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Boltzmann Equation for Heavy Ion Collisions

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Abstract

The sensitivity of inclusive observables in heavy ion collisions to the nuclear equation of state can be tested with the Boltzmann equation. We solve the Boltzmann equation, including mean field and Pauli blocking effects, by a method that follows closely the cascade model. We find that the inclusive pion production is insensitive to the nuclear equation of state, contrary to recent claims.

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A recent article by Stock, et al.,[1] suggests that the nuclear equation of state might be measured by the pion production cross section in heavy ion collisions. Interpretation of the pion yield data is based on comparison with cascade models of heavy ion collisions; a disagreement between the cascade prediction and the measured pion yield is taken as evidence for collective effects. Clearly, there is a need to develop a calculable model that goes beyond the cascade assumptions, and includes collective field effects and essential quantum effects.

An obvious candidate for a better theory is that based on the Boltzmann equation with a self-consistent potential field, and with a collision integral that respects the Pauli principle. The equation to be solved for the single-particle distribution function f(p,r) is

$$\frac{\partial f}{\partial t} + v\cdot\nabla_r f - \nabla U\cdot\nabla_p f = -\int \frac{d^3p_2 d^3p_1' d^3p_2'}{(2\pi)^9}\, \sigma v_{12}[f f_2(1-f_{1'})(1-f_{2'})$$

$$- f_{1'} f_{2'}(1-f)(1-f_2)](2\pi)^3 \delta^3(p+p_2-p_1'+p_2') \qquad (1)$$

The left-hand side of the equation, set equal to zero, is the Vlasov equation. The collision integral on the right-hand side depends on the nucleon-nucleon cross section, σ. The right-hand side differs from the classical collision integral[2] by the Pauli blocking factor (1-f)(1-f). A theory based on Eq. (1) has much to recommend it, as it exhibits proper behavior under a broad range of limiting conditions. When the collision term dominates, hydrodynamics becomes valid. Without the collision term, the theory is classical time-dependent Hartree theory, which reproduces quite well the behavior of the quantum theory.[3,4] The mean field U can be an arbitrary function of density, making it possible to model a variety of equations of state. A theory of heavy ion collisions should give correct physics for noninteracting nuclei, namely vanishing time-dependence to the internal properties. This is achieved easily with Eq. (1), putting a constraint on the function U. In contrast, there is no consistent way in the cascade description to treat Fermi motion.

Although the cascade model is inadequate as a complete theory, it provides a framework for solving the Boltzmann equation. The distribution function f(p,r) is represented by test particles, i.e. the density of the test particles is the measure of f, as in Ref. 4. Then the Vlasov equation is solved by propagating the particles according the Newtonian mechanics,

$$\dot{p} = -\nabla U \quad (2a)$$

$$\dot{x} = v \quad (2b)$$

If the number of test particles were equal to the number of cascade particles, the collision integral on the right-hand side of Eq. (1) would be given by the cascade collisions of the test particles, except for the Pauli-blocking factor. This difference is implemented by randomly permitting or blocking the

particle-particle collisions called for in the cascade according to the probability $(1-f_{1'})(1-f_{2'})$.

An individual heavy ion collision, simulated by the cascade, has too few particles to usefully determine either the mean field or the Pauli blocking probability. We shall use an ensemble of cascade simulations to build up an adequate number of test particles. Our cascade program evolves the collision by fixed time steps, so there is no difficulty in generating ensembles at definite times. In practice, cascade comparisons use ensembles of simulations anyway to improve the statistics, so the major change is in the ordering of the loops in the program.

We now address the question of how accurately U and f need to be calculated. Since we are testing inclusive effects of changes in the equation of state, a rather coarse mesh in U should suffice. We determine U using cubes of 2 fm on a side. The number of particles in each cube is used to determine the density and U for the entire cube. Statistical fluctuations in the particle number cause fluctuations in U, which produces a systematic error because of the nonlinearity in the $U(\rho)$ relationship. The results we present are based on ensembles of 10 simulations. At normal density, there is an average of 13 particles in a test cube, for a fluctuation of 30%. For our parametrization of U, Eq. (5) below, this produces less than a 20 MeV systematic bias in U over the densities of interest.

