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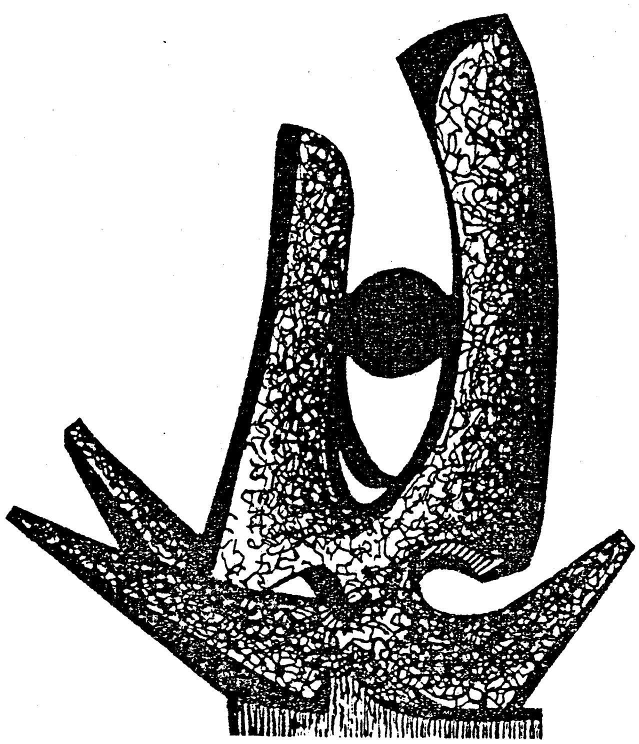

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ABSTRACT

The concept of an effective number of bosons in the Interacting Boson model is introduced and a procedure is proposed to calculate it. As an example, the effective number of bosons in the 50-82 shell for protons is calculated. The effect of the neutron-proton interaction is also discussed.

Recently there has been much interest in the derivation of the Interacting Boson Model[1]) (IBM) parameters from a microscopic model such as the shell model[2]) or Hartree-Fock[3,4]). One of the most important IBM parameters, the number of bosons, has however escaped attention. Since the bosons are regarded as representing the active nucleon pairs[5]), the number of bosons is always counted as the number of valence particle or hole pairs, whichever is less. In some cases the assignment of a valence shell is ambiguous. An example of this can be found in the 50-82 shell for protons where the gap in the single particle energies of the $d_{5/2}$, $g_{7/2}$ and the $h_{11/2}$, $d_{3/2}$, $s_{1/2}$ orbits causes $Z=64$ to act as a shell closure for near spherical nuclei[6]), while its effect is faded out almost completely for the deformed nuclei in this region. In order to deal with these cases it is necessary to introduce the concept of an effective number of bosons (N_{eff}) which can be calculated unambiguously from a microscopic theory for the bosons. In this paper a possible procedure for calculating N_{eff} will be outlined.

In the IBM the s and d bosons are considered as representing fermion pair degrees of freedom[5]). The equivalents of the s and d bosons in a shell model picture are collective two particle $J=0$ and $J=2$ states;

$$s^\dagger \sim S^\dagger = \sum_j \alpha_j \frac{\hat{j}}{2} (a_j^\dagger a_j^\dagger)^{(0)} \tag{1}$$

$$d^\dagger \sim D^\dagger = \sum_{jj'} \beta_{jj'} \frac{1}{2}\sqrt{1+\delta_{jj'}}\, (a_j^\dagger a_{j'}^\dagger)^{(2)}, \tag{2}$$

where $a_j^\dagger$ is a fermion creation operator. Microscopic models such a given in Refs. 2, 3 and 4 provide a framework for calculating the coefficients α_j and $\beta_{jj'}$ but will not be discussed in this paper. For the following it will be instructive to consider two extreme cases more closely: (A) the coefficients α_j are all equal, and (b) one is much larger than the other.

Case A: When all coefficients α_j in Eq. (1) are equal, for example $\alpha_j=1$, it can be shown that the $S^\dagger$ operator, together with its adjoint S and $S_0=\frac{1}{2}[S^\dagger,S]$ generate a quasi-spin algebra[7]). As a consequence the problem is completely equivalent to that of a large j shell with a pair degeneracy Ω, equal to the total of the orbits involved, $\Omega=\sum_j\Omega_j$ where $\Omega_j=(2j+1)/2$. The particles are distributed evenly over the orbits,

$$\langle S^N|\hat{n}_j|S^N\rangle = 2\Omega_j N/\Omega \tag{3}$$

where $\hat{n}_j$ is the number operator for particles in shell j. Physically this case corresponds to that of several degenerate orbits and the number of bosons is unambiguously defined as the number of particle or hole pairs, whichever is less, $N_B=\min(N,\Omega-N)$.

