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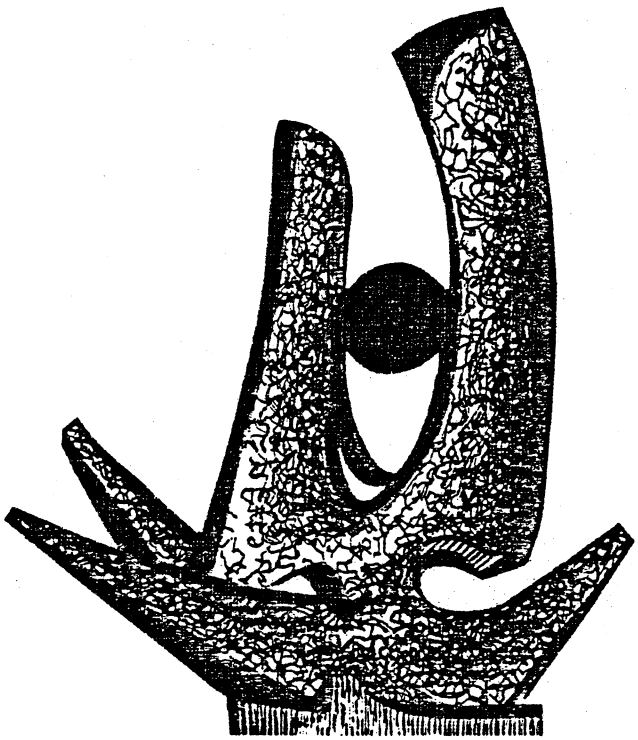

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ABSTRACT

From a comparison of time scales for establishing a mechanical instability and a liquid-gas instability in nuclear systems we find that for temperatures in excess of ~8 MeV, the liquid-gas instability may manifest itself. We consider also the effects of collision damping on the mechanical instability.

1. Introduction

The availability of heavy ion beams over a wide range of energies makes it possible to create nuclear systems with high temperatures and greater than normal nuclear densities in a laboratory environment. Such conditions may lead to phase transitions in nuclear matter conjectured to occur at greater than normal nuclear densities. It is also possible that during the process of rarefaction, following the initial compression, the system may enter a state of lower than normal nuclear density , where the conditions of temperature and density necessary for a liquid-gas coexistence may develop[1-3]. If heavy ions are to be adequate probes of these states, two conditions must be satisfied: the thermalization process and the time necessary for this state to develop must be less than or of the order of the disassembly time. Firstly, we assume that the system has a sufficiently short mean free path to allow the system to equilibrate in a time commensurate with the collision time of the heavy ions. Secondly, any phase transition, for example a liquid-gas phase instability, must establish itself prior to disassembly if it is to have a discernible experimental consequence. It is important therefor to estimate the time scale for the system to disassemble and compare with the time scale necessary for the establishment of chemical equilibrium.

2. Time Scales Neglecting Damping

To estimate the time scale for disassembly we consider a 'mechanical' instability resulting in a breakup of the hot zone[4]. The effects of damping are neglected initially, however these effects will be taken into account in the following section. The underlying argument begins by evaluating the excitation energy per nucleon as a function of the density. Firstly, the chemical potential must be derived from the density,

$$\rho = \frac{4}{(2\pi)^3} \int d^3k \left[1 + \exp\left\{ \left(\frac{k^2}{2m} + E_v - \mu\right)/T\right\}\right]^{-1} \qquad (1)$$

where the potential energy is obtained from the Skyrme interaction (ignoring effective mass corrections)[5]

$$\frac{E_v}{A} = - A \left(\frac{\rho}{\rho_o}\right) + B \left(\frac{\rho}{\rho_o}\right)^{5/3} \qquad (2)$$