The occupation factor f cannot be calculated in the same way, because a six-dimensional space divided into fixed cubes would require too much storage. Instead, we examine the neighborhood of the final state phase space whenever the cascade simulation calls for a two-particle collision. The phase space density in the vicinity of one of the final state particles is calculated as

$$f_{num}(p,r) = \frac{\sum_{i}^{N} n_i}{N\Omega(p,r)} \qquad (3)$$

Here (p,r) are the phase space coordinates of the particle, $\Omega(p,r)$ is a volume of phase space centered on (p,r), n_i is the number of particles in that volume in ith simulation, and N is the total number of simulations. The volume is chosen as a cube in momentum and coordinate spaces, which allows a rapid computation of Eq. (3). The particle collision is blocked with a probability P given by

$$P = 1-\max[0,1-f_{num}(1)]\cdot\max[0,1-f_{num}(2)] \qquad (4)$$

This procedure has a systematic error arising because f_{num} can exceed one but the blocking factor cannot. Some feeling for the importance of this can be obtained by examining the average blocking factor when f=1 for the distribution function. We choose as our test volume cubes of sides $p=\sim 1\ fm^{-1}$ and $\Delta r \tilde{=} 3$ fm, containing about four particles in the ensemble sum. The average blocking factor arising from the Poisson statistics

associated with four particles is P=0.96, which is adequate for our purposes.

The potential function U will be chosen with the Skyrme parametrization,

$$\text{stiff:} \quad U(\rho) = -124\ \rho/\rho_0 + 70.5(\rho/\rho_0)^2\ \text{MeV} \qquad (5a)$$

$$\text{soft:} \quad U(\rho) = -356\ \rho/\rho_0 + 303(\rho/\rho_0)^{7/6}\ \text{MeV} \qquad (5b)$$

Both these potentials produce proper saturation of nuclear matter, and both potentials are about 50 MeV deep at ordinary density. The stiff potential produces a compressibility coefficient K=375 MeV while the soft potential has a more realistic compressibility coefficient, K=200 MeV.

Collisions governed by the soft equation of state should reach higher central densities, and we found this to be the case, as may be seen from Fig. 1, showing the maximum density as a function of time. The kinetic energy should also be higher, but within the limitations of our statistics the effect was not significant. Our pion production is calculated in a delta resonance approximation, so we do not want to emphasize the absolute production rates here, but merely compare the effect of various model assumptions. Table I shows the pion production calculated for two of the energies reported by Stock, et al., 360 MeV/n and 722 Mev/n. The first column shows the results with Pauli blocking of nucleons, and the second column the results with the Pauli blocking turned off. Evidently, the Pauli blocking of nucleons has a small effect on the yield. At first this seems surprising; one might expect the Pauli blocking to have a substantial effect. However, if the system reaches equilibrium the pion abundance is determined by the temperature at freezeout. At the collision energies we consider, the Fermi gas temperature is only slightly higher than the temperature of a Boltzmann gas of the same energy. The predicted yield for the soft equation of state, including blocking, is shown in the third column. There is an increased yield for the soft equation of state, but the effect is rather small. We believe that the insensitivity is a consequence of the system reaching statistical equilibrium while expanding to densities below nuclear matter density. As long as the system is in thermal equilibrium at some intermediate density, it will be difficult to see residual traces of the physics at earlier stages of the collision. This parallels the situation in the study of the early universe: it is hard to look back farther than the point of last equilibrium at the time of nucleosynthesis. Of course, the situation is more promising for the heavy ion collisions because the total entropy production depends on the equation of state. From entropy considerations we expect and find an increase in pion yield for the softer equation of state.[8] However, the effects on pion production are evidently too small to measure.

Turning to the absolute magnitudes of the yields, we see from the comparison with experiment in the last column of Table I that our calculations predict too much cross section. This contrasts with a recent study by Kitazoe, et al.,[5] who found a smaller cross section using a more ad hoc recipe for the mean

field and Pauli effects. We are presently studying the various cascade assumptions, to understand the discrepancies between various codes.[6] For example, we do not Pauli-block a nucleon associated with a pion, since it is assumed to be in a delta state. However, some of the low-energy pion production is not mediated by deltas, and in that case the exclusion should be applied to both final state baryons. We have just learned of other work[9,10] on the pion production question using different techniques than the numerical solution of the Boltzmann equation.