Case B: Let us consider the case of three orbits with pair degeneracies Ω_c, Ω_v and Ω_u, coefficients α_j such that $\alpha_c^2 \gg \alpha_v^2 \gg \alpha_u^2$, and a number of fermion pairs N such that $\Omega_c<N<\Omega_c+\Omega_v$. The fermions are in this case not evenly distributed over the different orbits, but one has instead[8]),

$$\begin{aligned}\langle S^N|\hat{n}_c|S^N\rangle &= 2\Omega_c,\\ \langle S^N|\hat{n}_v|S^N\rangle &= 2(N-\Omega_c),\\ \langle S^N|\hat{n}_u|S^N\rangle &= 0\end{aligned} \tag{4}$$

where terms proportional to α_c^2/α_v^2, α_v^2/α_u^2 or α_c^2/α_u^2 have been omitted. The orbit Ω_c is thus completely filled and can be regarded as a core orbit, orbit Ω_v is only partially occupied and plays the role of a valence shell while orbit Ω_u is empty and can be regarded as an inactive, unoccupied orbit. Also in this case the number of bosons can be defined unambiguously as $N_B=\min(M,\Omega_v-M)$ where $M=N-\Omega_c$.

In the general case the coefficients α_j are not all equal nor are they vastly different. It is thus not possible to make a direct separation of the orbits involved into core and valence orbits, however for the definition of the number of bosons this is essential as was shown in the last example. There is however an indirect way which allows for such a separation. For case B it is possible to evaluate the matrix elements[8]) in leading order in the ratio's of α^2,

$$\begin{aligned}\langle S^{N-1}|SS^\dagger|S^{N-1}\rangle &= Z(\Omega_v-M+1)N^2/M\\ \langle S^{N-1}|SSS^\dagger S^\dagger|S^{N-1}\rangle &= Z^2(\Omega_v-M)(\Omega_v-M+1)N^2(N+1)^2/M(M+1)\end{aligned} \tag{5}$$

and

$$\langle S^{N-1}D|SS^\dagger|S^{N-1}D\rangle = Z(\Omega_v-M+1)N^2/M$$

where $|S^{N-1}D\rangle$ is a normalized $J=2$, $v=2$ state corresponding in the IBA model to the one d-boson state, and $M=N-\Omega_c$. The constant Z introduced in Eq. (5) plays the role of a normalization constant and is unimportant for the following discussion. Eqs. (5) are derived for the case of example B but are also valid in the more general case in which Ω_c, Ω_v and Ω_u each represent several degenerate orbits, as can be seen from example A. For the present purpose we will assume their validity even for the general case, in which the α_j coefficients cannot directly be separated into three different categories as in example B, and where they will be used to calculate the size of the effective core, Ω_c, and effective valence shell, Ω_v. In a microscopic model[2-4]), the matrix elements on the right-hand side of Eqs. (5) can be calculated and the equations can subsequently be used to solve for the three unknowns Z, Ω_v and M. The effective number of bosons can now be defined as $N_{eff}=\mathrm{Min}(M,\Omega_v-M)$. In IBM the number of bosons is necessarily an integer, but M as calculated from Eqs. (5) of course not. The number of bosons should therefore be taken as the nearest integer to N_{eff}. To calculate N_{eff} the particular set of Eqs. (5) has been taken since these are the simplest that unambiguously define a value for N_{eff}.

As an example the effective number of bosons in the 50-82 proton shell will be calculated. The microscopic structure of the S and D pair states, Eqs. (1,2) were obtained from a shell model-like calculation in a generalized seniority[7]) basis that includes states with generalized seniority $w \leq 2$[9,10]). The Hamiltonian was taken from a shell model calculation where the parameters have been optimized so as to give a best fit to the even and odd mass $N=82$ isotones[11]). The calculation of the S and D pair structure has been done in the generalized seniority basis, rather than a normal shell model basis, since in this basis the pair structure can directly be determined. It has been verified in Ref. 9 that the calculation in the much smaller generalized seniority basis yields similar results for the lowest few states of each spin as the shell model calculation. The obtained values for α_j as appear in Eq. (1) are plotted in Fig. 1. These values already give a strong indication that for values of $Z>66$, $Z=64$ can be considered as a rather good shell closure and this will be confirmed by the calculated values of N_{eff}. Using the shell model structure of the S- and D-pair state, the right-hand side of Eqs. (5) can be calculated numerically and the Eqs. (5) can be solved for Ω_v and M. The thus obtained values for $\Omega_c=N-M$, Ω_v and $N_{eff}=\mathrm{Min}(M,\Omega_v-M)$ are plotted in Fig. 2, together with the values one would obtain if $Z=64$ was taken as a full shell closure. Clearly the $Z=64$ shell closure has an important influence, without it Ω_c would be zero, $\Omega_v=16$ and $N_{eff}=8$ for $Z=66$. The $Z=64$ subshell closure can however not be regarded as a real shell closure, at $Z=64$ the effective number of bosons is 2.5 rather than zero.

