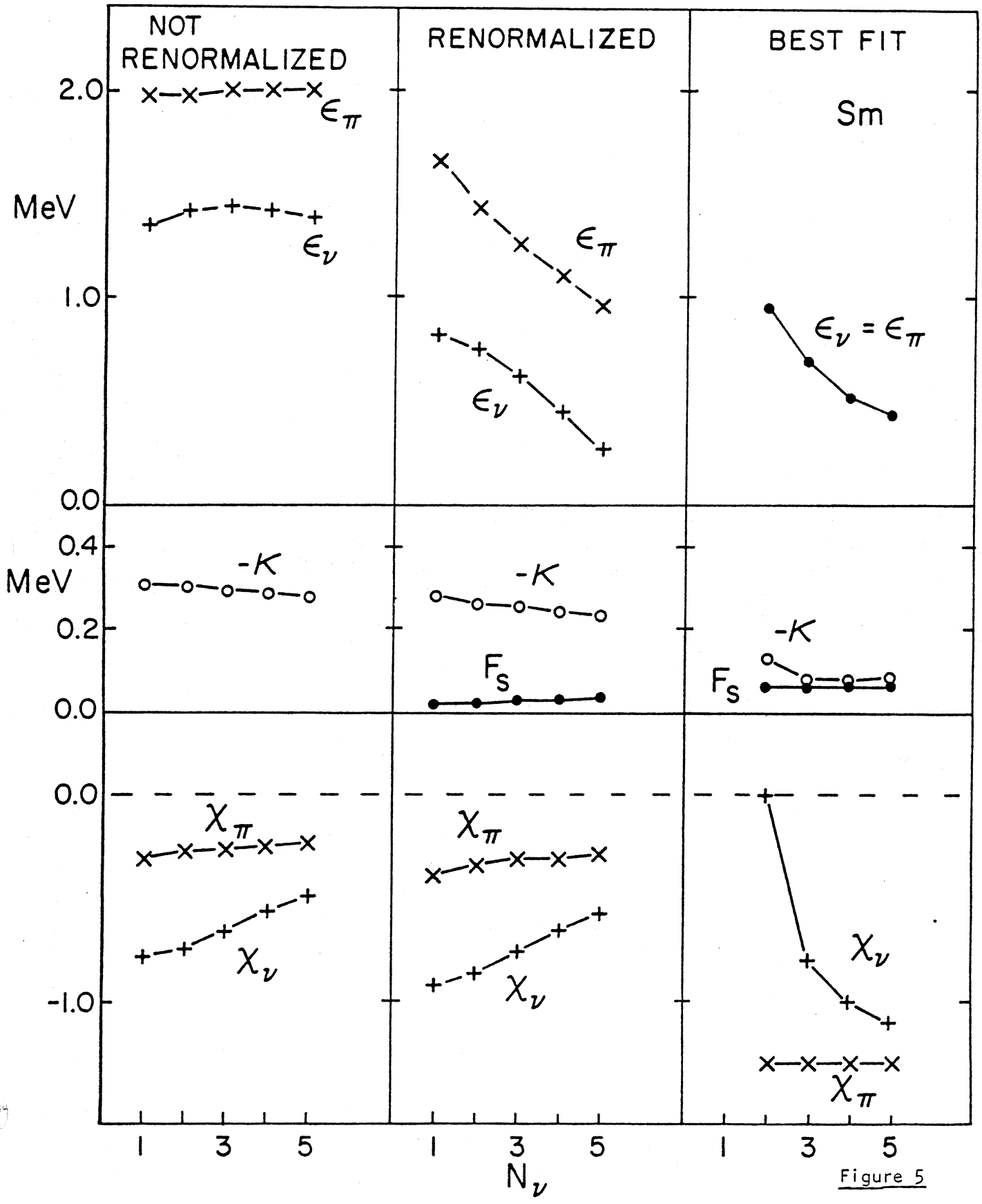
NOT RENORMALIZED
RENORMALIZED
BEST FIT
Sm
MeV
2.0
1.0
0.0
ϵ_π
ϵ_ν
$\epsilon_\nu = \epsilon_\pi$
0.4
0.2
0.0
-K
F_S
-1.0
χ_π
χ_ν
1
3
5
N_ν