The constants A and B can be determined from nuclear matter properties in the following manner. The total energy is the sum of the thermal and potential contributions ($E = E_T + E_v$), which for T=Ø has the form

$$\frac{E}{A} = \frac{3}{5} \varepsilon_F \left(\frac{\rho}{\rho_o}\right)^{2/3} - A \left(\frac{\rho}{\rho_o}\right) + B \left(\frac{\rho}{\rho_o}\right)^{5/3} \qquad (3)$$

where ε_F is the Fermi energy of normal nuclear matter ($\tilde{=}$ 38

MeV). Nuclear matter considerations require E=-16 MeV and $\partial(E/A)/\partial\rho=0$ at $\rho = \rho_o$ and T=Ø. The values thus obtained are A = 74.2 MeV and B = 35.4 MeV. The thermal contribution to the energy can be calculated using

$$\frac{E_T}{V} = \frac{4}{(2\pi)^3} \int d^3k \, \frac{k^2}{2m} \left[1 + \exp\left\{ \left(\frac{k^2}{2m} + E_v - \mu\right)/T\right\}\right]^{-1} \quad (4)$$

Utilizing thermodynamics one can calculate the entropy and pressure. The results are presented in FIG.1, where several isentropes are plotted as a function of the density. From the initial excited compressed state formed in a heavy ion collision the system will oscillate along an isentrope in a harmonic fashion similar to a monopole oscillation in the absence of collision damping. If, during the oscillation, the pressure becomes positive then the system will disassociate. In practice, the incident projectiles (protons or heavy ions) are expected to create conditions of normal or greater than normal nuclear densities respectively. It is therefore necessary to define an overstressed region such that under harmonic oscillation the system will enter the region where the pressure is positive on the return portion of the oscillation. The unstable (P>Ø) region defines the conditions of temperature and density for which the system will disassemble and is depicted in Fig. 1. The boundary for this region is not altered by damping considerations. In the absence of damping (Γ=Ø), the boundary of the overstressed region is as indicated in Fig. 1. To estimate the time

required for this type of disassembly to occur we make use of the known energy of the monopole oscillations in medium mass nuclei[6]:

$$E_{monopole} \sim \hbar \omega_o \sim 15 MeV \qquad (5)$$

resulting in

$$\tau_{osc} = \frac{\pi}{\omega_o} \sim 1.4 \times 10^{-22} sec. \qquad (6)$$

Estimates of the disassembly time derived from high energy cascade models give values of the order ~50 fm/c (1.7×10^{-22} sec.), consistent with the estimate from the monopole oscillation[7].

If a "chemical" instability, such as a liquid-gas phase instability, is to exist there must be sufficient time for equilibration to be established across the phase boundary. The time required for this equilibration to occur is of the same order as the evaporation time. To estimate the evaporation time we use the expression characteristic of thermionic emission[8]. Thus, the current density can be expressed as

$$J = \frac{em}{2\pi^2 \hbar^3} T^2 (1-r) e^{-W/T} \qquad (7)$$

where r is the quantum mechanical reflection coefficient (taken as 0), W is the work function (taken as 8 MeV) and T is the temperature. By definition

$$J = \frac{\Delta q}{\Delta t}\,\frac{1}{A} \tag{8}$$

where A is the surface area of the emitting source. If we set $q = e$ (equivalent to the emission of one nucleon) then $\Delta t = \tau_{evap}$. Assuming a spherical geometry, so that $A=4\pi R^2$ where $R \sim 3.5$ fm. as determined from the participant-spectator model for intermediate impact parameters[9] and consistent with determinations from pion interferometry measurements[10], the evaporation time is found to be

$$\tau_{evap} \simeq 3.5 \times 10^{-21} \cdot \frac{1}{T^2} \cdot e^{8/T} \tag{9}$$

The resulting values are given in Table 1 and are in good agreement with results deduced from an empirical fit to the measured widths of compound nuclei for A=20-100[11]. Comparing the evaporation time with the time required for disassembly it appears that for $T \gtrsim 8.1$ MeV (henceforth referred to as the breakeven temperature) the liquid-gas phase instability may develop.