As a second example the effective number of proton bosons is calculated for the Samarium isotopes. The structure of the pair states has been calculated in Ref. 4. In this calculation the influence of

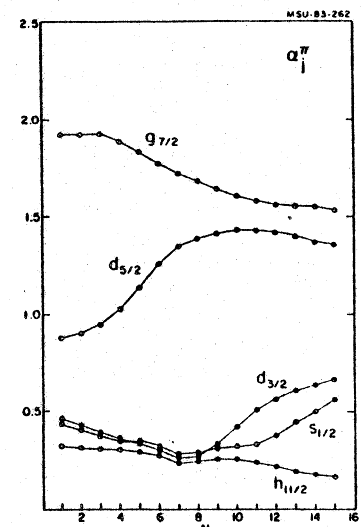

Fig. 1. The coefficients α_j as a function of the number of protons in the 50-82 major shell.

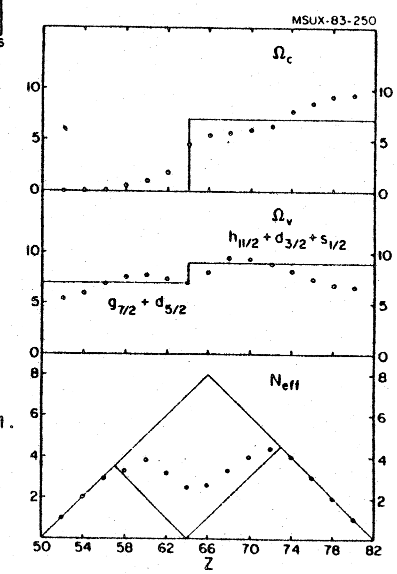

Fig. 2. The calculated values for Ω_c, Ω_v and N_{eff}, as discussed in the text, as a function of the number of protons in the 50-82 major shell.

the neutron-proton interaction on the structure of the bosons has been taken into consideration in a deformed HFB basis, which results in a dependence of the structure of the proton S and D pair state on the number of neutrons. If $Z = 64$ is taken as a shell closure there is only one proton boson in Sm, if $Z = 64$ would have no effect the number of bosons is 6. From the calculation for the $N = 82$ isotones one obtains $\Omega_v = 7.4$ and $N^{\pi}_{eff} = 3.2$ for ^{144}Sm, which is about halfway in between. As can be seen from Table I, the effective number of proton bosons is strongly influenced by the neutron proton interaction. In the deformed region (N>90), $N^{\pi}_{eff} \approx 5.0$, which means that the neutron proton interaction effectively has washed out the effects of the gap in the single particle energies at $Z = 64$.

Table I - The values obtained for Ω^{π}_{v} and N^{π}_{eff} for the Samarium ($Z = 62$) isotopes as a function of the number of neutrons. The structure of the S_{π} and D_{π} fermion pair states was taken from Ref. 4.

N	Ω^{π}_{v}	N^{π}_{eff}
86	9.0	3.6
88	9.7	4.2
90	10.0	4.5
92	10.3	4.8
94	10.6	5.0
96	10.8	5.2
98	11.0	5.3

In this paper the concept of an effective number of bosons is introduced and a procedure is proposed to calculate it. In Ref. 12 an effective pair degeneracy Ω_{eff} was introduced which interpretation is equivalent to Ω_v as introduced in this paper. Although the method for calculating Ω_v is completely different from Ref. 12, the calculated values are approximately the same.

In the 50-82 major shell for protons the effective number of bosons is considerably different from the values that would have been obtained if the shell is taken as degenerate. On the other hand the effective number of bosons also significantly deviates from the estimate obtained if $Z = 64$ is considered to be a real shell closure as was suggested by Wolf et al.[13]) on the basis of a calculation of g-factors. The effects of the $Z = 64$ subshell closure are substantially washed out when one is going away from the $N = 82$ closed shell due to the strong neutron-proton interaction. The fact that it is important in the IBA model to include the proper number of bosons, even for the calculation of low-lying states, is clearly demonstrated in Ref. 14.

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