It appears that the present technique will be workable for solving the Boltzmann equation down to much lower energies, where presently neither the cascade nor the time-dependent Hartree theory is justified. If so, this would provide a very useful tool for analyzing intermediate energy heavy ion collisions.

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Table I. Yields of negative pions in Ar + KCl collisions

Lab energy (MeV/n)	Equation of State: Stiff	Equation of State: Stiff, no Pauli Blocking	Equation of State: Soft	Cugnon Cascade[a]	Exp[b]
360	0.68 ± .06	0.77 ± .03	0.75 ± .05	0.8	0.2
722	2.1 ± .06	2.2 ± .08	2.1 ± .08	3.3	1.6

[a]Ref. 7, quoted in Ref. 1

[b]Ref. 1

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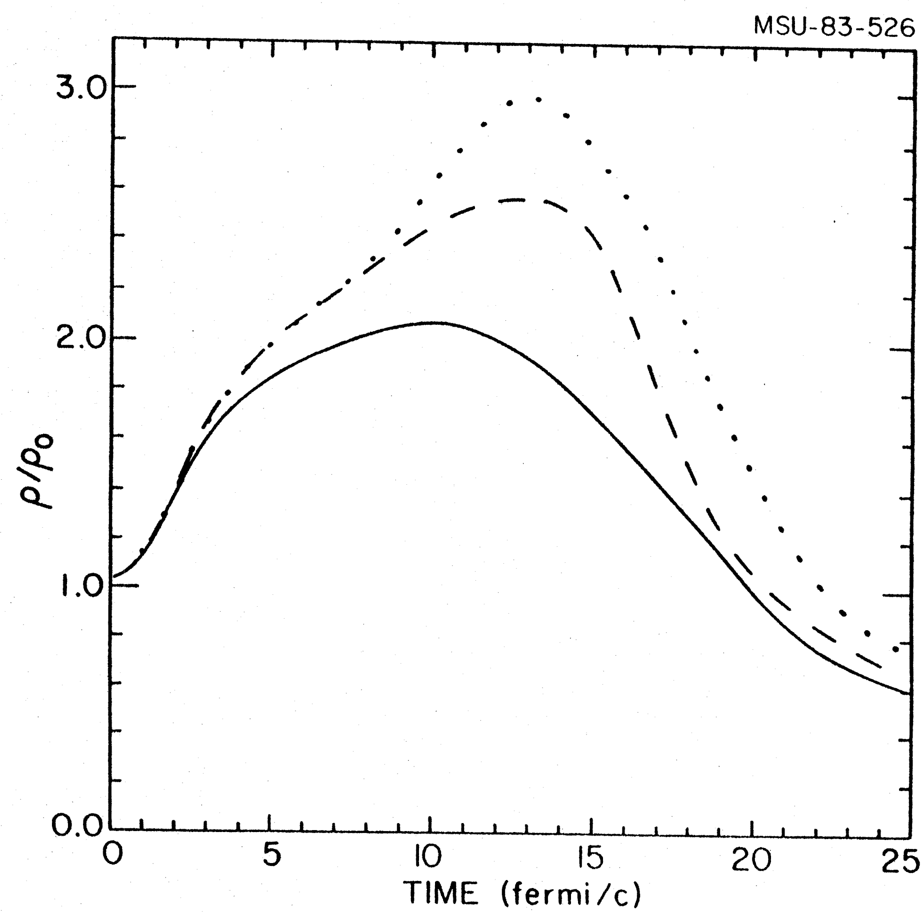

Figure 1. Maximum density for $^{40}Ca + ^{40}Ca$ collisions at $E_{lab} = 360$ MeV/n as a function of time. The solid, dashed lines show the densities for the stiff and soft equations of state. The result for a super soft equation of state, with $U(\rho > \rho_0 = U(\rho_0)$, is shown as the dotted line.