Since a liquid-gas phase instability exists only for temperatures below the critical temperature and above the breakeven temperature, it is likely that if the breakeven temperature were higher than the critical temperature, the liquid-gas instability would never develop. The critical temperature of 20 MeV, predicted in reference 1 assumed a binding energy per nucleon in nuclear matter of 16 MeV/u compared to the phenomenological binding energy per nucleon

of 8 MeV for finite nuclei. A more thorough treatment of this question and effective mass considerations is given in reference 2, where it is shown that in finite nuclei the predicted critical temperature lies between 13.4 MeV and 8.1 MeV depending on the choice of effective mass. Thus for temperatures above 8 MeV the liquid- gas instability may develop and there is sufficient time for it to do so.

3. Damping Effects

Collision damping has been neglected throughout this discussion. A simple approach to the problem begins with the equation for a damped non-driven oscillator,

$$\ddot{x} + \gamma \dot{x} + \omega_o^2 x = 0 \qquad (10)$$

$$x = \rho - \rho_{min}(S) \qquad (11)$$

$$\Gamma = \frac{\gamma}{\sqrt{\omega_o^2 - \frac{1}{4}\gamma^2}} \qquad (12)$$

where γ is the damping coefficient (assumed constant) and ω_o is the undamped harmonic oscillator frequency. The dimensionless damping coefficient is introduced in equation (12) and will be deduced from experimental measurements of the monopole oscillation characteristics. The variable ρ_{min} is the value of the density for which the excitation energy is a minimum for a given value of the entropy. The damping

constant determines the rate at which energy is transferred to thermal energy from the collective motion thus determining the temperature . For a given value of the density, the entropy can be calculated thus fixing the value of ρ_{min}. The Skyrme interaction does not produce a true harmonic oscillation in the density coordinate but for reasonable excursions from the equilibrium density we shall use this approximation. A solution to such an equation is of the form

$$x \sim Ae^{-\gamma t/2}\cos(\omega t) \tag{13}$$

where

$$\omega^2 = \omega_o^2 - \frac{1}{4}\gamma^2 \tag{14}$$

The time required for disassembly is then

$$\tau \simeq \frac{\pi}{\sqrt{\omega_o^2 - \frac{1}{4}\gamma^2}} \tag{15}$$

The empirical full width at half maximum (FWHM) of the monopole excitation is typically $\simeq 4$ MeV for an excitation energy of 15 MeV[6]. Table 2 illustrates how values of the damping constant influence the break-even temperature for the onset of the liquid-gas instability. In Fig. 1 the overstressed region is redefined assuming that the damping constant remains fixed at $\Gamma = 0.27$ (indicative of $T=0$ damping) and is not a function of temperature. One important point to

note is that the minimum temperature required for the system to reach the unstable region can be determined from Fig. 1, noting the minimum excitation energy of the overstressed region which occurs at the intersection of the overstressed region boundary and the S/A=Ø isentrope. Since energy is conserved, one can follow the line of constant energy to the boundary of the unstable region and then calculate the corresponding temperature from the corresponding density and energy. The resulting minimum observable temperature for a disassembly of this nature is 5.5 MeV. This is the minimum amount of energy transferred from the collective motion to the random thermal motion consistent with the damping deduced from the monopole oscillation width. This damping process also generates entropy. The minimum observable entropy can be calculated in a fashion similar to the minimum temperature and the final entropy at the unstable region boundary is Ø.9. Hence a mechanical instability generates a minimum temperature of 5.5 MeV and minimum entropy of Ø.9 prior to disassembly. The minimum excitation energy of the overstressed region is 4.75 MeV above the binding energy of normal nuclear matter (E = -16 MeV) and $\rho \sim 1.5$ times normal nuclear density. The minimum incident energy required for an equal mass projectile and target system is 4 times the excitation energy, i.e. 19 MeV per nucleon. If 19 MeV per nucleon incident energy is insufficent to generate a system of 1.5 times normal nuclear density then the experimental minimum will be even larger. Initial entropies of

greater than 2.0 appear to be affected much less than lower values of the initial entropy; hence this consideration does not contradict the cascade model calculations performed at higher energies which predict that entropy remains essentially constant during the expansion process[12].