Figure 5

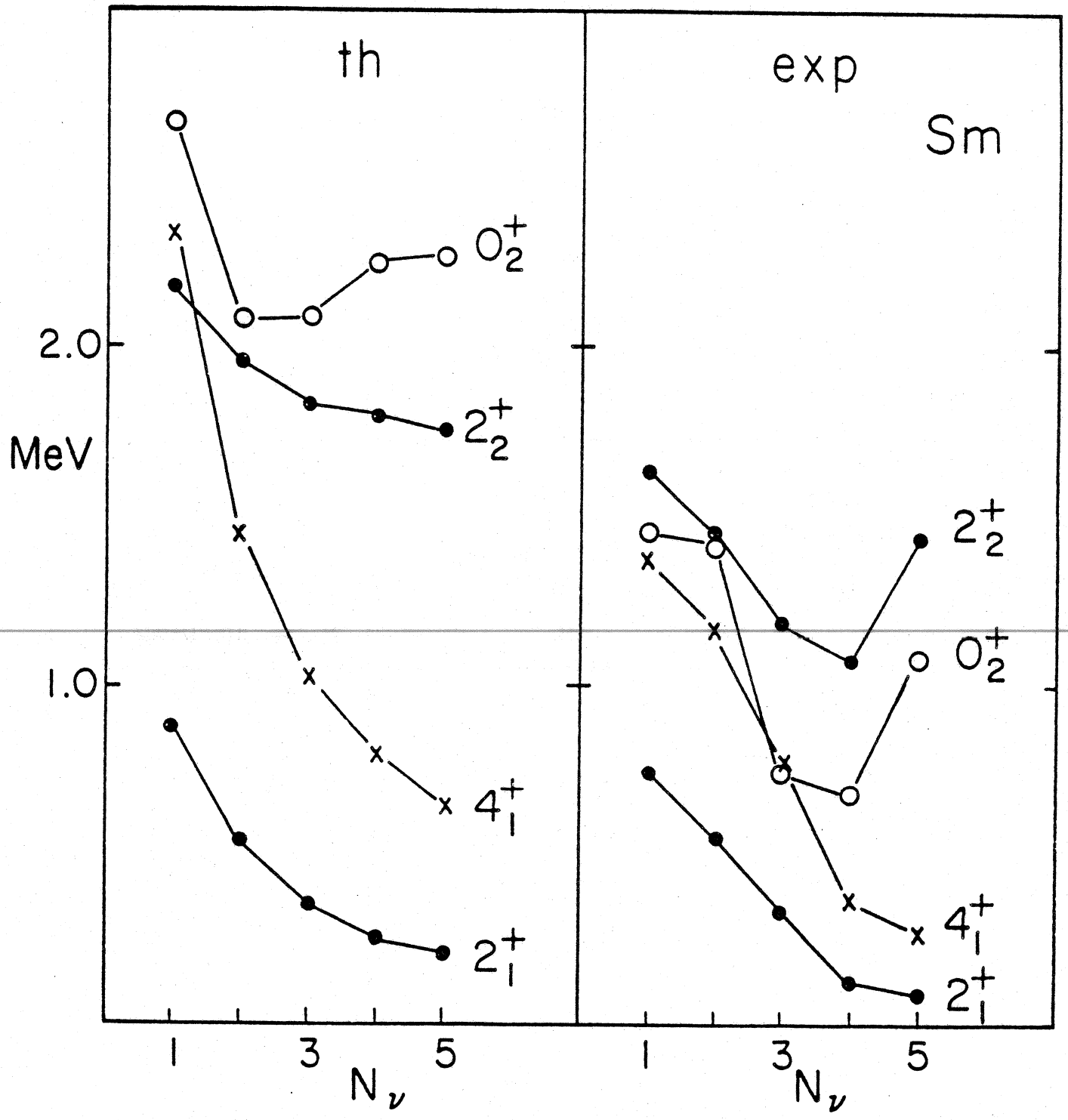

Figure 6

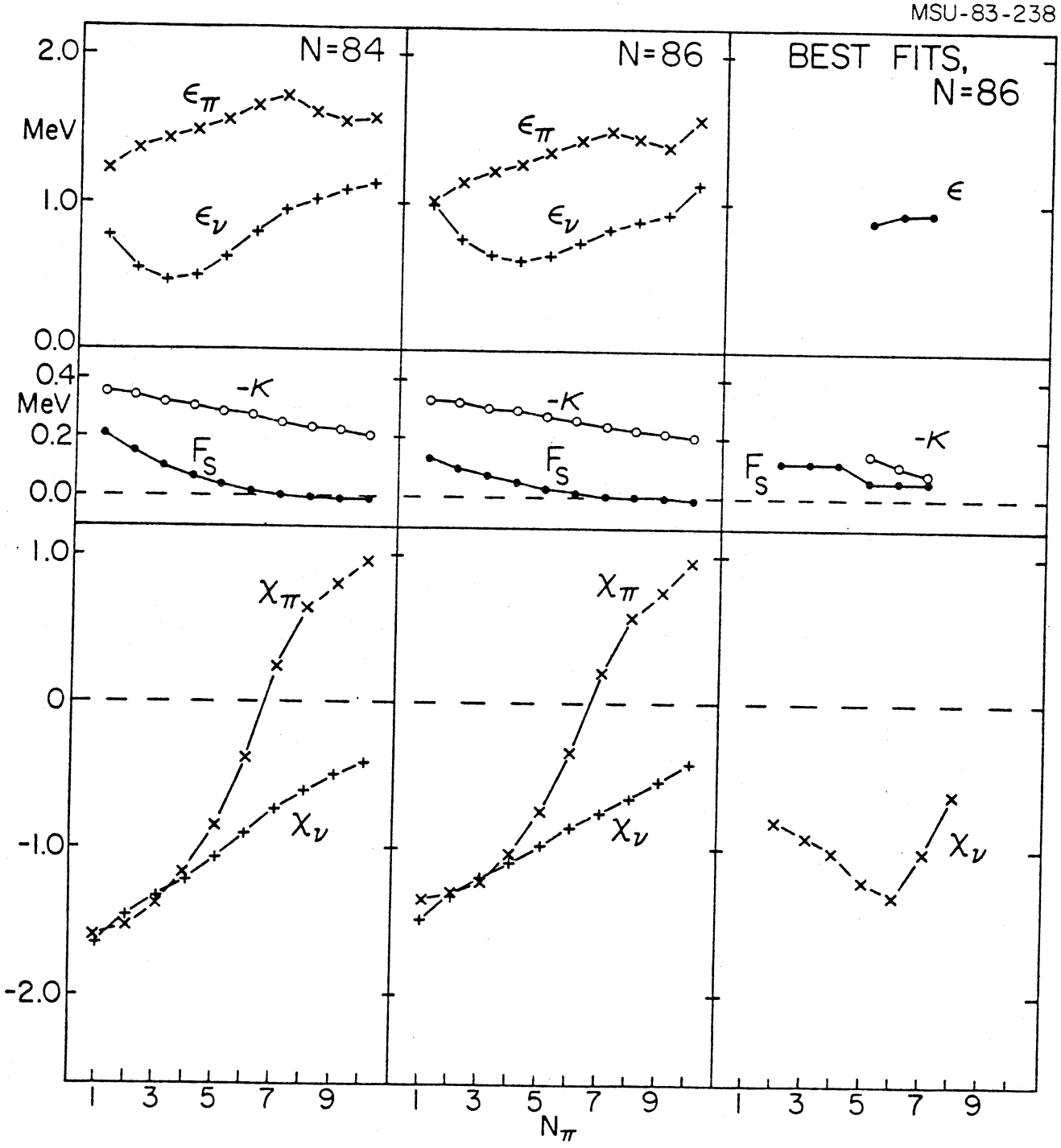
N=84
N=86
BEST FITS, N=86
MeV
2.0
1.0
0.0
0.4
0.2
1.0
0
-1.0
-2.0
ϵ_π
ϵ_ν
ϵ
-K
F_S
χ_π
χ_ν
1 3 5 7 9
N_π

Figure 7