The damping factor results in an enhancement of the time required for the system to reach the breakup region. The enhancement factor is

$$\Delta\tau \equiv \tau - \tau(\Gamma = 0) = \left(\sqrt{1 + \frac{1}{4}\Gamma^2} - 1\right) \tau\,(\Gamma = 0) \qquad (16)$$

resulting in an enhancement of 1-2%. Since Pauli blocking decreases with increasing temperature, the damping constant should also increase, further increasing the time available for chemical equilibration.

4. Conclusion

In summary, for temperatures greater than 8 MeV and less than the critical temperature a liquid-gas phase instability may manifest itself. If damping, as deduced from the monopole excitation, is taken into account, it appears that the oscillation time is increased by 1-2%. However, with increasing temperature, the damping constant becomes larger, and the available time for chemical equilibration will be greater.

The damping also modifies the conditions for a mechanical instability particularly at low temperatures and low entropies. The redefinition of the overstressed boundary necessitates a corresponding increase in the incident energy of the projectile to permit the system to enter the unstable region illustrated in Fig. 1. For equal mass projectile and target, the minimum incident energy necessary is 19 MeV per nucleon in the lab frame which corresponds to an excitation energy of 4.75 MeV per nucleon. Injecting 4.75 MeV per nucleon of excitation energy with a compression of 1.5 ρ_o, corresponding to S=Ø and T=Ø, will just place the system within the overstressed region defined for minimal damping, i.e. Γ=Ø.27. Subsequently, the system expands until it just reaches the unstable boundary where it has a temperature of 5.5 MeV and an entropy of Ø.9, whereupon the system disassembles since the pressure is positive. Consequently, observation of a mechanical instability, unperturbed by liquid-gas considerations, should be confined to temperatures greater than 5.5 MeV and less than ~8 MeV.

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REFERENCES

1) M.W. Curtin, H.Toki and D.K.Scott, Phys. Lett. 123B (1983) 289

2) H.Jaqaman, A.Mekjain and L.Zamick, Rutgers University Preprint RU-83-61 (1983)

3) P.J.Siemens, to be published

4) G.F.Bertsch, Proc. Int. Conf. on Nucleus-Nucleus Collisions, ed. G.F. Bertsch, C.K. Gelbke and D.K. Scott, (East Lansing, 1982) Nucl. Phys. A 400 (1983), 221 C; G.F.Bertsch and P.J.Siemens, to be published

5) L. Zamick, Phys. Lett. 45B, (1973) 313

6) D.H.Youngblood et al,Phys. Rev. C23. (1981) 1997

7) J. Cugnon, T. Mizutani and J. Vandermeulen, Nucl. Phys. A352 (1981) 505; H. Kruse, private communication

8) J.S.Blakemore, in Solid State Physics (W.B.Saunders, Philadelphia, 1974) p.190

9) D.K.Scott, in Dynamics of Heavy Ion Collisions, ed. N.Cindro, R.A.Ricci and W.Greiner (North-Holland, Amsterdam, 1981) p.241

10) S. Nagamiya et al., Phys. Rev. C24 (1981) 971.

11) H.Bohning, Proc. Int. Conf. on Nucl. React. Induced by Heavy Ions, 1970, ed. R.Bock and W.R.Hering (North-Holland, Amsterdam, 1970) p.633

12) G.F.Bertsch and J.Cugnon, Phys. Rev. C24 (1981) 2514

TABLE I

Nucleon evaporation times as a function of temperature

T (MeV)	5	10	15	20
t (10^{-22} sec)	6.9	0.77	0.27	0.13

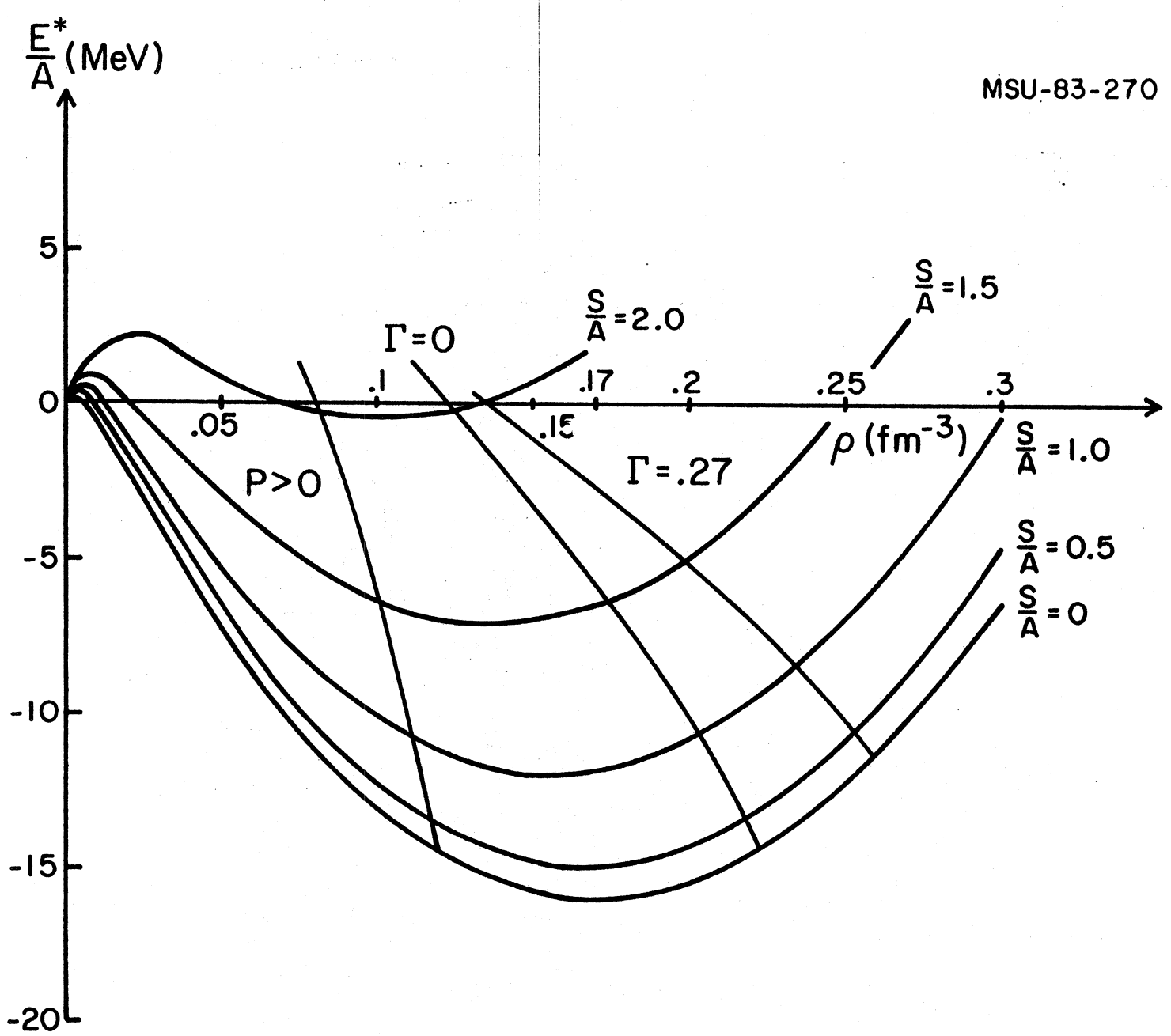
TABLE 2

Breakeven temperatures for various values of the dimensionless damping constant

$\Gamma / \Gamma (T \simeq \emptyset)$	0	5	10	20
$T_{break\text{-}even}$ (MeV)	8.1	7.6	6.9	5.8

FIGURE CAPTIONS

Fig. 1. The excitation energy per nucleon versus density for various values of the entropy per nucleon. The unstable region ($P>0$) is indicated. Two boundaries for the overstressed region are drawn: no damping (labelled $\Gamma=0$) and minimal damping ($\Gamma=0.27$).


E*/A (MeV)
MSU-83-270
5
0
-5
-10
-15
-20
.05
.1
.15
.17
.2
.25
.3
ρ (fm⁻³)
Γ=0
Γ=.27
P>0
S/A=2.0
S/A=1.5
S/A=1.0
S/A=0.5
S/A=0